Exercise 1.5 Page No: 1.49

1. Show that the following numbers are irrational.

(i) 1/√2

Solution:

Consider 1/√2 is a rational number
Let us assume 1/√2 = r where r is a rational number
On further calculation we get
1/r = √2
Since r is a rational number, 1/r = √2 is also a rational number
But we know that √2 is an irrational number
So our supposition is wrong.
Hence, 1/√2 is an irrational number.

(ii) 7√5

Solution:

Let's assume on the contrary that 7√5 is a rational number. Then, there exist positive integers a and b such that

7√5 = a/b where, a and b, are co-primes
⇒ √5 = a/7b
⇒ √5 is rational [∵ 7, a and b are integers ∴ a/7b is a rational number]

This contradicts the fact that √5 is irrational. So, our assumption is incorrect.
Hence, 7√5 is an irrational number.

(iii) 6 + √2

Solution:

Let's assume on the contrary that 6+√2 is a rational number. Then, there exist co prime positive integers a and b such that

6 + √2 = a/b
⇒ √2 = a/b – 6
⇒ √2 = (a – 6b)/b
⇒ √2 is rational [∵ a and b are integers ∴ (a-6b)/b is a rational number]

This contradicts the fact that √2 is irrational. So, our assumption is incorrect.
Hence, 6 + √2 is an irrational number.

(iv) 3 − √5

Solution:

Let's assume on the contrary that 3-√5 is a rational number. Then, there exist co prime positive integers a and b such that

3-√5 = a/b
⇒ √5 = a/b + 3
⇒ √5 = (a + 3b)/b
⇒ √5 is rational [∵ a and b are integers ∴ (a+3b)/b is a rational number]

This contradicts the fact that √5 is irrational. So, our assumption is incorrect.

Hence, 3-√5 is an irrational number.

2. Prove that the following numbers are irrationals.

(i) 2/√7

Solution:

Let's assume on the contrary that 2/√7 is a rational number. Then, there exist co-prime positive integers a and b such that

2/√7 = a/b
⇒ √7 = 2b/a
⇒ √7 is rational [∵ 2, a and b are integers ∴ 2b/a is a rational number]

This contradicts the fact that √7 is irrational. So, our assumption is incorrect.
Hence, 2/√7 is an irrational number.

(ii) 3/(2√5)

Solution:

Let's assume on the contrary that 3/(2√5) is a rational number. Then, there exist co – prime positive integers a and b such that

3/(2√5) = a/b
⇒ √5 = 3b/2a
⇒ √5 is rational [∵ 3, 2, a and b are integers ∴ 3b/2a is a rational number]

This contradicts the fact that √5 is irrational. So, our assumption is incorrect.
Hence, 3/(2√5) is an irrational number.

(iii) 4 + √2

Solution:

Let's assume on the contrary that 4 + √2 is a rational number. Then, there exist co prime positive integers a and b such that

4 + √2 = a/b
⇒ √2 = a/b - 4
⇒ √2 = (a - 4b)/b
⇒ √2 is rational [∵ a and b are integers ∴ (a - 4b)/b is a rational number]

This contradicts the fact that √2 is irrational. So, our assumption is incorrect.
Hence, 4 + √2 is an irrational number.

(iv) 5√2

Solution:

Let's assume on the contrary that 5√2 is a rational number. Then, there exist positive integers a and b such that

$5\sqrt{2} = a/b$ where, a and b, are co-primes
⇒ $\sqrt{2} = a/5b$
⇒ $\sqrt{2}$ is rational [∵ a and b are integers ∴ a/5b is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $5\sqrt{2}$ is an irrational number.

3. Show that $2 - \sqrt{3}$ is an irrational number.

Solution:

Let's assume on the contrary that $2 - \sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$2 - \sqrt{3} = a/b$
⇒ $\sqrt{3} = 2 - a/b$
⇒ $\sqrt{3} = (2b - a)/b$
⇒ $\sqrt{3}$ is rational [∵ a and b are integers ∴ (2b - a)/b is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.
Hence, $2 - \sqrt{3}$ is an irrational number.

4. Show that $3 + \sqrt{2}$ is an irrational number.

Solution:

Let's assume on the contrary that $3 + \sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that

$3 + \sqrt{2} = a/b$
⇒ $\sqrt{2} = a/b - 3$
⇒ $\sqrt{2} = (a - 3b)/b$
⇒ $\sqrt{2}$ is rational [∵ a and b are integers ∴ (a - 3b)/b is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $3 + \sqrt{2}$ is an irrational number.

5. Prove that $4 - 5\sqrt{2}$ is an irrational number.

Solution:

Let's assume on the contrary that $4 - 5\sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that

$4 - 5\sqrt{2} = a/b$
⇒ $5\sqrt{2} = 4 - a/b$
⇒ $\sqrt{2} = (4b - a)/(5b)$
⇒ $\sqrt{2}$ is rational [∵ 5, a and b are integers ∴ (4b - a)/5b is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $4 - 5\sqrt{2}$ is an irrational number.

