MPBSE Class 12th Maths Question Paper With Solutions 2019

QUESTION PAPER CODE E - 236

SECTION - A

Question 1: Choose and write the correct options. **1* 5 = 5M**

[i] Let A = {1, 2, 3}, then number of equivalence relations containing (1, 2) is:

(A) l (B) 2 (C) 3 (D) 4

Solution: (B)

Total possible pairs = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

Reflexive means (a, a) must be in relation.

Hence (1, 1), (2, 2), (3, 3) must be in a relation.

Symmetric means if (a, b) is in relation, then (b, a) must be in relation.

Therefore, if (1, 2) is in relation, then (2, 1) must also be in relation.

Transitive means if (a, b) is in relation, and (b, c) is in relation, then (a, c) is in relation.

If (1, 2) is in relation, and (2, 1) is in relation, then (1, 1) must be in relation.

Relation R_1 = {(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)}

Total possible pairs = {(1, 1) , (1, 2), (1, 3), (2, 1) , (2, 2), (2, 3), (3, 1) , (3, 2), (3, 3)}

The smallest relation is R_1 = {(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)}

Add (3, 2) also, as it is symmetric but, as (1, 2) and (2, 3) are there, (1, 3) is to be added as it is transitive.

Since we are adding (1, 3), (3, 1) also is to be added, as it is symmetric.

Relation R_2 = {(1, 2), (2, 1), (1, 1), (2, 2), (3, 3), (2, 3), (3, 2), (1, 3), (3, 1)}

So, only 2 possible relations are there which are equivalence.

[ii] If $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$ (B) $-\pi/2 \le y \le \pi/2$

(C) $0 < y < \pi$ (D) $-\pi/2 < y < \pi/2$

Solution: (B)

$\sin^{-1} x = y$

$y = \sin^{-1} x$

The range of principal value of sin is $[-\pi/2, \pi/2]$.

So, $-\pi/2 \le y \le \pi/2$ is the correct answer.

[iii] If $$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$ **and A + A' = I, then the value of α is**

(A) $\pi/6$ (B) $\pi/3$

(C) π (D) $3\pi/2$

Solution: (B)

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A + A' = I

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since the matrices are equal, the corresponding elements can be equated.

2 cos α = 1

cos α = 1 / 2

cos α = cos 60^o

α = **π** / 3

[iv] Let A be a nonsingular square matrix of order 3 x 3, then adj |A| is equal to

(A) $|A|^2$ (B) |A| (C) $|A|^3$ (D) 3|A|

Solution: A

A (adj A) = |A| I

Taking determinants both sides

|A (adj A)| = ||A| I|

|A (adj A)| = |A| |adj A| $(|AB|=|A||B|)$

$||A|I| = |A|^3|I| = |A|^3$

|A (ad jA)| = ||A|I|

Putting values

$|A|\ |adj\ (A)| = |A|^3$

$|adj\ (A)| = |A|^3 / |A|$

$|adj\ (A)| = |A|^2$

B is the correct answer.

[v] Function f (x) = |x| at x = 0 is

(A) Continuous but not differentiable

(B) Discontinuous and not differentiable
(C) Discontinuous and differentiable
(D) Continuous and differentiable

Solution: (A)
f (x) = |x|

$$f(x) = \begin{cases} -x, if\ x < 0 \\ x, if\ x \geq 0 \end{cases}$$

f is continuous at x = 0 if the left-hand limit is equal to the right-hand limit.
$\lim_{x\to 0-} f(x) = \lim_{x\to 0+} f(x) = f(0)$
LHL = $\lim_{x\to 0-} f(x)$
= $\lim_{x\to 0-} |x|$
= $\lim_{x\to 0} (-x)$
= - (0)
= 0
RHL = $\lim_{x\to 0+} f(x)$
= $\lim_{x\to 0+} |x|$
= $\lim_{x\to 0} (x)$
= 0
LHL = RHL and f (0) = 0.
Hence $\lim_{x\to 0-} f(x) = \lim_{x\to 0+} f(x) = f(0)$.
f is continuous at x = 0.

Question 2: Fill in the blanks. **1 * 5 = 5M**

[i] The direction cosine of the vector 3i - 2j + 6k are _____. **[3 / 7, -2 / 7, 6 / 7]**

[ii] If $y = x + e^x$, then d^2y / dx^2 = _____. **[$d^2y / dx^2 = e^x$]**
Solution:
$y = x + e^x$
$dy / dx = 1 + e^x$
$d^2y / dx^2 = e^x$

[iii] Area bounded by the curve $y = x^2$, x - axis and x = 1 and x = 2, is _____. **[7 / 3]**

Solution:

$\int_1^2 x^2\, dx$

$= (x^3 / 3)_1^2$

$= 1 / 3\ (2^3 - 1^3)$

$= 7 / 3$

[iv] Direction ratio of two parallel lines will be in _____. **[proportion]**

[v] Intercept of 2x + y - z = 5 on x-axis is _____. **[5 / 2]**

Solution:

2x + y - z = 5

Since it cuts the x-axis, y and z = 0.

