

JEE Main 2020 Paper



Date : 6th September 2020

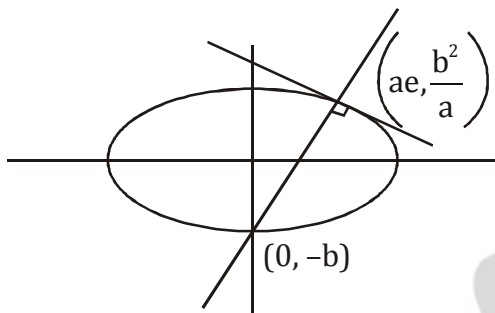
Time : 02 : 00 pm - 05 : 00 pm

Subject : Maths

Q.1 If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies:

- (1) $e^4 + 2e^2 - 1 = 0$ (2) $e^2 + 2e - 1 = 0$ (3) $e^4 + e^2 - 1 = 0$ (4) $e^2 + e - 1 = 0$

Sol. (3)



Equation of normal at $\left(ae, \frac{b^2}{a}\right)$

$$\frac{a^2x}{ae} - \frac{b^2y}{\frac{b^2}{a}} = a^2e^2$$

$$\frac{ax}{e} - ay = a^2e^2 \Rightarrow \frac{x}{e} - y = ae^2$$

It passes through $(0, -b)$

$$-b = ae^2 \Rightarrow b^2 = a^2e^4$$

$$a^2(1 - e^2) = a^2e^4$$

$$\Rightarrow e^4 + e^2 - 1 = 0$$

Q.2 The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x) \cdot (\lambda + \sin x)$,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is:

- (1) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

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Sol.

(1)

$$f(x) = (1 - \cos^2 x)(\lambda + \sin x)$$

$$f(x) = \sin^2 x (\lambda + \sin x)$$

$$f'(x) = 2\sin x \cos x (\lambda + \sin x) + \sin^2 x (\cos x)$$

$$= \sin 2x \left(\lambda + \sin x + \frac{\sin x}{2} \right)$$

$$= \sin 2x (2\lambda + 3\sin x)$$

$$\text{For extreme value } f'(x) = 0$$

$$\sin 2x = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0 \rightarrow \text{One point}$$

$$2\lambda + 3\sin x = 0$$

$$\Rightarrow \sin x = \frac{-2\lambda}{3}$$

$$\sin x \in (-1, 1) - \{0\}$$

$$-1 < \frac{-2\lambda}{3} < 1 \Rightarrow \frac{-3}{2} < \lambda < \frac{3}{2}$$

$$\lambda \in \left(\frac{-3}{2}, \frac{3}{2} \right) - \{0\}$$

Q.3

The probabilities of three events A, B and C are given by $P(A)=0.6$, $P(B)=0.4$ and $P(C)=0.5$. If $P(A \cup B)=0.8$, $P(A \cap C)=0.3$, $P(A \cap B \cap C)=0.2$, $P(B \cap C)=\beta$ and $P(A \cup B \cup C)=\alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval:

(1) $[0.36, 0.40]$ (2) $[0.25, 0.35]$ (3) $[0.35, 0.36]$ (4) $[0.20, 0.25]$

Sol.

(2)

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\alpha = 0.6 + 0.4 + 0.5 - P(A \cap B) - \beta - 0.3 + 0.2$$

$$\alpha = 1.4 - P(A \cap B) - \beta \Rightarrow \alpha + \beta = 1.4 - P(A \cap B) \quad \dots\dots\dots(1)$$

again

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.6 + 0.4 - P(A \cap B)$$

$$P(A \cap B) = 0.2 \quad \dots\dots\dots(2)$$

Put the value $P(A \cap B)$ in equation (1)

$$\alpha + \beta = 1.2$$

$$\alpha = 1.2 - \beta$$

$$0.85 \leq \alpha \leq 0.95 \Rightarrow 0.85 \leq 1.2 - \beta \leq 0.95$$

$$\beta \in [0.25, 0.35]$$

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- Q.4** The common difference of the A.P. b_1, b_2, \dots, b_m is 2 more than the common difference of A.P. a_1, a_2, \dots, a_n . If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to:
 (1) -127 (2) 81 (3) 127 (4) -81

