

EXERCISE 6.6

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1. Write the following squares of binomials as trinomials:

(i) $(x + 2)^2$

(ii) $(8a + 3b)^2$

(iii) $(2m + 1)^2$

(iv) $(9a + 1/6)^2$

(v) $(x + x^2/2)^2$

(vi) $(x/4 - y/3)^2$

(vii) $(3x - 1/3x)^2$

(viii) $(x/y - y/x)^2$

(ix) $(3a/2 - 5b/4)^2$

(x) $(a^2b - bc^2)^2$

(xi) $(2a/3b + 2b/3a)^2$

(xii) $(x^2 - ay)^2$

Solution:

(i) $(x + 2)^2$

Let us express the given expression in trinomial

$$x^2 + 2(x)(2) + 2^2$$

$$x^2 + 4x + 4$$

(ii) $(8a + 3b)^2$

Let us express the given expression in trinomial

$$(8a)^2 + 2(8a)(3b) + (3b)^2$$

$$64a^2 + 48ab + 9b^2$$

(iii) $(2m + 1)^2$

Let us express the given expression in trinomial

$$(2m)^2 + 2(2m)(1) + 1^2$$

$$4m^2 + 4m + 1$$

(iv) $(9a + 1/6)^2$

Let us express the given expression in trinomial

$$(9a)^2 + 2(9a)(1/6) + (1/6)^2$$

$$81a^2 + 3a + 1/36$$

(v) $(x + x^2/2)^2$

Let us express the given expression in trinomial

$$(x)^2 + 2(x)(x^2/2) + (x^2/2)^2$$

$$x^2 + x^3 + 1/4x^4$$

(vi) $(x/4 - y/3)^2$

Let us express the given expression in trinomial

$$(x/4)^2 - 2(x/4)(y/3) + (y/3)^2$$
$$1/16x^2 - xy/6 + 1/9y^2$$

(vii) $(3x - 1/3x)^2$

Let us express the given expression in trinomial

$$(3x)^2 - 2(3x)(1/3x) + (1/3x)^2$$
$$9x^2 - 2 + 1/9x^2$$

(viii) $(x/y - y/x)^2$

Let us express the given expression in trinomial

$$(x/y)^2 - 2(x/y)(y/x) + (y/x)^2$$
$$x^2/y^2 - 2 + y^2/x^2$$

(ix) $(3a/2 - 5b/4)^2$

Let us express the given expression in trinomial

$$(3a/2)^2 - 2(3a/2)(5b/4) + (5b/4)^2$$
$$9/4a^2 - 15/4ab + 25/16b^2$$

(x) $(a^2b - bc^2)^2$

Let us express the given expression in trinomial

$$(a^2b)^2 - 2(a^2b)(bc^2) + (bc^2)^2$$
$$a^4b^2 - 2a^2b^2c^2 + b^2c^4$$

(xi) $(2a/3b + 2b/3a)^2$

Let us express the given expression in trinomial

$$(2a/3b)^2 + 2(2a/3b)(2b/3a) + (2b/3a)^2$$
$$4a^2/9b^2 + 8/9 + 4b^2/9a^2$$

(xii) $(x^2 - ay)^2$

Let us express the given expression in trinomial

$$(x^2)^2 - 2(x^2)(ay) + (ay)^2$$
$$x^4 - 2x^2ay + a^2y^2$$

2. Find the product of the following binomials:

(i) $(2x + y)(2x + y)$

(ii) $(a + 2b)(a - 2b)$

(iii) $(a^2 + bc)(a^2 - bc)$

(iv) $(4x/5 - 3y/4)(4x/5 + 3y/4)$

(v) $(2x + 3/y)(2x - 3/y)$

(vi) $(2a^3 + b^3)(2a^3 - b^3)$

(vii) $(x^4 + 2/x^2)(x^4 - 2/x^2)$

(viii) $(x^3 + 1/x^3)(x^3 - 1/x^3)$

Solution:

(i) $(2x + y)(2x + y)$

Let us find the product of the given expression

$$2x(2x + y) + y(2x + y)$$

$$4x^2 + 2xy + 2xy + y^2$$

$$4x^2 + 4xy + y^2$$

(ii) $(a + 2b)(a - 2b)$

Let us find the product of the given expression

$$a(a - 2b) + 2b(a - 2b)$$

$$a^2 - 2ab + 2ab - 4b^2$$

$$a^2 - 4b^2$$

(iii) $(a^2 + bc)(a^2 - bc)$

Let us find the product of the given expression

$$a^2(a^2 - bc) + bc(a^2 - bc)$$

$$a^4 - a^2bc + bca^2 - b^2c^2$$

$$a^4 - b^2c^2$$

(iv) $(4x/5 - 3y/4)(4x/5 + 3y/4)$

Let us find the product of the given expression

$$4x/5(4x/5 + 3y/4) - 3y/4(4x/5 + 3y/4)$$

$$16/25x^2 + 12/20yx - 12/20xy - 9y^2/16$$

$$16/25x^2 - 9/16y^2$$

(v) $(2x + 3/y)(2x - 3/y)$

Let us find the product of the given expression

$$2x(2x - 3/y) + 3/y(2x - 3/y)$$

$$4x^2 - 6x/y + 6x/y - 9/y^2$$

$$4x^2 - 9/y^2$$

(vi) $(2a^3 + b^3)(2a^3 - b^3)$

Let us find the product of the given expression

$$2a^3(2a^3 - b^3) + b^3(2a^3 - b^3)$$

$$4a^6 - 2a^3b^3 + 2a^3b^3 - b^6$$

$$4a^6 - b^6$$

(vii) $(x^4 + 2/x^2)(x^4 - 2/x^2)$

Let us find the product of the given expression

$$x^4(x^4 - 2/x^2) + 2/x^2(x^4 - 2/x^2)$$

$$x^8 - 2x^2 + 2x^2 - 4/x^4$$

$$(x^8 - 4/x^4)$$

(viii) $(x^3 + 1/x^3)(x^3 - 1/x^3)$

Let us find the product of the given expression

$$x^3(x^3 - 1/x^3) + 1/x^3(x^3 - 1/x^3)$$

$$x^6 - 1 + 1 - 1/x^6$$

$$x^6 - 1/x^6$$

3. Using the formula for squaring a binomial, evaluate the following:

(i) $(102)^2$

(ii) $(99)^2$

(iii) $(1001)^2$

(iv) $(999)^2$

(v) $(703)^2$

Solution:

(i) $(102)^2$

We can express 102 as $100 + 2$

So, $(102)^2 = (100 + 2)^2$

Upon simplification we get,

$$(100 + 2)^2 = (100)^2 + 2(100)(2) + 2^2$$

$$= 10000 + 400 + 4$$

$$= 10404$$

(ii) $(99)^2$

We can express 99 as $100 - 1$

So, $(99)^2 = (100 - 1)^2$

Upon simplification we get,

$$(100 - 1)^2 = (100)^2 - 2(100)(1) + 1^2$$

$$= 10000 - 200 + 1$$

$$= 9801$$

(iii) $(1001)^2$

We can express 1001 as $1000 + 1$

So, $(1001)^2 = (1000 + 1)^2$

Upon simplification we get,

$$\begin{aligned}(1000 + 1)^2 &= (1000)^2 + 2(1000)(1) + 1^2 \\ &= 1000000 + 2000 + 1 \\ &= 1002001\end{aligned}$$

(iv) $(999)^2$

We can express 999 as $1000 - 1$

So, $(999)^2 = (1000 - 1)^2$

Upon simplification we get,

$$\begin{aligned}(1000 - 1)^2 &= (1000)^2 - 2(1000)(1) + 1^2 \\ &= 1000000 - 2000 + 1 \\ &= 998001\end{aligned}$$

(v) $(703)^2$

We can express 703 as $700 + 3$

So, $(703)^2 = (700 + 3)^2$

Upon simplification we get,

$$\begin{aligned}(700 + 3)^2 &= (700)^2 + 2(700)(3) + 3^2 \\ &= 490000 + 4200 + 9 \\ &= 494209\end{aligned}$$

