JEE Main 2020 Paper

Date : 6th September 2020

Time : 09 : 00 am - 12 : 00 pm

Subject : Maths

Q.1 The region represented by $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ is also given by the inequality:
$\{z = x + iy \in C : |z| - Re(z) \le 1\}$

(1) $y^2 \le 2\left(x + \frac{1}{2}\right)$ (2) $y^2 \le x + \frac{1}{2}$ (3) $y^2 \ge 2(x + 1)$ (4) $y^2 \ge x + 1$

Sol. **1**

$\{z = x + iy \in C : |z| - Re(z) \le 1\}$

$|z| = \sqrt{x^2 + y^2}$

$Re(z) = x$

$|z| - Re(z) \le 1$

$\Rightarrow \sqrt{x^2 + y^2} - x \le 1$

$\Rightarrow \sqrt{x^2 + y^2} \le 1 + x$

$\Rightarrow x^2 + y^2 \le 1 + x^2 + 2x$

$\Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$

Q.2 The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to:
(1) $p \wedge \sim q$ (2) $\sim p \vee \sim q$ (3) $\sim p \vee q$ (4) $\sim p \wedge \sim q$

Sol. **4**

$p \vee (\sim p \wedge q)$

$(p \vee \sim p) \wedge (p \vee q)$

$t \wedge (p \vee q)$

$p \vee q$

$\sim (p \vee (\sim p \wedge q)) = \sim (p \vee q)$

$= (\sim p) \wedge (\sim q)$

Q.3 The general solution of the differential equation $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$ is:
(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

Sol. **3**

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$

$$\sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$$

$$\frac{\sqrt{(1+x^2)}dx}{x} = -\frac{y}{\sqrt{1+y^2}}dy$$

Integrate the equation

$$\int\frac{\sqrt{1+x^2}}{x}dx = -\int\frac{y}{\sqrt{1+y^2}}dy$$

$1 + x^2 = t^2$ $\quad\quad$ $1 + y^2 = z^2$

$2xdx = 2tdt$

$dx = \frac{t}{x}dt$ $\quad\quad$ $2ydy = 2zdz$

$$\int\frac{t.tdt}{t^2-1} = -\int\frac{zdx}{z}$$

$$\int\frac{t^2-1+1}{t^2-1}dt = -z + c$$

$$\int 1dt + \int\frac{1}{t^2-1}dt = -z + c$$

$$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) = -z + c$$

$$\sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) = -\sqrt{1+y^2} + c$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\ln\left(\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}-1}\right) + c$$

Q.4 Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line:

(1) $x + 2y = 0$ $\quad$ (2) $x + 2 = 0$ $\quad$ (3) $2x + 1 = 0$ $\quad$ (4) $x + 3 = 0$

Sol. **4**

Let t_1 tangent of $y^2 = 4(x + 1)$

$L_1 : t_1y = (x + 1) + t_1^2$ ....(i)

and t_2 tangent of $y^2 = 8 (x + 2)$

$L_2 : t_2y = (x + 2) + 2 t_2^2$

$L_1 \perp L_2$

$\frac{1}{t_1}\cdot\frac{1}{t_2} = -1$

$t_1t_2 = -1$

$t_2(i) - t_1\ (ii)$

$t_1t_2y = t_2\ (x + 1) + t_2.\ t_1^2$

$t_1t_2y = t_1\ (x + 2) + 2t_2^2.\ t_1$

$-\qquad -\qquad -$

$(t_2-t_1)\ x + (t_2-2t_1) + t_2t_1(t_1-2t_2) = 0$

$(t_2 - t_1)\ x + (t_2 - 2t_1) - (t_1 - 2t_2) = 0$

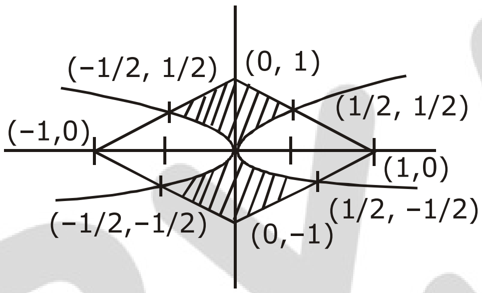
$(t_2 - t_1)\ x + 3t_2 - 3t_1 = 0$

$\Rightarrow x + 3 = 0$

Q.5 The area (in sq. units) of the region $A = \{(x, y): |x| + |y| \le 1, 2y^2 \ge |x|\}$