Sol. (4)
 A.P. ($a_1, a_2, a_3, \dots, a_n$) (CD = D_a)
 ($b_1, b_2, b_3, \dots, b_m$) (CD = D_b)
 $D_b = D_a + 2$
 $a_{40} = -159$
 $a_1 + 39 D_a = -159$ -----(1)
 $a_{100} = -399$
 $a_1 + 99 D_a = -399$ -----(2)
 Eqn (1) - (2)
 $-60 D_a = 240 \Rightarrow D_a = -4$
 $D_b = -4 + 2 = -2$
 $a_1 + 39(-4) = -159 \Rightarrow a_1 = -3$
 $b_{100} = a_{70}$
 $b_1 + 99 D_b = a_1 + 69 D_a$
 $b_1 + 99(-2) = (-3) + 69(-4)$
 $b_1 = -81$

- Q.5** The integral $\int_1^2 e^x \cdot x^x (2 + \log_e x) dx$ equal :
 (1) $e(4e-1)$ (2) $e(4e+1)$ (3) $4e^2-1$ (4) $e(2e-1)$

Sol. (1)
 $\int_1^2 e^x \cdot x^x (2 + \ln x) dx$
 $e^x \cdot x^x = t \therefore$ Upper Limit = $e^2 \cdot 2^2$, Lower Limit = e
 $(e^x \cdot x^x + e^x x^x (1 + \ln x)) dx = dt$
 $e^x \cdot x^x (2 + \ln x) dx = dt$
 $\int_e^{4e^2} dt = [t]_e^{4e^2} = 4e^2 - e = e(4e-1)$

- Q.6** If the tangent to the curve, $y=f(x)=x \log_e x$, ($x>0$) at a point $(c, f(c))$ is parallel to the line-segment joining the points $(1,0)$ and (e,e) , then c is equal to:

- (1) $e^{\left(\frac{1}{1-e}\right)}$ (2) $\frac{e-1}{e}$ (3) $\frac{1}{e-1}$ (4) $e^{\left(\frac{1}{e-1}\right)}$

Sol. (4)
 $y = f(x) = x \ln x$
 $m_1 = \frac{dy}{dx} \Big|_{(c, f(c))} = (\ln x + 1) \Big|_{(c, f(c))} = \ln c + 1$
 slope of the line joining $(1, 0), (e, e)$
 $m_2 = \left(\frac{e}{e-1} \right)$
 $m_2 = m_1 \Rightarrow \ln c + 1 = \frac{e}{e-1}$
 $\ln c = \frac{e}{e-1} - 1 = \frac{1}{e-1}$

$$\boxed{c = e^{\left(\frac{1}{e-1}\right)}}$$

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Q.7 If $y = \left(\frac{2}{\pi}x - 1\right) \operatorname{cosec} x$ is the solution of the differential equation,

$\frac{dy}{dx} + p(x)y = \frac{2}{\pi} \operatorname{cosec} x$, $0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to:

- (1) $\operatorname{cosec} x$ (2) $\cot x$ (3) $\tan x$ (4) $\sec x$

Sol. 2

$$y = \left(\frac{2}{\pi}x - 1\right) \operatorname{cosec} x$$

Differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{2}{\pi} \operatorname{cosec} x - \left(\frac{2x}{\pi} - 1\right) \operatorname{cosec} x \cdot \cot x$$

$$\frac{dy}{dx} + \left(\frac{2x}{\pi} - 1\right) \operatorname{cosec} x \cdot \cot x = \frac{2}{\pi} \operatorname{cosec} x$$

$$\frac{dy}{dx} + y \cot x = \frac{2}{\pi} \operatorname{cosec} x$$

Compare this differential equation with given differential equation
 $p(x) = \cot x$

Q.8 If α and β are the roots of the equation $2x(2x+1)=1$, then β is equal to:

- (1) $2\alpha(\alpha - 1)$ (2) $-2\alpha(\alpha + 1)$ (3) $2\alpha^2$ (4) $2\alpha(\alpha + 1)$