4. Simplify the following using the formula: $(a - b)(a + b) = a^2 - b^2$:

(i) $(82)^2 - (18)^2$

(ii) $(467)^2 - (33)^2$

(iii) $(79)^2 - (69)^2$

(iv) 197×203

(v) 113×87

(vi) 95×105

(vii) 1.8×2.2

(viii) 9.8×10.2

Solution:

(i) $(82)^2 - (18)^2$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}(82)^2 - (18)^2 &= (82 - 18)(82 + 18) \\ &= 64 \times 100 \\ &= 6400\end{aligned}$$

(ii) $(467)^2 - (33)^2$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}(467)^2 - (33)^2 &= (467 - 33)(467 + 33) \\ &= (434)(500) \\ &= 217000\end{aligned}$$

(iii) $(79)^2 - (69)^2$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}(79)^2 - (69)^2 &= (79 + 69)(79 - 69) \\ &= (148)(10) \\ &= 1480\end{aligned}$$

(iv) 197×203

We can express 203 as $200 + 3$ and 197 as $200 - 3$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}197 \times 203 &= (200 - 3)(200 + 3) \\ &= (200)^2 - (3)^2 \\ &= 40000 - 9 \\ &= 39991\end{aligned}$$

(v) 113×87

We can express 113 as $100 + 13$ and 87 as $100 - 13$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}113 \times 87 &= (100 + 13)(100 - 13) \\ &= (100)^2 - (13)^2 \\ &= 10000 - 169 \\ &= 9831\end{aligned}$$

(vi) 95×105

We can express 95 as $100 - 5$ and 105 as $100 + 5$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}95 \times 105 &= (100 - 5)(100 + 5) \\ &= (100)^2 - (5)^2 \\ &= 10000 - 25 \\ &= 9975\end{aligned}$$

(vii) 1.8×2.2

We can express 1.8 as $2 - 0.2$ and 2.2 as $2 + 0.2$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}1.8 \times 2.2 &= (2 - 0.2)(2 + 0.2) \\ &= (2)^2 - (0.2)^2 \\ &= 4 - 0.04 \\ &= 3.96\end{aligned}$$

(viii) 9.8×10.2

We can express 9.8 as $10 - 0.2$ and 10.2 as $10 + 0.2$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}9.8 \times 10.2 &= (10 - 0.2)(10 + 0.2) \\ &= (10)^2 - (0.2)^2 \\ &= 100 - 0.04 \\ &= 99.96\end{aligned}$$

5. Simplify the following using the identities:

(i) $((58)^2 - (42)^2)/16$

(ii) $178 \times 178 - 22 \times 22$

(iii) $(198 \times 198 - 102 \times 102)/96$

(iv) $1.73 \times 1.73 - 0.27 \times 0.27$

(v) $(8.63 \times 8.63 - 1.37 \times 1.37)/0.726$

Solution:

(i) $((58)^2 - (42)^2)/16$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}((58)^2 - (42)^2)/16 &= ((58-42)(58+42))/16 \\ &= ((16)(100))/16 \\ &= 100\end{aligned}$$

(ii) $178 \times 178 - 22 \times 22$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}178 \times 178 - 22 \times 22 &= (178)^2 - (22)^2 \\ &= (178-22)(178+22) \\ &= 200 \times 156 \\ &= 31200\end{aligned}$$

(iii) $(198 \times 198 - 102 \times 102)/96$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}(198 \times 198 - 102 \times 102)/96 &= ((198)^2 - (102)^2)/96 \\ &= ((198-102)(198+102))/96 \\ &= (96 \times 300)/96 \\ &= 300\end{aligned}$$

(iv) $1.73 \times 1.73 - 0.27 \times 0.27$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}1.73 \times 1.73 - 0.27 \times 0.27 &= (1.73)^2 - (0.27)^2 \\ &= (1.73-0.27)(1.73+0.27) \\ &= 1.46 \times 2 \\ &= 2.92\end{aligned}$$

(v) $(8.63 \times 8.63 - 1.37 \times 1.37)/0.726$

Let us simplify the given expression using the formula $(a - b)(a + b) = a^2 - b^2$