(1) $\frac{1}{6}$ (2) $\frac{5}{6}$ (3) $\frac{1}{3}$ (4) $\frac{7}{6}$

Sol. **2**



Total area = $4\int_0^{1/2}\left[(1-x)-\left(\sqrt{\frac{x}{2}}\right)\right]dx$

$= 4\left[x - \frac{x^2}{2} - \frac{1}{\sqrt{2}}\frac{x^{3/2}}{3/2}\Big|_0^{1/2}\right]$

$= 4\left[\frac{1}{2} - \frac{1}{8} - \frac{\sqrt{2}}{3}\left(\frac{1}{2}\right)^{3/2}\right]$

$= 4\times\frac{5}{24} = \frac{5}{6}$

Q.6 The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is:

(1) 1 (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2}$

Sol. **3**

Plane through line of intersection is

$x + y + z + 1 + \lambda(2x - y + z + 3) = 0$

It should be parallel to given line

$0(1 + 2\lambda) - 1(1 - \lambda) + 1(1 + \lambda) = 0 \Rightarrow \lambda = 0$

Plane: $x + y + z + 1 = 0$

Shortest distance of $(1, -1, 0)$ from this plane

$$= \frac{|1 - 1 + 0 + 1|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$$

Q.7 Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then:

(1) a, c, p are in G.P. (2) a, b, c, d are in G.P.

(3) a, b, c, d are in A.P. (4) a, c, p are in A.P.

Sol. **2**

$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$

$(a^2p^2 - 2abp + b^2] + [b^2p^2 - 2bcp + c^2] + [c^2p^2 - 2cdp + d^2] = 0$

$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$ap = b$ $\qquad \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$

$bp = c$

$cp = d$ $\qquad$ a, b, c, d are in G.P.

Q.8 Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

(1) 2! 3! 4! (2) $(3!)^3 \cdot (4!)$ (3) $3!\,(4!)^3$ (4) $(3!)^2 \cdot (4!)$

Sol. **2**

Total numbers in three familes = 3 + 3 + 4 = 10

so total arrangement = 10!

Family 1	Family 2	Family 3
3	3	4

Favourable cases

$$= \underset{\text{Arrangement of 3 Families}}{3!} \times \underset{\text{Interval Arrangement of families memebers}}{3! \times 3! \times 4!}$$

$= (3!)^3 \cdot (4!)$

Q.9 The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively:

(1) 6 and 8 (2) 5 and 8 (3) 5 and 7 (4) 4 and 9

Sol. **2**

x + y + z = 2
x + 2y + 3z = 5
x + 3y + λz = μ
has infinitely many solutions

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \lambda - 1 \end{vmatrix} = 0$$

$(\lambda - 1 - 4) = 0$
$\Rightarrow \lambda = 5$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & \mu - 2 \end{vmatrix} = 0$$

$(\mu - 2 - 6) = 0$
$\Rightarrow \mu = 8$
$\lambda = 5, \mu = 8$

Q.10 Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to:
(1) (−3, −1) (2) (−4, −1) (3) (1, 3) (4) (−3, 3)

Sol. **1**

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_3 \to R_3 - R_2$

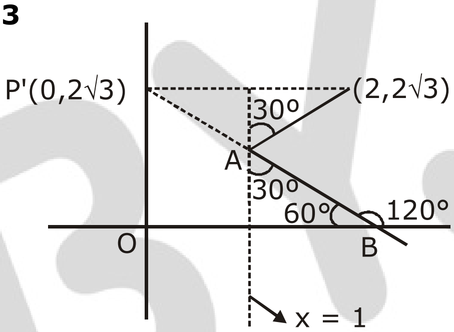
$$\begin{vmatrix} -1 & 1 & 0 \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ -1 & 0 & 1 \end{vmatrix}$$

$\Rightarrow -1(\sin^2 x) -1(1 + \cos^2 x + \sin 2x)$
$\Rightarrow -\sin^2 x - \cos^2 x - 1 - \sin 2x$
$= -2 - \sin 2x$
$\therefore$ minimum value when $\sin 2x = 1$
$m = -2-1 = -3$
$\therefore$ Maximum value when $\sin 2x = -1$
$(m, M) = (-3, -1)$

Q.11 A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30º on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $(4, -\sqrt{3})$ (2) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (3) $(3, -\sqrt{3})$ (4) $\left(4, -\frac{\sqrt{3}}{2}\right)$