2x = 5

x = 5 / 2

Question 3: Write true/false in the following statements. 1 * 5 = 5M

(i) If E_1 and E_2 are exclusive events, then $P(E_1 \cap E_2)$ is 0. **[True]**

(ii) If P (A) = 1 / 2, P (B) = 0, then P (A / B) is not defined. **[True]**

(iii) The objective function of an LPP is always linear. **[True]**

(iv) The feasible region of a linear programming problem is always a linear Polygon. **[True]**

(v) The value of $\int_0^\pi \cos^3 x\, dx$ is 0. **[True]**

Question 4: Match the correct pairs. 1 * 5 = 5M

(i)	$\int \frac{dx}{x^2 - a^2}$ (का मान)	(a)	$\sin^{-1}\frac{x}{a} + C$
(ii)	$\int \frac{dx}{\sqrt{a^2 - x^2}}$ (का मान)	(b)	$\frac{1}{2a}\log\frac{a+x}{a-x} + C$
(iii)	$\int \frac{1}{x\sqrt{x^2 - a^2}}dx$	(c)	$\frac{1}{2a}\log\frac{x-a}{x+a} + C$
(iv)	$\int \frac{1}{\sqrt{x^2 - a^2}}dx$	(d)	$\frac{1}{a}\sec^{-1}\frac{x}{a}$
(v)	$\int \tan x\, dx$	(e)	$\log\left(x - \sqrt{x^2 - a^2}\right)$
		(f)	$\log\left(x + \sqrt{x^2 - a^2}\right)$
		(g)	$\log \sec x$

Solution:

(i) - (c)
(ii) - (a)
(iii) - (d)
(iv) - (f)
(v) - (g)

Question 5: Write the answers in one word/sentence each. 1 * 5 = 5M

(i) The maximum value of $x^{1/x}$ is

Solution:

$y = x^{1/x}$

$\log y = (1 / x) \log x$

$(1 / y) (dy / dx) = (1 - \log x) / x^2$

$dy / dx = 0$

$1 - \log x = 0$

$\log x = 1$

$x = e$

The maximum value is $e^{1/e}$.

(ii) Rate of change in the area of a circle having radius r when r = 5 cm.

Solution:

Area of circle = $A = \pi r^2$

$dA / dr = d (\pi r^2) / dr$

$= 2\pi r$

For r = 5,

$dA / dr = 2\pi * 5 = 10\pi$

(iii) The slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at x = 0.

Solution:

$y = 2x^2 + 3 \sin x$

$dy / dx = 4x + 3 \cos x$

Slope of tangent * Slope of normal = -1

$(4x + 3 \cos x)$ * Slope of normal = -1

Slope of normal = $-1 / (4x + 3 \cos x)$

At x = 0,

Slope of normal = $-1 / (4 * 0 + 3 \cos 0)$

$= -1 / 3$

(iv) The minimum value of 3 sin θ + 4 cos θ is

Solution:

Let $f(\theta) = 3 \sin \theta + 4 \cos \theta$

$f'(\theta) = 3 \cos \theta - 4 \sin \theta$ ---- (1)

$f''(\theta) = - 3 \sin \theta - 4 \cos \theta$ ---- (2)

Now for maximum of minimum value of $f(\theta)$, $f'(\theta) = 0$.

$3 \cos \theta - 4 \sin \theta = 0$

$\sin \theta = \pm 3 / 5$; $\cos \theta = \pm 4 / 5$
From (2), $\sin \theta = - 3 / 5$; $\cos \theta = - 4 / 5$
$f''(\theta) > 0$
The minimum value of $f(\theta) = - (3^2 + 4^2) / 5 = -5$.

(v) Derive the equation of tangent line at (1, 1) on curve $y = x^3$.
Solution:
$dy / dx = 3x^2$
At (1, 1), $dy / dx = 3$
$y - 1 = 3 (x - 1)$
$y - 1 = 3x - 3$
$y = 3x - 2$

SECTION - B

Question 6: [i] If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, **then find the value of A . B.**

OR

[ii] If $A = \begin{bmatrix} 1 & 5 & 6 \\ -6 & 7 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -5 & 7 \\ 8 & -7 & 7 \end{bmatrix}$, **then find the value of A - B.**

Solution:

[i] $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$,

$$= \begin{bmatrix} 2+8 & 6+20 \\ 3+4 & 9+10 \end{bmatrix}$$
$$= \begin{bmatrix} 10 & 26 \\ 7 & 19 \end{bmatrix}$$

OR

[ii] $A = \begin{bmatrix} 1 & 5 & 6 \\ -6 & 7 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -5 & 7 \\ 8 & -7 & 7 \end{bmatrix}$,

$$A - B = \begin{bmatrix} 0 & 10 & -1 \\ -14 & 14 & -7 \end{bmatrix}$$

Question 7: [i] Check the continuity of the function f given by f (x) = 2x + 3 at x = 1.

OR

[ii] Prove that the modulus function f (x) = |x| is not differentiable at x = 0.

Solution:

[i] The function is continuous at x = 1 if lim $_{x\to1}$ f (x) = f (1).

LHS = lim $_{x\to1}$ f (x)

= lim $_{x\to1}$ (2x + 3)

= 2 * 1 + 3

= 5

RHS = f (1)

= 2 * 1 + 3

= 5

LHS = RHS

The function is continuous.

OR

[ii] |x| = f (x) = x x ≥ 0

-x x < 0

f (x) is not differentiable at x = 0.
f ' (x) = x / |x|

Question 8: [i] Find the following integrals $\int(x^{2/3} + 1)\, dx$.

OR

[ii] Find the following integrals $\int[(1 - \sin x) / \cos^2 x]\, dx$.