Sol. (2)

$$2x(2x+1) = 1$$

If α & β are the roots i.e α & β satisfy this equation

$$2\alpha(2\alpha + 1) = 1 \quad \Rightarrow \quad \alpha(2\alpha + 1) = \frac{1}{2}$$

$$4x^2 + 2x - 1 = 0$$

$$\alpha + \beta = \frac{-1}{2} = -\alpha(2\alpha + 1)$$

$$\beta = -\alpha(2\alpha + 1) - \alpha = -\alpha(2\alpha + 2) = -2\alpha(\alpha + 1)$$

Q.9 For all twice differentiable functions $f: \mathbb{R} \rightarrow \mathbb{R}$, with $f(0)=f(1)=f'(0)=0$,

- (1) $f''(x)=0$, at every point $x \in (0,1)$ (2) $f''(x) \neq 0$, at every point $x \in (0,1)$
(3) $f''(x)=0$, for some $x \in (0,1)$ (4) $f''(0)=0$

Sol. (3)

Applying Rolle's theorem in $[0,1]$ for function $f(x)$

$$f'(c) = 0, c \in (0,1)$$

again applying Rolle's theorem in $[0,c]$ for function $f'(x)$ s

$$f''(c_1) = 0, c_1 \in (0,c)$$

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Q.10 The area (in sq.units) of the region enclosed by the curves $y=x^2-1$ and $y=1-x^2$ is equal to :

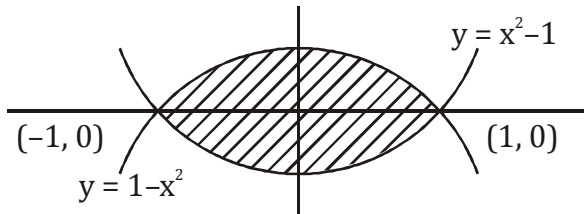
(1) $\frac{4}{3}$

(2) $\frac{7}{2}$

(3) $\frac{16}{3}$

(4) $\frac{8}{3}$

Sol. (4)



$$\text{Total area} = 4 \int_0^1 (1-x^2) dx = 4 \left[x - \frac{x^3}{3} \right]_0^1$$

$$= 4 \left[1 - \frac{1}{3} \right] = \frac{8}{3} \text{ sq.unit}$$

Q.11 For a suitably chosen real constant a , let a function, $f: \mathbb{R} - \{-a\} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{a-x}{a+x}. \text{ Further suppose that for any real number } x \neq -a \text{ and } f(x) \neq -a,$$

$$(f \circ f)(x) = x. \text{ Then } f\left(-\frac{1}{2}\right) \text{ is equal to:}$$

(1) -3

(2) 3

(3) $\frac{1}{3}$

(4) $-\frac{1}{3}$

Sol. (2)

$$f(x) = \frac{a-x}{a+x}$$

$$f(f(x)) = \frac{a-f(x)}{a+f(x)} = x$$

$$\frac{a-ax}{1+x} = f(x) = \frac{a-x}{a+x}$$

$$a \left(\frac{1-x}{1+x} \right) = \frac{a-x}{a+x}$$

$$\Rightarrow a = 1$$

$$\text{So } f(x) = \frac{1-x}{1+x}$$

$$f\left(-\frac{1}{2}\right) = 3$$

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Q.12 Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. If $B = A + A^4$, then $\det(B)$:

- (1) is one (2) lies in (1,2) (3) lies in (2,3) (4) is zero

Sol. (2)

$$A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$B = A + A^4$$

$$A^2 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta - \sin^2 \theta & 2 \sin \theta \cos \theta \\ -2 \sin \theta \cos \theta & -\sin^2 \theta + \cos^2 \theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Similarly

$$A^4 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$B = A^4 + A = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix} + \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$B = A^4 + A = \begin{bmatrix} \cos 4\theta + \cos \theta & \sin 4\theta + \sin \theta \\ -\sin 4\theta - \sin \theta & \cos 4\theta + \cos \theta \end{bmatrix}$$

$$\begin{aligned} B &= (\cos 4\theta + \cos \theta)^2 + (\sin 4\theta + \sin \theta)^2 \\ &= \cos^2 4\theta + \cos^2 \theta + 2 \cos 4\theta \cos \theta \\ &\quad + \sin^2 4\theta + \sin^2 \theta + 2 \sin 4\theta \sin \theta \\ &= 2 + 2 (\cos 4\theta \cos \theta + \sin 4\theta \sin \theta) \\ &= 2 + 2 \cos 3\theta \end{aligned}$$

$$\text{at } \theta = \frac{\pi}{5}$$

$$|B| = 2 + 2 \cos \frac{3\pi}{5} = 2 - (1 - \sin 18)$$

$$|B| = 2 \left(1 - \frac{\sqrt{5}-1}{4} \right) = 2 \left(\frac{5-\sqrt{5}}{4} \right) = \frac{5-\sqrt{5}}{2}$$