Sol. **3**



Equation of line P'B passing through $(0, 2\sqrt{3})$
$(y - y_1) = m(x - x_1)$
$(y - 2\sqrt{3}) = \tan 120° (x - 0)$
$y - 2\sqrt{3} = -\sqrt{3}x$
$\sqrt{3}x + y = 2\sqrt{3}$
Check options
$(3, -\sqrt{3})$ satisfy the line

Q.12 Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:

(1) $\frac{10}{99}$ (2) $\frac{5}{33}$ (3) $\frac{15}{101}$ (4) $\frac{5}{101}$

Sol. **2**

Case-1

E, O, E, O, E, O, E, O, E, O, E

$2b = a + c \rightarrow$ Even

$\Rightarrow$ Both a and c should be either even or odd.

$$P = \frac{{}^{6}C_2 + {}^{5}C_2}{{}^{11}C_3} = \frac{5}{33}$$

Case -2

O, E, O, E, O, E, O, E, O, E, O

$$P = \frac{{}^{5}C_2 + {}^{6}C_2}{{}^{11}C_3} = \frac{5}{33}$$

Total probability = $\frac{1}{2} \times \frac{5}{33} + \frac{1}{2} \times \frac{5}{33} = \frac{5}{33}$

Q.13 If $f(x + y) = f(x)\, f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, x, y$\in$N, where N is the set of all natural number, then the value of $\frac{f(4)}{f(2)}$ is :

(1) $\frac{2}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{3}$ (4) $\frac{4}{9}$

Sol. **4**

$f(x + y) = f(x)\, f(y)$

* Put x = 1, y = 1

$f(2) = (f(1))^2$

* Put x = 2, y = 1

$f(3) = f(2).\, f(1) = f((1))^3$

* Put x = 2, y = 2

$f(4) = f((2))^2 = f((1))^4$

$f(n) = (f(1))^n$

$$\sum_{x=1}^{\infty} f(x) = f(1) + f(2) + f(3) + \ldots\ldots. = 2$$

$\Rightarrow f(1) + f((1))^2 + f((1))^3 \ldots\ldots. = 2$

$$\frac{f(1)}{1 - f(1)} = 2$$

$f(1) = 2/3$

$$f(2) = \left(\frac{2}{3}\right)^2, \; f(4) = \left(\frac{2}{3}\right)^4$$

$$\frac{f(4)}{f(2)} = \frac{(2/3)^4}{(2/3)^2} = \frac{4}{9}$$

Q.14 If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :

(1) $\frac{5}{8}$ (2) $\frac{1}{8}$ (3) $\frac{7}{8}$ (4) $\frac{3}{8}$

Sol. **2**

$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{9^{100}}{8}\right\} = \left\{\frac{(8+1)^{100}}{8}\right\}$

$\left\{\frac{{}^{100}C_0 1^{100} + {}^{100}C_1(8)1^{99} + {}^{100}C_2(8^2)1^{98} + \dots + {}^{100}C_{100}8^{100}}{8}\right\}$

$= \left\{\frac{{}^{100}C_0 1^{100} + 8k}{8}\right\}$

$= \left\{\frac{1+8k}{8}\right\} = \left\{\frac{1}{8} + k\right\} K \in I$

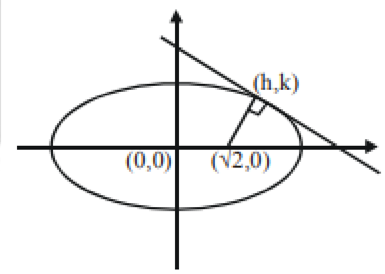
$= \frac{1}{8}$

Q.15 Which of the following points lies on the locus of the foot of perpedicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci ?