Solution:

[i] $\int(x^{2/3} + 1)\, dx$

$= \int(x^{2/3} + x^0)\, dx$

$= \int x^{2/3}\, dx + x^0\, dx$

$= \int x^{(2/3) + 1} / [(2 / 3) + 1] + \int x^{0 + 1} / [0 + 1] + c$

$= x^{(5/3)} / [(5 / 3)] + x + c$

$= (3x^{5/3} / 5) + x + c$

OR

[ii]

$$\int \frac{1-\sin(x)}{\cos^2(x)} dx$$

$$= \int \frac{1}{\cos^2(x)} - \frac{\sin(x)}{\cos^2(x)} dx$$

$$= \int \frac{1}{\cos^2(x)} dx - \int \frac{\sin(x)}{\cos^2(x)} dx$$

$$\int \frac{1}{\cos^2(x)} dx = \tan(x)$$

$$\int \frac{\sin(x)}{\cos^2(x)} dx = \sec(x)$$

$$= \tan(x) - \sec(x)$$

$$= \tan(x) - \sec(x) + C$$

Question 9: [i] Find the unit vector in the direction of vector a = 2i + 3j + k.

OR

[ii] Find the projection of the vector a = 2i + 3j + 2k on the vector b = i + 2j + k.

Solution:

[i] a = 2i + 3j + k = 2i + 3j + 1k

|a| = $\sqrt{2^2 + 3^2 + 1^2}$

= $\sqrt{4 + 9 + 1}$

= $\sqrt{14}$

Unit vector = a / |a|

= 2i + 3j + 1k / $\sqrt{14}$

= (2 / $\sqrt{14}$) i + (3 / $\sqrt{14}$) j + (1 / $\sqrt{14}$) k

OR

[ii] a = 2i + 3j + 2k

b = i + 2j + k

(a . b) = (2 * 1) + (3 * 2) + (2 * 1)

= 2 + 6 + 2

= 10

|b| = $\sqrt{1^2 + 2^2 + 1}$

= $\sqrt{1 + 4 + 1}$

= $\sqrt{6}$

Projection of vector a on b = (a . b) / |b|

= (1 / $\sqrt{6}$) (10)

= 10 / $\sqrt{6}$

= (10 / $\sqrt{6}$) ($\sqrt{6}$ / $\sqrt{6}$)

= (10 / 6) ($\sqrt{6}$)

= (5 / 3) ($\sqrt{6}$)

Question 10: [i] Find the equation of the plane whose intercepts on the coordinate axes are -4, 2, 3.

OR

[ii] Find the angle between the line (x + 1) / 2 = (y / 3) = (z - 3) / 6 and the plane 3x + y + z = 7.

Solution:

[i] The intercepts of the plane equation are - 4, 2, 3.

(x / - 4) + (y / 2) + (z / 3)

OR

[ii] (x + 1) / 2 = (y / 3) = (z - 3) / 6

The plane is 3x + y + z = 7.

sin ϕ = (2 * 3 + 3 * 1 + 6 * 1) / (√4 + 9 + 36) (9 + 1 + 1)

= (6 + 3 + 6) / 7 * √11

= 15 / 7√11

ϕ = $\sin^{-1}$ (15 / 7√11)

Question 11: [i] The radius of an air bubble is increasing at the rate 1 / 2 cm per second. At what rate is the volume of the bubble increasing when the radius is 1 cm.

OR

[ii] Find the slope of the normal to the curve x = 1 - a sin θ, y = $b\cos^2\theta$ at θ = π / 2.

Solution:

[i] Consider r is the radius of the bubble.

It is given that dr / dt = (1 / 2) cm / s --- (1)

The bubble is in the shape of a sphere.

So, the volume of the bubble = volume of the sphere = (4 / 3) πr^3

dv / dt at r = 1 cm

dv / dt = d ((4 / 3) πr^3) / dt

= [(4 / 3) π] $3r^2$ (dr / dt)

= $4\pi r^2$ (1 / 2)

= $2\pi r^2$

When radius is 1 cm, dv / dt = 2π $(1)^2$

$= 2\pi\ cm^3 / sec$

OR

[ii] $x = 1 - a \sin\theta$, $y = b\cos^2\theta$

$dx / d\theta = 0 - a \cos\theta$

$dy / d\theta = -2b \cos\theta \sin\theta$

$dy / dx = (dy / d\theta) / (dx / d\theta)$

$= (-2b \cos\theta \sin\theta) / (-a \cos\theta)$

$= (2b / a) \sin\theta$

The slope of the tangent at $\theta = \pi / 2$ is given by,

$(dy / dx)_{\theta = \pi / 2} = [(2b / a) \sin\theta]_{\theta = \pi / 2}$

$= (2b / a) \sin(\pi / 2)$

$= 2b / a$

The slope of the normal at $\theta = \pi / 2$ is given by,

$1 / (\text{slope of the tangent at } \theta = \pi / 4) = -1 / (2b / a)$

$= -a / 2b$

Question 12: [i] For what value of x is y = x (5 - x) maximum or minimum?

OR

[ii] Use differentials to find the value of √49.5.

Solution:

[i] $y = x (5 - x)$

$y = 5x - x^2$

$dy / dx = 5 - 2x$

$dy / dx = 0$

$5 - 2x = 0$

$x = 5 / 2$

$d^2y / dx^2 = -2$ (negative)

$x = 5 / 2$ is maximum and no minimum.

OR

[ii] Consider x = 49 and δx = 0.5.