We get,

$$\begin{aligned}(8.63 \times 8.63 - 1.37 \times 1.37)/0.726 &= ((8.63)^2 - (1.37)^2)/0.726 \\ &= ((8.63-1.37)(8.63+1.37))/0.726 \\ &= (7.26 \times 10)/0.726 \\ &= 72.6/0.726 \\ &= 100\end{aligned}$$

6. Find the value of x, if:

(i) $4x = (52)^2 - (48)^2$

(ii) $14x = (47)^2 - (33)^2$

(iii) $5x = (50)^2 - (40)^2$

Solution:

(i) $4x = (52)^2 - (48)^2$

Let us simplify to find the value of x by using the formula $(a - b)(a + b) = a^2 - b^2$

$$\begin{aligned}4x &= (52)^2 - (48)^2 \\4x &= (52 - 48)(52 + 48) \\4x &= 4 \times 100 \\4x &= 400 \\x &= 100\end{aligned}$$

(ii) $14x = (47)^2 - (33)^2$
Let us simplify to find the value of x by using the formula $(a - b)(a + b) = a^2 - b^2$

$$\begin{aligned}14x &= (47)^2 - (33)^2 \\14x &= (47 - 33)(47 + 33) \\14x &= 14 \times 80 \\x &= 80\end{aligned}$$

(iii) $5x = (50)^2 - (40)^2$
Let us simplify to find the value of x by using the formula $(a - b)(a + b) = a^2 - b^2$

$$\begin{aligned}5x &= (50)^2 - (40)^2 \\5x &= (50 - 40)(50 + 40) \\5x &= 10 \times 90 \\5x &= 900 \\x &= 180\end{aligned}$$

7. If $x + 1/x = 20$, find the value of $x^2 + 1/x^2$.

Solution:

We know that $x + 1/x = 20$

So when squaring both sides, we get

$$\begin{aligned}(x + 1/x)^2 &= (20)^2 \\x^2 + 2 \times x \times 1/x + (1/x)^2 &= 400 \\x^2 + 2 + 1/x^2 &= 400 \\x^2 + 1/x^2 &= 398\end{aligned}$$

8. If $x - 1/x = 3$, find the values of $x^2 + 1/x^2$ and $x^4 + 1/x^4$.

Solution:

We know that $x - 1/x = 3$

So when squaring both sides, we get

$$\begin{aligned}(x - 1/x)^2 &= (3)^2 \\x^2 - 2 \times x \times 1/x + (1/x)^2 &= 9 \\x^2 - 2 + 1/x^2 &= 9 \\x^2 + 1/x^2 &= 9 + 2 \\x^2 + 1/x^2 &= 11\end{aligned}$$

Now again when we square on both sides we get,

$$(x^2 + 1/x^2)^2 = (11)^2$$

$$x^4 + 2 \times x^2 \times 1/x^2 + (1/x^2)^2 = 121$$

$$x^4 + 2 + 1/x^4 = 121$$

$$x^4 + 1/x^4 = 121 - 2$$

$$x^4 + 1/x^4 = 119$$

$$\therefore x^2 + 1/x^2 = 11$$

$$x^4 + 1/x^4 = 119$$

9. If $x^2 + 1/x^2 = 18$, find the values of $x + 1/x$ and $x - 1/x$.

Solution:

We know that $x^2 + 1/x^2 = 18$

When adding 2 on both sides, we get

$$x^2 + 1/x^2 + 2 = 18 + 2$$

$$x^2 + 1/x^2 + 2 \times x \times 1/x = 20$$

$$(x + 1/x)^2 = 20$$

$$x + 1/x = \sqrt{20}$$

When subtracting 2 from both sides, we get

$$x^2 + 1/x^2 - 2 \times x \times 1/x = 18 - 2$$

$$(x - 1/x)^2 = 16$$

$$x - 1/x = \sqrt{16}$$

$$x - 1/x = 4$$

10. If $x + y = 4$ and $xy = 2$, find the value of $x^2 + y^2$

Solution:

We know that $x + y = 4$ and $xy = 2$

Upon squaring on both sides of the given expression, we get

$$(x + y)^2 = 4^2$$

$$x^2 + y^2 + 2xy = 16$$

$$x^2 + y^2 + 2(2) = 16 \quad (\text{since } xy=2)$$

$$x^2 + y^2 + 4 = 16$$

$$x^2 + y^2 = 16 - 4$$

$$x^2 + y^2 = 12$$

11. If $x - y = 7$ and $xy = 9$, find the value of $x^2 + y^2$

Solution:

We know that $x - y = 7$ and $xy = 9$

Upon squaring on both sides of the given expression, we get

$$\begin{aligned}(x - y)^2 &= 7^2 \\ x^2 + y^2 - 2xy &= 49 \\ x^2 + y^2 - 2(9) &= 49 \quad (\text{since } xy=9) \\ x^2 + y^2 - 18 &= 49 \\ x^2 + y^2 &= 49 + 18 \\ x^2 + y^2 &= 67\end{aligned}$$

12. If $3x + 5y = 11$ and $xy = 2$, find the value of $9x^2 + 25y^2$

Solution:

We know that $3x + 5y = 11$ and $xy = 2$

Upon squaring on both sides of the given expression, we get

$$\begin{aligned}(3x + 5y)^2 &= 11^2 \\ (3x)^2 + (5y)^2 + 2(3x)(5y) &= 121 \\ 9x^2 + 25y^2 + 2(15xy) &= 121 \quad (\text{since } xy=2) \\ 9x^2 + 25y^2 + 2(15(2)) &= 121 \\ 9x^2 + 25y^2 + 60 &= 121 \\ 9x^2 + 25y^2 &= 121 - 60 \\ 9x^2 + 25y^2 &= 61\end{aligned}$$

13. Find the values of the following expressions:

- (i) $16x^2 + 24x + 9$ when $x = 7/4$
- (ii) $64x^2 + 81y^2 + 144xy$ when $x = 11$ and $y = 4/3$
- (iii) $81x^2 + 16y^2 - 72xy$ when $x = 2/3$ and $y = 3/4$

Solution:

(i) $16x^2 + 24x + 9$ when $x = 7/4$

Let us find the values using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$\begin{aligned}(4x)^2 + 2(4x)(3) + 3^2 \\ (4x + 3)^2\end{aligned}$$

Evaluating when $x = 7/4$

$$\begin{aligned}[4(7/4) + 3]^2 \\ (7 + 3)^2 \\ 100\end{aligned}$$

(ii) $64x^2 + 81y^2 + 144xy$ when $x = 11$ and $y = 4/3$

Let us find the values using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$(8x)^2 + 2(8x)(9y) + (9y)^2 \quad (8x + 9y)$$

Evaluating when $x = 11$ and $y = 4/3$

$$\begin{aligned}[8(11) + 9(4/3)]^2 \\ (88 + 12)^2\end{aligned}$$

$$(100)^2$$

$$10000$$

(iii) $81x^2 + 16y^2 - 72xy$ when $x = 2/3$ and $y = 3/4$

Let us find the values using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$(9x)^2 + (4y)^2 - 2(9x)(4y)$$

$$(9x - 4y)^2$$

Putting $x = 2/3$ and $y = 3/4$

$$[9(2/3) - 4(3/4)]^2$$

$$(6 - 3)^2$$

$$3^2$$

$$9$$

14. If $x + 1/x = 9$ find the value of $x^4 + 1/x^4$.

Solution:

We know that $x + 1/x = 9$

So when squaring both sides, we get

$$(x + 1/x)^2 = (9)^2$$

$$x^2 + 2 \times x \times 1/x + (1/x)^2 = 81$$

$$x^2 + 2 + 1/x^2 = 81$$

$$x^2 + 1/x^2 = 81 - 2$$

$$x^2 + 1/x^2 = 79$$

Now again when we square on both sides we get,

$$(x^2 + 1/x^2)^2 = (79)^2$$

$$x^4 + 2 \times x^2 \times 1/x^2 + (1/x^2)^2 = 6241$$

$$x^4 + 2 + 1/x^4 = 6241$$

$$x^4 + 1/x^4 = 6241 - 2$$

$$x^4 + 1/x^4 = 6239$$

$$\therefore x^4 - 1/x^4 = 6239$$

15. If $x + 1/x = 12$ find the value of $x - 1/x$.

Solution:

We know that $x + 1/x = 12$

So when squaring both sides, we get

$$(x + 1/x)^2 = (12)^2$$

$$x^2 + 2 \times x \times 1/x + (1/x)^2 = 144$$

$$x^2 + 2 + 1/x^2 = 144$$

$$x^2 + 1/x^2 = 144 - 2$$

$$x^2 + 1/x^2 = 142$$

When subtracting 2 from both sides, we get

$$x^2 + 1/x^2 - 2 \times x \times 1/x = 142 - 2$$

$$(x - 1/x)^2 = 140$$

$$x - 1/x = \sqrt{140}$$

16. If $2x + 3y = 14$ and $2x - 3y = 2$, find value of xy . [Hint: Use $(2x+3y)^2 - (2x-3y)^2 = 24xy$]

Solution:

We know that the given equations are

$$2x + 3y = 14 \dots \text{equation (1)}$$

$$2x - 3y = 2 \dots \text{equation (2)}$$

Now, let us square both the equations and subtract equation (2) from equation (1), we get,

$$(2x + 3y)^2 - (2x - 3y)^2 = (14)^2 - (2)^2$$

$$4x^2 + 9y^2 + 12xy - 4x^2 - 9y^2 + 12xy = 196 - 4$$

$$24xy = 192$$

$$xy = 8$$

\therefore the value of xy is 8.

17. If $x^2 + y^2 = 29$ and $xy = 2$, find the value of

(i) $x + y$

(ii) $x - y$

(iii) $x^4 + y^4$

Solution:

(i) $x + y$

We know that

$$x^2 + y^2 = 29$$

$$x^2 + y^2 + 2xy - 2xy = 29$$

$$(x + y)^2 - 2(2) = 29$$

$$(x + y)^2 = 29 + 4$$

$$x + y = \pm \sqrt{33}$$

(ii) $x - y$

We know that

$$x^2 + y^2 = 29$$

$$x^2 + y^2 + 2xy - 2xy = 29$$

$$(x - y)^2 + 2(2) = 29$$

$$(x - y)^2 + 4 = 29$$

$$(x - y)^2 = 25$$

$$(x - y) = \pm 5$$

(iii) $x^4 + y^4$

We know that

$$x^2 + y^2 = 29$$

Squaring both sides, we get

$$(x^2 + y^2)^2 = (29)^2$$

$$x^4 + y^4 + 2x^2y^2 = 841$$

$$x^4 + y^4 + 2(2)^2 = 841$$

$$x^4 + y^4 = 841 - 8$$

$$x^4 + y^4 = 833$$

18. What must be added each of the following expression to make it a whole square?

(i) $4x^2 - 12x + 7$

(ii) $4x^2 - 20x + 20$

Solution:

(i) $4x^2 - 12x + 7$

$$(2x)^2 - 2(2x)(3) + 3^2 - 3^2 + 7$$

$$(2x - 3)^2 - 9 + 7$$

$$(2x - 3)^2 - 2$$

∴ 2 must be added to the expression to make it a whole square.

(ii) $4x^2 - 20x + 20$

$$(2x)^2 - 2(2x)(5) + 5^2 - 5^2 + 20$$

$$(2x - 5)^2 - 25 + 20$$

$$(2x - 5)^2 - 5$$

∴ 5 must be added to the expression to make it a whole square.

19. Simplify:

(i) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

(ii) $(2x - 1)(2x + 1)(4x^2 + 1)(16x^4 + 1)$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(v) $(m^2 - n^2m)^2 + 2m^3n^2$

Solution:

(i) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

B7 grouping the values

$$(x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$$

$$[(x^2)^2 - (y^2)^2] (x^4 + y^4)$$

$$(x^4 - y^4) (x^4 - y^4)$$

$$[(x^4)^2 - (y^4)^2]$$

$$x^8 - y^8$$

(ii) $(2x - 1) (2x + 1) (4x^2 + 1) (16x^4 + 1)$

Let us simplify the expression by grouping

$$[(2x)^2 - (1)^2] (4x^2 + 1) (16x^4 + 1)$$

$$(4x^2 - 1) (4x^2 + 1) (16x^4 + 1) 1$$

$$[(4x^2)^2 - (1)^2] (16x^4 + 1) 1$$

$$(16x^4 - 1) (16x^4 + 1) 1$$

$$[(16x^4)^2 - (1)^2] 1$$

$$256x^8 - 1$$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

Upon expansion

$$(7m)^2 + (8n)^2 - 2(7m)(8n) + (7m)^2 + (8n)^2 + 2(7m)(8n)$$

$$(7m)^2 + (8n)^2 - 112mn + (7m)^2 + (8n)^2 + 112mn$$

$$49m^2 + 64n^2 + 49m^2 + 64n^2$$

By grouping the similar expression we get,

$$98m^2 + 64n^2 + 64n^2$$

$$98m^2 + 128n^2$$

(iv) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

Upon expansion

$$(2.5p)^2 + (1.5q)^2 - 2(2.5p)(1.5q) - (1.5p)^2 - (2.5q)^2 + 2(1.5p)(2.5q)$$

$$6.25p^2 + 2.25q^2 - 2.25p^2 - 6.25q^2$$

By grouping the similar expression we get,

$$4p^2 - 6.25q^2 + 2.25q^2$$

$$4p^2 - 4q^2$$

$$4(p^2 - q^2)$$

(v) $(m^2 - n^2m)^2 + 2m^3n^2$

Upon expansion using $(a + b)^2$ formula

$$(m^2)^2 - 2(m^2)(n^2)(m) + (n^2m)^2 + 2m^3n^2$$

$$m^4 - 2m^3n^2 + (n^2m)^2 + 2m^3n^2$$

$$m^4 + n^4m^2 - 2m^3n^2 + 2m^3n^2$$

$$m^4 + m^2n^4$$

20. Show that:

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$

(ii) $(9a - 5b)^2 + 180ab = (9a + 5b)^2$

(iii) $(4m/3 - 3n/4)^2 + 2mn = 16m^2/9 + 9n^2/16$

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Solution:

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$

Let us consider LHS $(3x + 7)^2 - 84x$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$(3x)^2 + (7)^2 + 2(3x)(7) - 84x$$

$$(3x)^2 + (7)^2 + 42x - 84x$$

$$(3x)^2 + (7)^2 - 42x$$

$$(3x)^2 + (7)^2 - 2(3x)(7)$$

$$(3x - 7)^2 = \text{R.H.S}$$

Hence, proved

(ii) $(9a - 5b)^2 + 180ab = (9a + 5b)^2$

Let us consider LHS $(9a - 5b)^2 + 180ab$

By using the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$(9a)^2 + (5b)^2 - 2(9a)(5b) + 180ab$$

$$(9a)^2 + (5b)^2 - 90ab + 180ab$$

$$(9a)^2 + (5b)^2 + 9ab$$

$$(9a)^2 + (5b)^2 + 2(9a)(5b)$$

$$(9a + 5b)^2 = \text{R.H.S}$$

Hence, proved

(iii) $(4m/3 - 3n/4)^2 + 2mn = 16m^2/9 + 9n^2/16$

Let us consider LHS $(4m/3 - 3n/4)^2 + 2mn$

$$(4m/3)^2 + (3n/4)^2 - 2mn + 2mn$$

$$(4m/3)^2 + (3n/4)^2$$

$$16/9m^2 + 9/16n^2 = \text{R.H.S}$$

Hence, proved

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

Let us consider LHS $(4pq + 3q)^2 - (4pq - 3q)^2$

$$(4pq)^2 + (3q)^2 + 2(4pq)(3q) - (4pq)^2 - (3q)^2 + 2(4pq)(3q)$$

$$24pq^2 + 24pq^2$$

$$48pq^2 = \text{RHS}$$

Hence, proved

$$(v) (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Let us consider LHS $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$

By using the identity $(a - b)(a + b) = a^2 - b^2$

We get,

$$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)$$

$$a^2 - b^2 + b^2 - c^2 + c^2 - a^2$$

$$0 = \text{R.H.S}$$

Hence, proved