(1) $(-1, \sqrt{3})$ (2) $(-2, \sqrt{3})$ (3) $(-1, \sqrt{2})$ (4) (1, 2)

Sol. **1**

Let foot of perpendicular is (h,k)



$\frac{x^2}{4} + \frac{y^2}{2} = 1$ (Given)

$a = 2, b = \sqrt{2}, e = \sqrt{1 - \frac{2}{4}} = \frac{1}{\sqrt{2}}$

$\therefore$ Focus $(ae, 0) = (\sqrt{2}, 0)$

Equation of tangent

$y = mx + \sqrt{a^2m^2 + b^2}$

$y = mx + \sqrt{4m^2 + 2}$

Passes throguh (h,k) $(k-mh)^2 = 4m^2+2$

line perpendicular to tangent will have slope

$-\frac{1}{m}$

$$y-0=-\frac{1}{m}(x-\sqrt{2})$$

$$\mathrm{my}=-\mathrm{x}+\sqrt{2}$$

$$(\mathrm{h}+\mathrm{mk})^2=2$$

Add equaiton (1) and (2) $\mathrm{k}^2\left(1+\mathrm{m}^2\right)+\mathrm{h}^2\left(1+\mathrm{m}^2\right)=4\left(1+\mathrm{m}^2\right)$

$h^2+k^2=4$

$x^2+y^2=4$ (Auxilary circle)

$\therefore (-1,\sqrt{3})$ lies on the locus.

Q.16 $\lim\limits_{x\to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

(1) is equal to 1 (2) is equal to $\frac{1}{2}$ (3) does not xist (4) is equal to $-\frac{1}{2}$

Sol **Bouns**

$\lim\limits_{x\to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

Using L-Hopital rule

$$=\lim_{x\to 1}\frac{2(x-1)\cdot(x-1)^2\cos(x-1)^4-0}{(x-1)\cdot\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{2(x-1)^3\cdot\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$=\frac{0}{1+1}=0$$

Q.17 If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n, a > 1) then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is :

(1) $n\sqrt{a-1}$ (2) $\sqrt{na-1}$ (3) a – 1 (4) $\sqrt{a-1}$

Sol. **4**

S.D. = $\sqrt{\frac{\Sigma(x_i - a)^2}{n} - \left(\frac{\Sigma(x_i - a)}{n}\right)^2}$

= $\sqrt{\left(\frac{na}{n}\right) - \left(\frac{n}{n}\right)^2}$ = $\sqrt{a-1}$

Q.18 If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is :

(1) 1 (2) 3 (3) 2 (4) 4

Sol. **3**

$x^2 - 64x + 256 = 0$

$\alpha + \beta = 64$

$\alpha\beta = 256$

$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$

$= \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = \frac{64}{32} = 2$

Q.19 The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

(1) $(t_1 + t_2)/2$ (2) $2a(t_1 + t_2) + b$ (3) $(t_2 - t_1)/2$ (4) $a(t_2 - t_1) + b$

Sol. **1**

$f'(t) = V_{av} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$

$= \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$

$= a(t_1 + t_2) + b = 2at + b$

$t = \frac{t_1 + t_2}{2}$

Q.20 If $I_1 = \int_0^1 (1 - x^{50})^{100}\,dx$ and $I_2 = \int_0^1 (1 - x^{50})^{101}\,dx$ such that $I_2 = \alpha I_1$ then α equals to :

(1) $\frac{5050}{5049}$ (2) $\frac{5050}{5051}$ (3) $\frac{5051}{5050}$ (4) $\frac{5049}{5050}$

Sol. **2**

$I_1 = \int_0^1 (1 - x^{50})^{100}\, dx$

$I_2 = \int_0^1 (1 - x^{50})(1 - x^{50})^{100} dx$

$= \int_0^1 (1 - x^{50})^{100}\, dx - \int_0^1 x^{50}(1 - x^{50})^{100}\, dx$

$I_2 = I_1 - \int_0^1 \underbrace{x}_{I} \cdot \underbrace{x^{49}(1 - x^{50})^{100}}_{II}\, dx$

By using by parts

$1 - x^{50} = t$

$\Rightarrow x^{49} dx = \frac{-dt}{50}$

$I_2 = I_1 - \left[x\left(\frac{-1}{50}\right)\frac{(1 - x^{50})^{101}}{101}\right]_0^1 + \int_0^1 \left(\frac{-1}{50}\right)\frac{(1 - x^{50})^{101}}{101} dx$

$I_2 = I_1 - 0 + \frac{\int_0^1 (1 - x^{50})^{101}}{(-5050)} dx$

$I_2 = I_1 - \frac{I_2}{5050}$

$\frac{5051}{5050} I_2 = I_1$

$I_2 = \frac{5050}{5051} I_1$

$\alpha = \frac{5050}{5051}$

Q.21 If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\,|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is______.