6. Show that $5 - 2\sqrt{3}$ is an irrational number.

Solution:

Let's assume on the contrary that $5 - 2\sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$5 - 2\sqrt{3} = a/b$

⇒ $2\sqrt{3} = 5 - a/b$

⇒ $\sqrt{3} = (5b - a)/(2b)$

⇒ $\sqrt{3}$ is rational [∵ 2, a and b are integers ∴ $(5b - a)/2b$ is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $5 - 2\sqrt{3}$ is an irrational number.

7. Prove that $2\sqrt{3} - 1$ is an irrational number.

Solution:

Let's assume on the contrary that $2\sqrt{3} - 1$ is a rational number. Then, there exist co prime positive integers a and b such that

$2\sqrt{3} - 1 = a/b$

⇒ $2\sqrt{3} = a/b + 1$

⇒ $\sqrt{3} = (a + b)/(2b)$

⇒ $\sqrt{3}$ is rational [∵ 2, a and b are integers ∴ $(a + b)/2b$ is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $2\sqrt{3} - 1$ is an irrational number.

8. Prove that $2 - 3\sqrt{5}$ is an irrational number.

Solution:

Let's assume on the contrary that $2 - 3\sqrt{5}$ is a rational number. Then, there exist co prime positive integers a and b such that

$2 - 3\sqrt{5} = a/b$

⇒ $3\sqrt{5} = 2 - a/b$

⇒ $\sqrt{5} = (2b - a)/(3b)$

⇒ $\sqrt{5}$ is rational [∵ 3, a and b are integers ∴ $(2b - a)/3b$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.

Hence, $2 - 3\sqrt{5}$ is an irrational number.

9. Prove that $\sqrt{5} + \sqrt{3}$ is irrational.

Solution:

Let's assume on the contrary that $\sqrt{5} + \sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$\sqrt{5} + \sqrt{3} = a/b$

⇒ $\sqrt{5} = (a/b) - \sqrt{3}$

⇒ $(\sqrt{5})^2 = ((a/b) - \sqrt{3})^2$ [Squaring on both sides]

⇒ $5 = (a^2/b^2) + 3 - (2\sqrt{3}a/b)$

⇒ $(a^2/b^2) - 2 = (2\sqrt{3}a/b)$

⇒ $(a/b) - (2b/a) = 2\sqrt{3}$

⇒ $(a^2 - 2b^2)/2ab = \sqrt{3}$

⇒ $\sqrt{3}$ is rational [∵ a and b are integers ∴ $(a^2 - 2b^2)/2ab$ is rational]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{5} + \sqrt{3}$ is an irrational number.

10. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Solution:

Let's assume on the contrary that $\sqrt{2} + \sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$\sqrt{2} + \sqrt{3} = a/b$

⇒ $\sqrt{2} = (a/b) - \sqrt{3}$

⇒ $(\sqrt{2})^2 = ((a/b) - \sqrt{3})^2$ [Squaring on both sides]

⇒ $2 = (a^2/b^2) + 3 - (2\sqrt{3}a/b)$

⇒ $(a^2/b^2) + 1 = (2\sqrt{3}a/b)$

⇒ $(a/b) + (b/a) = 2\sqrt{3}$

⇒ $(a^2 + b^2)/2ab = \sqrt{3}$

⇒ $\sqrt{3}$ is rational [∵ a and b are integers ∴ $(a^2 + 2b^2)/2ab$ is rational]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{2} + \sqrt{3}$ is an irrational number.

11. Prove that for any prime positive integer p, $\sqrt{p}$ is an irrational number.

Solution:

Consider $\sqrt{p}$ as a rational number

Assume $\sqrt{p} = a/b$ where a and b are integers and $b \neq 0$

By squaring on both sides

$p = a^2/b^2$

$pb = a^2/b$

p and b are integers $pb= a^2/b$ will also be an integer

But we know that a^2/b is a rational number so our supposition is wrong

Therefore, $\sqrt{p}$ is an irrational number.

12. If p, q are prime positive integers, prove that $\sqrt{p} + \sqrt{q}$ is an irrational number.

Solution:

Let's assume on the contrary that $\sqrt{p} + \sqrt{q}$ is a rational number. Then, there exist co prime positive integers a and b such that

$\sqrt{p} + \sqrt{q} = a/b$

⇒ $\sqrt{p} = (a/b) - \sqrt{q}$

⇒ $(\sqrt{p})^2 = ((a/b) - \sqrt{q})^2$ [Squaring on both sides]

⇒ $p = (a^2/b^2) + q - (2\sqrt{q}\ a/b)$

⇒ $(a^2/b^2) - (p+q) = (2\sqrt{q}\ a/b)$

⇒ $(a/b) - ((p+q)b/a) = 2\sqrt{q}$

⇒ $(a^2 - b^2(p+q))/2ab = \sqrt{q}$

⇒ $\sqrt{q}$ is rational [∵ a and b are integers ∴ $(a^2 - b^2(p+q))/2ab$ is rational]

This contradicts the fact that $\sqrt{q}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{p} + \sqrt{q}$ is an irrational number.