$y = x^{1/2}$

$dy / dx = 1 / 2\sqrt{x}$

= 1 / 2 * (49)

= 1 / (2 * 7)

= 1 / 14

δy = (dy / dx) (δx)

= (1 / 14) (0.5)

= 1 / 28

$\sqrt{49.5} = y + \delta y$

= 7 + (1 / 28)

= 7.036

Question 13: [i] If a + b + c = 0, then prove that a x b = b x c = c x a.

OR

[ii] Find the area of parallelogram whose adjacent sides are given by the vectors a = 3i + j + 4k and b = i - j + k.

Solution:

[i] a + b + c = 0

a x (a + b + c) = a x 0

a x a + a x b + a x c = a x 0

0 + a x b + a x c = a x 0

a x b + a x c = 0

a x b = - a x c

a x b = c x a

Similarly

a + b + c = 0

b x (a + b + c) = b x 0

b x a + b x b + b x c = b x 0

b x a + b x b + b x c = b x 0

b x a + b x c = 0

b x c = - b x a

b x c = a x b

a x b = b x c = c x a

OR

[ii] a = 3i + j + 4k
b = i - j + k
Area of the parallelogram = |a x b|

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= i\,(1 \times 1 - (-1) \times 4) - j\,(3 \times 1 - 1 \times 4) + k\,(3 \times -1 - 1 \times 1)$
$= i\,(1 - (-4)) - j\,(3 - 4) + k\,(-3 - 1)$
$= i\,(1 + 4) - j\,(-1) + k\,(-4)$
$= 5i + j - 4k$
Magnitude of a x b = $\sqrt{(5^2 + 1^2 + (-4)^2)}$
|a x b| = $\sqrt{(25 + 1 + 16)}$
= $\sqrt{42}$
Area of parallelogram = |a x b| = $\sqrt{42}$

Question 14: [i] Find the minimum distance between the line l_1 and l_2 given by
r = i + 2j - 4k + ƛ (2i + 3j + 6k)
r = 3i + 3j - 5k + μ (2i + 3j + 6k)

OR

[ii] Find the distance of the plane 2x - 3y + 4z - 6 = 0 from the origin.

Solution:
[i] r = i + 2j - 4k + ƛ (2i + 3j + 6k)
r = 3i + 3j - 5k + μ (2i + 3j + 6k)
The above two lines passes through the points having position vectors,
a_1 = i + 2j - 4k
a_2 = 3i + 3j - 5k and are parallel to the vector b = 2i + 3j + 6k
$a_2 - a_1$ = 2i + j - k
$(a_2 - a_1)$ b = (2i + j - k) (2i + 3j + 6k)

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

= 9i - 14j + 4k

$|(a_2 - a_1)\ b| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 16 + 16}$

$= \sqrt{293}$

$|b| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

= 7

Shortest distance = $|(a_2 - a_1)\ b| / |b|$

$= \sqrt{293} / 7$

OR

[ii] The equation of the plane is 2x - 3y + 4z - 6 = 0.

2x - 3y + 4z = 6 --- (1)

The direction ratios are given by a = 2, b = - 3 and c = 4.

$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + (-3)^2 + 4^2}$

$= \sqrt{4 + 9 + 16}$

$= \sqrt{29}$

The direction cosines are $l = 2 / \sqrt{29}$, $m = -3 / \sqrt{29}$, $n = 4 / \sqrt{29}$.

The equation of the plane is lx + my + nz = d.

$(2 / \sqrt{29})\, x + (-3 / \sqrt{29})\, y + (4 / \sqrt{29})\, z = d$

$2x - 3y + 4z = d\sqrt{29}$

On comparing with equation (1),

$6 = d\sqrt{29}$

$d = 6 / \sqrt{29}$

Question 15: [i] Show that relation “is equal to" in sets is an equivalence relation.

OR

[ii] If f (x) = x^2 and g (x) = x + 3. x ∈ R , then find the value of (g o f) x , (f o g) x , $(f \circ g)^2$.

Solution:

[i] The relation "is equal to", denoted "=", is an equivalence relation on the set of real numbers since for any x, y, z ∈ R:

a. Reflexivity: x = x,

b. Symmetry: if x = y then y = x,

c. Transitivity: if x = y and y = z then x = z.

All of these are true.

OR

[ii] f (x) = x^2 and g (x) = x + 3

(g o f) x = g f (x)

= f (x) + 3

= $x^2 + 3$

(f o g) x = f (g (x))

= $(x + 3)^2$

= $x^2 + 9 + 6x$

$(f \circ g)^2 = (2 + 3)^2$

= 5^2

= 25

Question 16: [i] Show that $\sin^{-1} (3 / 5) - \sin^{-1} (8 / 17) = \cos^{-1} (84 / 85)$.

OR

[ii] Prove that $\cos^{-1} x = 2 \cos^{-1} \sqrt{(1 + x) / 2}$.