Sol. **4**

$\sqrt{3}\,|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$

$= \sqrt{3}(\sqrt{2 + 2\cos\theta}) + \sqrt{2 - 2\cos\theta}$

$= \sqrt{6}(\sqrt{1 + \cos\theta}) + \sqrt{2}(\sqrt{1 - \cos\theta})$

$\sqrt{6}\left(\sqrt{1 + 2\cos^2\frac{\theta}{2} - 1}\right) + \sqrt{2}\left(\sqrt{1 - 1 + 2\sin^2\frac{\theta}{2}}\right)$

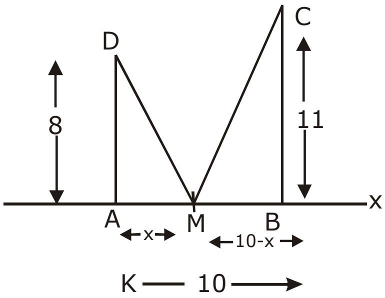
$\sqrt{6}\left(\sqrt{2\cos^2\frac{\theta}{2}}\right) + \sqrt{2}\left(\sqrt{2\sin^2\frac{\theta}{2}}\right)$

{ greatest value of trigonometric function a $\cos\theta$ + b $\sin\theta \le \sqrt{a^2 + b^2}$ }

$= 2\sqrt{3}\left|\cos\frac{\theta}{2}\right| + 2\left|\sin\frac{\theta}{2}\right| \le \sqrt{(2\sqrt{3})^2 + (2)^2} = 4$

Q.22 Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is ______.

Sol. **5**



$(MD)^2 = x^2 + 8^2 = x^2 + 64$

$(MC)^2 = (10-x)^2 + (11)^2 = (x-10)^2 + 121$

$f(x) = (MD)^2 + (MC)^2 = x^2 + 64 + (x-10)^2 + (121)$

Differentiate

$f'(x) = 0$

$2x + 2(x-10) = 0$

$4x = 20 \Rightarrow x = 5$

$f''(x) = 4 > 0$

at x = 5 point of minima

Q.23 Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is _______.

Sol. **5**

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x, & x < 0 \\ 0, & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x, & x > 0 \end{cases}$$

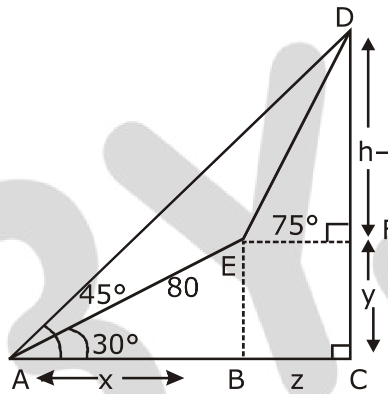
$$f''(x) = \begin{cases} 20x^3 \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) - 3x^2 \cos\left(\frac{1}{x}\right) - x \sin\left(\frac{1}{x}\right) + 10, & x < 0 \\ 0, & x = 0 \\ 20x^3 \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) + 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) + 2\lambda, & x > 0 \end{cases}$$

if f'' (0) exists then
$f''(0^+) = f''(0^-)$
$2\lambda = 10 \Rightarrow \lambda = 5$

Q.24 The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is ______.

Sol. **80**

x =



$x = 80 \cos 30° = 40\sqrt{3}$
$y = 80 \sin 30° = 40$
In ΔADC

$\tan 45° = \frac{h}{x+z} \Rightarrow h = x + z$

$\Rightarrow h = 40\sqrt{3} + z$ ....(i)
In ΔEDF

$\tan 75° \; \frac{h-y}{z}$

$2 + \sqrt{3} = \frac{h-40}{z} \Rightarrow z = \frac{h-40}{2+\sqrt{3}}$ .....(ii)
Put the value of z from (i)

$h - 40\sqrt{3} = \frac{h-40}{2+\sqrt{3}}$

$h(1+\sqrt{3}) = 40(2\sqrt{3}+3-1)$

$h(1+\sqrt{3}) = 80(1+\sqrt{3})$

$h = 80$

Q.25 Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _______.

Sol. **28**

A & B are two sets having m, n element respectively

No. of subsets of A = 2^m

No. of subsets of B = 2^n

$2^m = 2^n + 112$

$2^m - 2^n = 112$

$2^n(2^{m-n}-1) = 112$

$2^n(2^{m-n}-1) = 2^4(2^3-1)$

$n = 4$ $\quad\quad$ $m - n = 3$

$m - 4 = 3 \Rightarrow m = 7$

$m.\ n = 28$