Q.13 The centre of the circle passing through the point (0,1) and touching the parabola $y=x^2$ at the point (2,4) is :

- (1) $\left(\frac{3}{10}, \frac{16}{5} \right)$ (2) $\left(\frac{6}{5}, \frac{53}{10} \right)$ (3) $\left(\frac{-16}{5}, \frac{53}{10} \right)$ (4) $\left(\frac{-53}{10}, \frac{16}{5} \right)$

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Sol. (3)

Circle passing through point (0,1) and touching curve $y = x^2$ at (2,4) tangent at (2,4) is

$$\frac{(y+4)}{2} = x(2)$$

$$\Rightarrow y - 4x + 4 = 0$$

Equation of circle

$$(x-2)^2 + (y-4)^2 + \lambda(4x-y-4) = 0$$

Passing through (0,1)

$$4 + 9 + \lambda(-5) = 0$$

$$\lambda = \frac{13}{5}$$

Circle is

$$x^2 - 4x + 4 + y^2 - 8y + 16 + \frac{13}{5} [4x - y - 4] = 0$$

$$x^2 + y^2 + \left(\frac{52}{5} - 4\right)x - \left(8 + \frac{13}{5}\right)y + 20 - \frac{52}{5} = 0$$

$$x^2 + y^2 + \frac{32}{5}x - \frac{53}{5}y + \frac{48}{5} = 0$$

$$\text{Centre is } \left(-\frac{16}{5}, \frac{53}{10}\right)$$

Q.14 A plane P meets the coordinate axes at A, B and C respectively. The centroid of $\triangle ABC$ is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is:

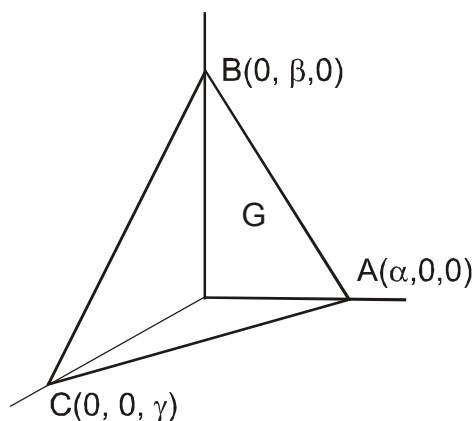
$$(1) \frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$$

$$(2) \frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

$$(3) \frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$$

$$(4) \frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$$

Sol. (2)



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$$G = \left(\frac{\alpha}{3}, \frac{\beta}{3}, \frac{\gamma}{3} \right) = (1, 1, 2)$$

$\alpha = 3, \beta = 3, \gamma = 6$
Equation of plane is

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

$$\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$$

$$2x + 2y + z = 6$$

Required line perpendicular to plane is $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

Q.15 Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max \{x, x^2\}$. Let S denote the set of all points in \mathbb{R} , where f is not differentiable. Then

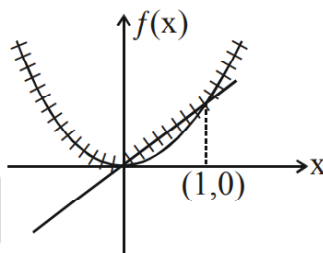
(1) $\{0, 1\}$

(2) ϕ (an empty set)

(3) $\{1\}$

(4) $\{0\}$

Sol. (1)



Function is not differentiable at two point
 $\{0, 1\}$

Q.16 The angle of elevation of the summit of a mountain from a point on the ground is 45° . After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60° . Then the height (in km) of the summit from the ground is:

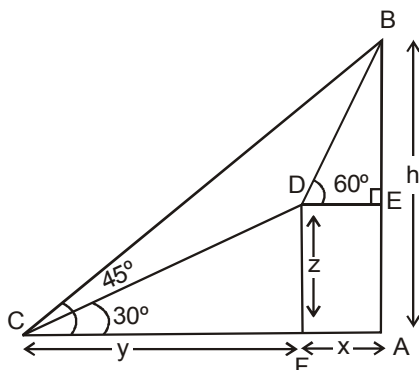
(1) $\frac{1}{\sqrt{3}+1}$

(2) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(4) $\frac{1}{\sqrt{3}-1}$

Sol. (4)



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If ΔCDF

$$\sin 30^\circ = \frac{z}{1} \Rightarrow z = \frac{1}{2} \text{ km}$$

$$\cos 30^\circ = \frac{y}{1} \Rightarrow y = \frac{\sqrt{3}}{2} \text{ km}$$

Now in ΔABC

$$\tan 45^\circ = \frac{h}{x+y} \Rightarrow h = x + y$$

$$x = h - \frac{\sqrt{3}}{2}$$

Now in ΔBDE

$$\tan 60^\circ = \frac{h-z}{x}$$

$$\sqrt{3}x = h - \frac{1}{2}$$

$$\sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2} \Rightarrow h = \frac{1}{\sqrt{3}-1} \text{ km}$$

Q.17 If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals:

(1) 1

(2) 9

(3) 2

(4) 3

Sol. (4)

$${}^{10}C_r \left(\frac{-k}{x^2}\right)^r (\sqrt{x})^{10-r}$$

$${}^{10}C_r (-k)^r (x)^{5-\frac{5r}{2}}$$

For constant term

$$5 - \frac{5r}{2} = 0 \Rightarrow r = 2$$

$$T_3 = {}^{10}C_2 k^2 = 405$$

$$k^2 = \frac{405}{45} = \frac{81}{9} = 9$$

$$|k| = 3$$

Q.18 Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the

(1) line, $y = x$

(2) real axis

(3) imaginary axis

(4) line, $y = -x$

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Sol. (1)

$$\begin{aligned} z &= x + iy \\ z^2 &= i |z|^2 \\ x^2 - y^2 + 2i xy &= i(x^2 + y^2) \\ \text{equating real terms} \\ x^2 - y^2 &= 0 \Rightarrow x^2 = y^2 \\ \text{equating imaginary terms} \\ 2xy &= x^2 + y^2 \\ (x - y)^2 &= 0 \Rightarrow x = y \end{aligned}$$

Q.19 Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1, -4)$ in this line is:

- (1) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (2) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (3) $\left(\frac{29}{5}, \frac{11}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

Sol. (1)

Let (h, k) is the image of the point $(-1, -4)$ in the line

$$\frac{x}{3} + \frac{y}{1} = 1$$

$$x + 3y = 3$$

$$L_2: 3x - y + \lambda = 0$$

$$-3 + 4 + \lambda = 0$$

$$\lambda = -1$$

$$3x - y = 1$$

(h, k) satisfy the equation of line L_2

$$3h - k = 1 \quad (1)$$

$$\left| \frac{-1 - 12 - 3}{\sqrt{1+9}} \right| = \left| \frac{h + 3k - 3}{\sqrt{1+9}} \right|$$

$$16 = |h + 3k - 3|$$

$$h + 3k = 19 \quad (2)$$

$$h + 3k = -13 \quad (3)$$

From equation (2) & (3) put the value of h in equation (1)

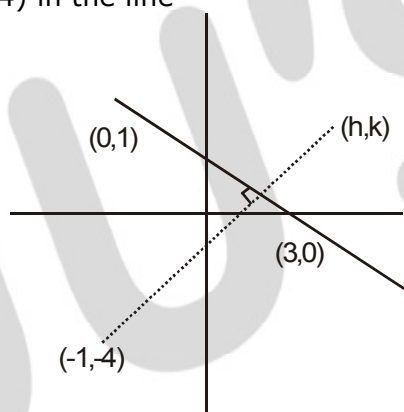
$$h = 19 - 3k, \quad h = -13 - 3k$$

$$3(19 - 3k) - k = 1 \quad 3(-13 - 3k) - k = 1$$

$$-10k = -56 = \frac{28}{5} \quad -10k = 40 \Rightarrow k = -4$$

$$k = \frac{28}{5}, h = 19 - 3\left(\frac{28}{5}\right) = \frac{95 - 84}{5} = \frac{11}{5}$$

$$\text{Image} = \left(\frac{11}{5}, \frac{28}{5}\right)$$



Q.20 Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is:

- (1) For an integer n, if n is even, then $n^3 - 1$ is even
 (2) For an integer n, if n is odd, then $n^3 - 1$ is even
 (3) For an integer n, if $n^3 - 1$ is not even, then n is not odd.
 (4) For an integer n, if n is even, then $n^3 - 1$ is odd

Sol. (4)

$P: n^3 - 1$ is even, $q: n$ is odd

Contrapositive of $p \rightarrow q = \sim q \rightarrow \sim p$

\Rightarrow If n is not odd then $n^3 - 1$ is not even

\Rightarrow For an integer n, if n is even, then $n^3 - 1$ is odd

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Q.21 The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____

Sol. 120

Consonants → LTTR

Vowels → EE

$$\text{Total No of words} = \frac{6!}{2!2!} = 180 \quad (\text{T and E are repeated})$$

Total no of words if vowels are together

$$= \frac{5!}{2!} = 60 \quad (\text{E is repeated})$$

$$\text{Required} = 180 - 60 = 120$$

Q.22 If \vec{x} and \vec{y} be two non-zero vectors such that $|\vec{x} + \vec{y}| = |\vec{x}|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to \vec{y} , then the value of λ is _____

Sol. 1

$$|\vec{x} + \vec{y}|^2 = |\vec{x}|^2$$

$$\Rightarrow |\vec{y}|^2 + 2\vec{x} \cdot \vec{y} = 0 \quad (1)$$

$$\text{and } (2\vec{x} + \lambda\vec{y}) \cdot \vec{y} = 0$$

$$\Rightarrow \lambda (|\vec{y}|^2) + 2\vec{x} \cdot \vec{y} = 0 \quad (2)$$

by comparing (1) & (2)
we get $\lambda = 1$

Q.23 Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$, respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _____

Sol. 6

X_i (observation)	0	2	2^2	2^n
f_i (frequency)	nC_0	nC_1	nC_2	nC_n

$$\bar{x} = \frac{\sum f_i X_i}{\sum f_i}$$

$$= \frac{0 \times {}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 + \dots + 2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + \dots + {}^nC_n}$$

$$= \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$$3^n = 729 = 3^6$$

$$n = 6$$

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Q.24 Suppose that function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y)=f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(1)=3$.

If $\sum_{i=1}^n f(i) = 363$, then n is equal to

Sol. 5

$$f(x+y) = f(x) f(y)$$

$$f(x) = a^x$$

$$\Rightarrow f(1) = a = 3$$

$$\text{So } f(x) = 3^x$$

$$\sum_{i=1}^n f(i) = 363$$

$$\Rightarrow 3 + 3^2 + 3^3 + \dots + 3^n = 363 \Rightarrow \frac{3(3^n - 1)}{2} = 363 \Rightarrow n = 5$$

Q.25 The sum of distinct values of λ for which the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$$

has non-zero solutions, is _____

Sol. 3

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ 0 & \lambda - 3 & -\lambda + 3 \\ 3 - \lambda & 0 & \lambda - 3 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 + C_3$$

$$\begin{vmatrix} 3\lambda - 1 & 3\lambda + 1 & 2\lambda \\ -\lambda + 3 & \lambda - 3 & -\lambda + 3 \\ 0 & 0 & \lambda - 3 \end{vmatrix} = 0$$

$$(\lambda - 3) [(3\lambda - 1)(\lambda - 3) - (3 - \lambda)(3\lambda + 1)] = 0$$

$$(\lambda - 3) [3\lambda^2 - 10\lambda + 3 - (8\lambda - 3\lambda^2 + 3)] = 0$$

$$(\lambda - 3) (6\lambda^2 - 18\lambda) = 0$$

$$(6\lambda) (\lambda - 3)^2 = 0$$

$$\lambda = 0, 3$$

$$\text{sum of values of } \lambda = 0 + 3 = 3$$