Solution:

[i] $\sin^{-1}(3 / 5) = x$ and $\sin^{-1}(8 / 17) = y$

sinx = 3 / 5 ; siny = 8 / 17

$\cos x = \sqrt{(1 - \sin^2 x)}$

$= \sqrt{(1 - (3 / 5)^2)}$

$= \sqrt{(1 - 9/25)}$
$= 4/5$
$\cos y = \sqrt{(1 - \sin^2 y)}$
$= \sqrt{(1 - (8/17)^2)}$
$= \sqrt{(1 - 64/289)}$
$= 15/17$
$\cos(x - y) = \cos x \cos y + \sin x \sin y$
$= 4/5 \times 15/17 + 3/5 \times 8/17 = 60/85 + 24/85 = 84/85$
$\Rightarrow x - y = \cos^{-1}(84/85)$
$\Rightarrow \sin^{-1}(3/5) - \sin^{-1}(8/17) = \cos^{-1}(84/85)$

OR

[ii] $\cos^{-1} x = 2\cos^{-1}\sqrt{(1 + x)/2}$
Put $x = \cos\theta$
$\cos^{-1}\cos\theta = 2\cos^{-1}\sqrt{(1 + \cos\theta)/2}$
$\theta = 2\cos^{-1}\sqrt{[1 + 2\cos^2\theta/2]/2}$
$= 2\cos^{-1}\cos(\theta/2)$
$= \theta$
$\theta = \theta$
LHS = RHS

Question 17: [i] Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

OR

[ii] Find the area of the triangle whose vertices are (3, 8), (- 4, 2) and (5, 1).

Solution:

[i] Applying $R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} b+c-\mathbf{b}-\mathbf{c} & a-c-a-c & a-b-a-b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} \mathbf{0} & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= 0\begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix} - (-2c)\begin{vmatrix} b & b \\ c & a+b \end{vmatrix} - 2b\begin{vmatrix} b & c+a \\ c & c \end{vmatrix}$$

= 0 + 2c (b (a + b) – cb) – 2b (cb – c(c + a))

= 2c (ab + b^2 – cb) – 2b (cb – c^2 – ca)

= 2abc + 2cb^2 – 2bc^2 – 2b^2c + 2bc^2 + 2abc

= 2abc + 2abc + 2cb^2 – 2cb^2 – 2bc^2 + 2bc^2

= 4abc + 0 + 0

= 4abc

OR

[ii] Area of the triangle = (1 / 2) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Δ = (1 / 2) $\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

= (1 / 2) (3 (2 - 1) - 8 (- 4 - 5) + 1 (- 4 - 10))

= (1 / 2) (3 + 72 - 14)

= 61 / 2

Question 18: [i] Find the equation of the plane the coordinate point (1, - 1, 2) and is perpendicular to each of the planes 2x + 3y - 2z = 5 and x + 2y - 3z = 8.

OR

[ii] Show that the angle between any two diagonals of a cube is $\cos^{-1}(1 / 3)$.

Solution:

[i] The equation of any plane passing through the point is (1, -1, 2) is

a (x - 1) + b (y + 1) + c (z - 2) = 0 ---- (1)

It is given that (1) is perpendicular to the plane 2x + 3y - 2z = 5.

2a + 3b - 2c = 0 ---- (2)

It is given that (1) is perpendicular to the plane x + 2y - 3z = 8.

a + 2b - 3c = 0 ---- (3)

On solving (1), (2), (3),

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2 & 3 & -2 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

- 5 (x - 1) + 4 (y + 1) + 1 (z - 2) = 0

5x - 4y - z = 7

OR

[ii] Consider a to be the edge of the cube with vertex at the origin.

There exist our diagonals for a cube.

The direction ratios of the diagonals are given by

(a - 0), (a - 0), (a - 0) = a, a, a and (0 - a), (a - 0), (a - 0) = a, a, a

Let θ be the angle between the diagonals.

$\cos \theta = a(-a) + a(a) + a(a) / [\sqrt{a^2 + a^2 + a^2} \sqrt{-a^2 + a^2 + a^2}]$

$= a^2 / 3a^2$

= 1 / 3

cos θ = 1 / 3

$\theta = \cos^{-1}(1 / 3)$

Question 19: [i] Draw the graph of the inequality 3x + 2y ≤ 6.

OR

[ii] Find the minimum value of P = 2x + 4y, subject to constraints:

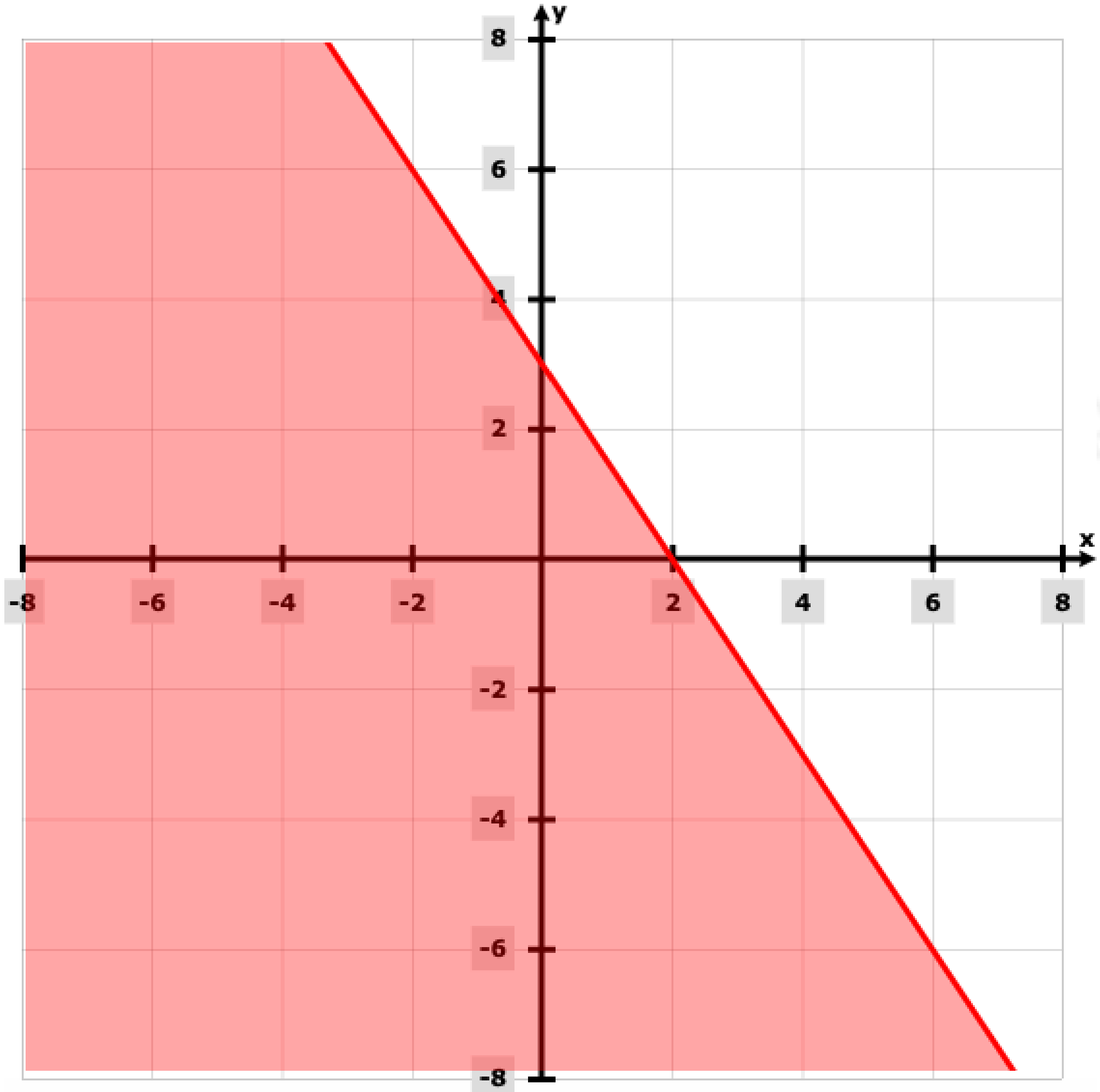
4x + 3y ≤ 12

$x + 2y \geq 4$

$x, y \geq 0$

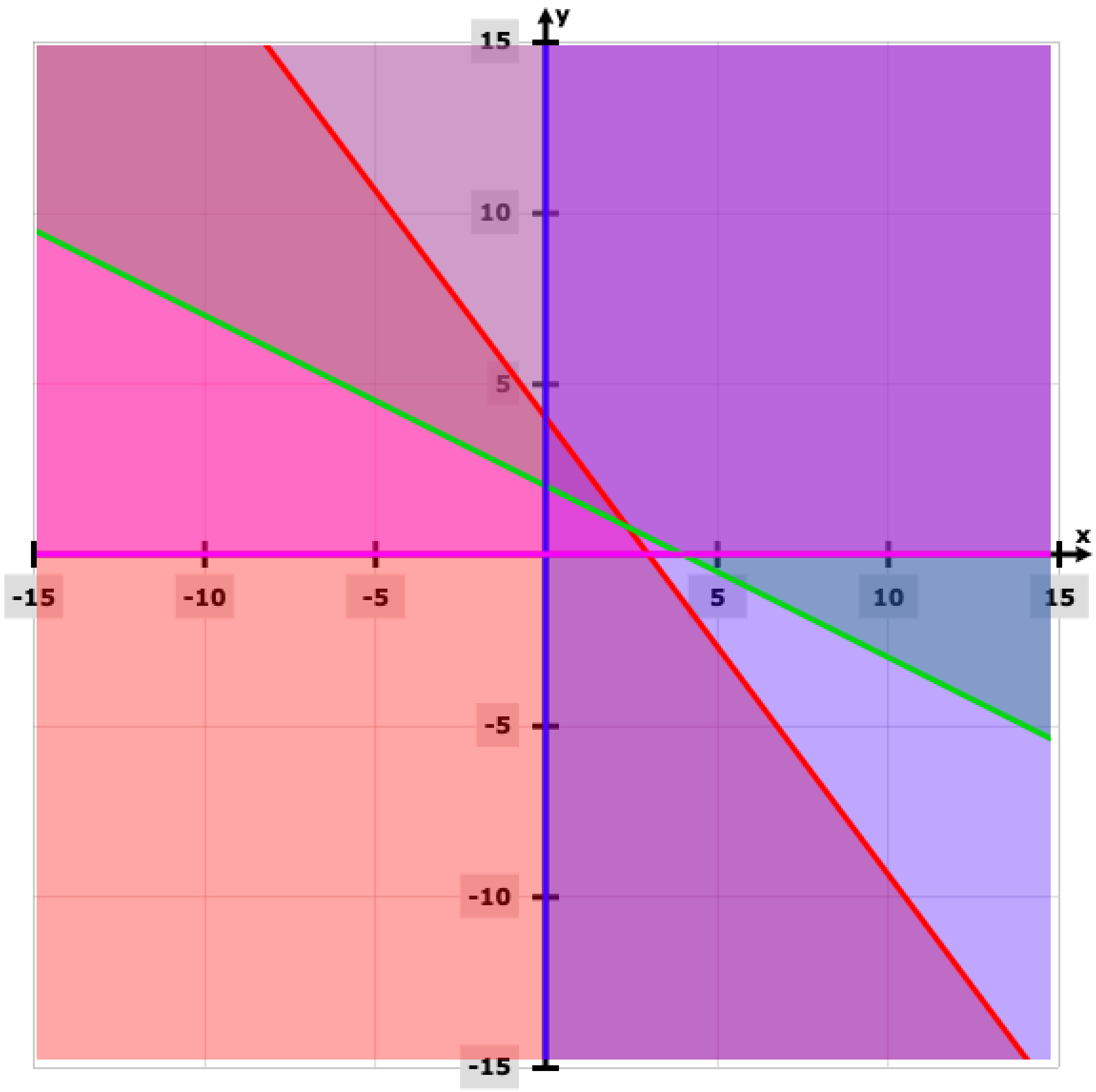
Solution:

[i]



OR

[ii]



Points	P = 2x + 4y
(0, 0)	0
(3, 0)	6
(12 / 5), (4 / 5)	8
(0, 2)	8

The value is minimum at the origin.

Question 20: [i] If P (A) = ½, P (B) = ¼ and P (A ∩ B) = ¼, then find the following:
(i) P (A / B)
(ii) P (B / A)

OR

[ii] In four throws of two dice what is the probability of getting the same figure on both dice?

Solution:
[i] P (A) = ½
P (B) = ¼
P (A ∩ B) = ¼
(i) P (A / B) = P (A ∩ B) / P (B)
= (1 / 4) / (1 / 4)
= 1
(ii) P (B / A) = P (A ∩ B) / P (A)
= (1 / 4) / (1 / 2)
= 1 / 2

OR

[ii] n = 4
p = 6 / 36 = 1 / 6
q =1 - (1 / 6) = 5 / 6
P (success) = 1 - P (no doublet)
= 1 - P (X = 0)
= $1 - {}^{4}C_{0} (5 / 6)^{4-0} (1 / 6)^{0}$
= $1 - (5 / 6)^{4}$
= 1 - (625 / 1296)
= 671 / 1296

Question 21: [i] A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

OR

[ii] Find the probability distribution of numbers of doublets in three throws of a pair of dice.

Solution:

[i] Sample space = S = {BB, BG, GB, GG} where B = Boy, G = Girl

A: at least one of the children is boy: {BB, BG, GB}

B: both are boys: {BB}

$P(B / A) = P(A \cap B) / P(A)$

= (1 / 4) / (3 / 4)

= 1 / 3

OR

[ii] The possible number of doublets possible on the throwing of 2 dice is

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

P (getting a doublet) = 6 / 36 = 1 / 6

P (not getting a doublet) = 1 – 1 / 6 = 5 / 6

When two dies are thrown thrice, 0 doublet or 1 doublet or 2 doublets or 3 doublets can be obtained.

X can take values 0, 1, 2, 3.

P (X = 0)

P (X = 0) = P (0 doublet on three throws)

= 5 / 6 × 5 / 6 × 5 / 6

= 125 / 216

P (X = 1)

P (X = 1) = P (one doublet on three throws)

= 1 / 6 × 5 / 6 × 5 / 6 + 5 / 6 × 1 / 6 × 5 / 6 + 5 / 6 × 5 / 6 × 1 / 6

= 3 × 5 / 6 × 5 / 6 × 1 / 6

= 75 / 216

P(X = 2)

P(X = 2) = P(two doublet on three throws)

= 1 / 6 × 1 / 6 × 5 / 6 + 1 / 6 × 5 / 6 × 1 / 6 + 5 / 6 × 1 / 6 × 1 / 6

= 3 × 1 / 6 × 1 / 6 × 5 / 6

= 15 / 216

P(X = 3)

P(X = 3) = P(three doublets on three throws)

= 1 / 6 × 1 / 6 × 1 / 6

= 1 / 216

The probability distribution is

X	0	1	2	3
P(X)	125 / 216	75 / 216	15 / 216	1 / 216

Question 22: [i] If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$**, then prove that A' . A = I.**

OR

[ii] If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, **then verify that**

(a) (A')' = A

(b) (A + B)' = A' + B'

Solution:

[i]

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$L.H.S : A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\ cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

$$= R.H.S$$

OR

[ii] (a)

$$A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix}$$

$$(A')' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix}' = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$= A$$

(b)

$$(A + B) = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2 & \sqrt{3}+(-1) & 2+2 \\ 4+1 & 2+2 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}$$

$$(A + B)' = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

Question 23: [i] Discuss the continuity of the following function

$$f(x) = \begin{cases} x \cdot \sin\left(\frac{1}{x}\right) & , \ x \neq 0 \\ 0 & \ \ x = 0 \end{cases}$$

OR

[ii] Prove that the function

$$f(x) = \begin{cases} x^2 - 1 & , \ \text{when} \ \ x \geq 1 \\ 1 - x & , \ \text{when} \ \ x < 1 \end{cases}$$

is not differentiable at x = 1.

Solution:

[i] Continuity is defined by

f (x) is continuous at x = a ⇔ $\lim_{x \to a}$ f (x) = f (a)

To show that $\lim_{x \to 0}$ (x) sin (1 / x) = f (0)

Let z = 1 / x, then as x → 0, z → ∞

$\lim_{x \to 0} (x) \sin(1/x) = \lim_{z \to \infty} (1/z) \sin z$

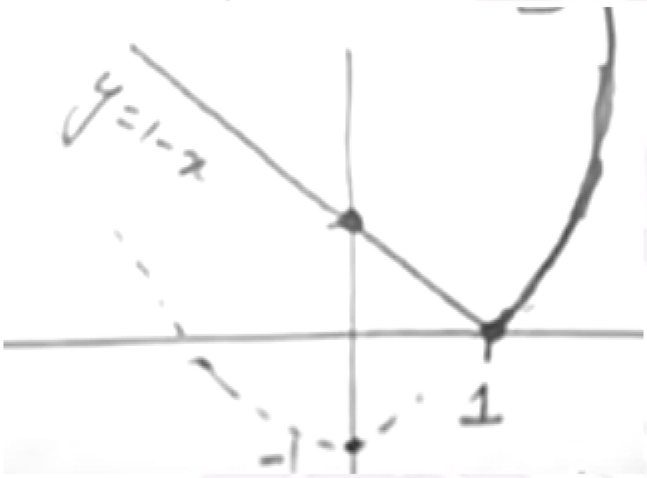
$= \lim_{z \to \infty} \sin z / z$

$= 0$

OR

[ii]

$$f(x) = \begin{cases} x^2 - 1, & \text{when } x \geq 1 \\ 1 - x, & \text{when } x < 1 \end{cases}$$



y = 1 - x

$y = x^2 - 1$

$x^2 = y + 1$

From the graph, the function is not differentiable at x = 1.

Question 24: [i] Evaluate $\int (xe^x) / (1 + x^2)\, dx$.

OR

[ii] Evaluate $\int_0^1 (\tan^{-1} x) / (1 + x^2)\, dx$.

Solution:

[i] Let $I = \int xe^x (1 + x)^2\, dx$

$I = \int (x + 1 - 1)e^x / (1 + x)^2\, dx$

$I = \int e^x (1 + x)\, dx - \int e^x (1 + x)^2\, dx$

Applying integration by parts in first integral,

$= e^x / (1 + x) - \int - (1 / (1 + x)^2)\, e^x\, dx - \int e^x / (1 + x)^2\, dx + C$

$= e^x / (1 + x) + \int (1 / (1 + x)^2)\, e^x\, dx - \int e^x / (1 + x)^2\, dx + C$

$I = e^x / 1 + x) + C$

OR

[ii] Let $\tan^{-1} x = t$

$dx / 1 + x^2 = dt$

When $x = 0$, $t = \tan^{-1} 0 = 0$

When $x = 1$, $t = \tan^{-1} 1 = \pi / 4$

$I = \int_0^{\pi/4} t\, dt$

$= [t^2 / 2]$

$= (1 / 2)\, [t^2]$

$= (1 / 2)\, (\pi / 4)^2 - 0$

$= \pi^2 / 32$

Question 25: [i] Find the area enclosed by circle $x^2 + y^2 = a^2$.

OR

[ii] Find the area of region bounded by the curves $y_1 = \sin x$ and $y_2 = \cos x$ between $x = 0$ and $x = \pi / 4$.

Solution:

[i] The equation of circle is: $x^2 + y^2 = a^2$

$y^2 = a^2 - x^2$

$y = \sqrt{a^2 - x^2}$

Area of circle = 4 × Area of first quadrant

$= 4\int^a_0 y\, dx$

$= 4\int^a_0 \sqrt{(a^2 - x^2)}\, dx$

$= 4\, [x / 2\, (\sqrt{(a^2 - x^2)}) + (a^2 / 2) \sin^{-1} x / a]^a_0$

$= 4\, [0 + (a^2 / 2) \sin^{-1} a / a - (0 + (a^2 / 2) \sin^{-1} 0 / a)]$

$= 4\, [(a^2 / 2) \sin^{-1} 1 - 0]$

$= 2a^2 (\pi / 2)$
$= \pi a^2$ square units

OR

[ii] $y_1 = \sin x$ and $y_2 = \cos x$
Area $= \int_0^{\pi/4} (\cos x - \sin x)\, dx$
$= [\sin x + \cos x]_0^{\pi/4}$
$= \sin (\pi / 4) + \cos (\pi / 4) - [\sin 0 + \cos 0]$
$= (1 / \sqrt{2}) + (1 / \sqrt{2}) - [0 + 1]$
$= (2 / \sqrt{2}) - 1$
$= \sqrt{2} - 1$ square units

Question 26: [i] Verify that the function $y = a \cos x + b \sin x$. where a, b $\in$ R is a solution of the differential equation $d^2y / dx^2 + y = 0$.

OR

[ii] Solve the differential equation $dy / dx = x \cdot \log x$.

Solution:
[i] $y = a \cos x + b \sin x$
$dy / dx = - a \sin x + b \cos x$
$d^2y / dx^2 = - a \cos x - b \sin x$
LHS $= d^2y / dx^2 + y$
$= - a \cos x - b \sin x + a \cos x + b \sin x$
$= 0$
= RHS

OR

[ii] $y = x^2 \log x - x^2 / 2 + e$
$y = [x^3 / 2] \log x - x^2 + c$
$y = (1 / 2) x^2 + (1 / 2) x^2 \log x + c$
$y = (x^2 / 2) \log x - x^2 / 4 + c$