TN Board Class 12th Maths Question Paper With Solutions 2019

QUESTION PAPER CODE 1312

PART - I

Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer. [20 * 1 = 20]

Question 1: The percentage error in the 11th root of the number 28 is approximately ______ times the percentage error in 28.

(1) 11

(2) 28

(3) 1 / 28

(4) 1 / 11

Solution: (4) Let $y = x^{1/11}$ Taking log on both sides log $y = (1 / 11) \log x$ dy / y = (1 / 11) (dx / x)dy / y * 100 = (1 / 11) (dx / x) * 100The percentage error in x = 28 is dx / x * 100 = 28The percentage error in y = (dy / y) * 100 = (1 / 11) * 28

Question 2: The line 5x - 2y + 4k = 0 is a tangent to $4x^2 - y^2 = 36$, then k is:

(1) 9 / 4 (2) 81 / 16 (3) 4 / 9 (4) 2 / 3

Solution: (1) The equation of the hyperbola is $4x^2 - y^2 = 36$ $4x^2 / 36 - y^2 / 36 = 1$ $4x^2 / 9 - y^2 / 36 = 1 ---- (1)$ $a^2 = 9$ $b^2 = 36$ The equation of the line is 5x - 2y + 4k = 0 2y = 5x + 4k y = 5 / 2 + 2k ---- (2)From equation (2) m = 5 / 2 c = 2k $c^2 = a^2 m^2 - b^2$ $(2k)^2 = 9 (5 / 2)^2 - 36$ $4k^2 = 9 \times (25 / 4) - 36$ $4k^2 = (225 - 144) / 4$ $k^2 = 81 / 16$ => k = 9 / 4

Question 3: In the multiplicative group of the cube root of unity, the order of ω^2 is : [ω is a complex cube root of unity]

(1) 2 (2) 1 (3) 4 (4) 3

Solution: (4) In the cube root of unity $\omega^3 = 1$ $1 + \omega + \omega^2 = 0$ $(\omega^2)^2 = \omega^4 = \omega^3 . \omega = \omega$ $(\omega^2)^3 = \omega^6 = (\omega^3)^2 = (1)^2 = 1$ $0 (\omega^2) = 3$

Question 4: If f (x) and g (x) are two functions as defined in Generalized law of mean then Lagrange's law of mean is a particular case of Generalised law of mean for:

(1) f'(x) = 0 (2) g'(x) = 0 (3) g (x) is an identity function(4) f (x) is an identity function

Solution: (3)

Question 5: If -x - iy lies in the first quadrant, then -ix + y lies in the:

(1) third quadrant(3) first quadrant

(2) fourth quadrant

(4) second quadrant

Solution: (4)

Question 6: Which of the following is a tautology?

(1) $p \lor (\sim p)$ (2) $p \land (\sim p)$ (3) $p \lor q$ (4) $p \land q$

Solution: (1)

Question 7: Variance of the random variable X is 4. Its mean is 2. Then E (X²) is:

(1) 6 (2) 8 (3) 2 (4) 4

Solution: (2) Variance = E (X²) - (E (X))² 4 = E (X²) - 2²4 + 4 = E (X²)

 $8 = E(X^2)$

Question 8: r = si - tk is the equation of :

(1) yz - plane
(2) xz - plane
(3) a straight line joining the points i and k
(4) xy - plane

Solution: (2)

Question 9: Which one of the following statements is true about the curve $y = x^{1/3}$?

(1) The curve has a point of inflection in which y' does not exist

- (2) The curve has more than one point of inflection
- (3) The curve has no point of inflection
- (4) The curve has a point of inflection in which y'' = 0

Solution: (1)

Question 10: If $z_1 = 1 + 2i$, $z_2 = 1 - 3i$ and $z_3 = 2 + 4i$ then, the points on the Argand diagram representing $z_1 z_2 z_3$, $2 z_1 z_2 z_3$, $-7 z_1 z_2 z_3$ are:

(1) Vertices of an isosceles triangle

(2) Collinear

(3) Vertices of a right-angled triangle

(4) Vertices of an equilateral triangle

Solution: (2)

Question 11: In the homogeneous system $\rho(A)$ is less than the number of unknowns, then the system has:

- (1) only non-trivial solutions
- (2) no solution
- (3) only trivial solution
- (4) trivial solution and infinitely many non-trivial solutions

Solution: (4)

Question 12: $y = cx - c^2$ is the general solution of the differential equation:

(1)
$$y' = c$$

(2) $(y')^2 + xy' + y = 0$
(3) $(y')^2 - xy' + y = 0$
(4) $y'' = 0$

Solution: (3)

Question 13: The order and degree of the differential equation $y' + (y'')^2 = x (x + y'')^2$ are:

(1) 1, 2 (2) 1, 1 (3) 2, 2 (4) 2, 1

Solution: (3)

Question 14: The value of $\int_0^{\pi/2} [\tan x - \cot x] / [1 + \tan x \cot x]$ is:

(1) $\pi / 4$ (2) π (3) $\pi / 2$ (4) 0

Solution: (4)

$$\begin{split} & \int_{0}^{\frac{\pi}{2}} \frac{\tan(x) - \cot(x)}{1 + \tan(x)\cot(x)} dx \\ & \frac{\frac{\sin(x)}{\cos(x)} - \frac{\cos(x)}{\sin(x)}}{1 + \frac{\sin(x)}{\cos(x)} \cdot \frac{\cos(x)}{\sin(x)}} : \frac{\sin^2(x) - \cos^2(x)}{2\cos(x)\sin(x)} \\ &= \int_{0}^{\frac{\pi}{2}} \frac{\sin^2(x) - \cos^2(x)}{2\cos(x)\sin(x)} dx \\ &= \frac{1}{2} \cdot \int_{0}^{\frac{\pi}{2}} \frac{\sin^2(x) - \cos^2(x)}{\cos(x)\sin(x)} dx \\ &= \frac{1}{2} \cdot \int_{0}^{\frac{\pi}{2}} \frac{\sin(x)}{\cos(x)} - \frac{\cos(x)}{\sin(x)} dx \\ &= \frac{1}{2} \left(\int_{0}^{\frac{\pi}{2}} \frac{\sin(x)}{\cos(x)} dx - \int_{0}^{\frac{\pi}{2}} \frac{\cos(x)}{\sin(x)} dx \right) \\ &= \text{diverges} \end{split}$$

Question 15: In a Poisson distribution if P (X = 2) = P (X = 3) then, the value of its parameter λ is:

(1) 3 (2) 0 (3) 6 (4) 2

Solution: (1)

Question 16: The surface area of the solid of revolution of the region bounded by $x^2 + y^2 = 4$, x = -2 and x = 2 about x-axis is:

(1) 64π (2) 32π (3) 8π (4) 16π

Solution: (4)

Question 17: If a + b + c = 0, |a| = 3, |b| = 4, |c| = 5 then, the angle between a and b is:

(1) $5\pi/3$ (2) $\pi/2$ (3) $\pi/6$ (4) $2\pi/3$

Solution: (2)

a + b + c = 0 |a| = 3 |b| = 4 |c| = 5 $\theta = \cos^{-1} [(a \cdot b) / |a| |b|]$ a + b + c = 0 a + b = -c $(a + b)^{2} = (-c)^{2}$ $|a|^{2} + |b|^{2} + 2ab = |c|^{2}$ $3^{2} + 4^{2} + 2ab = 5^{2}$ 9 + 16 + 2ab = 25 2ab = 0 ab = 0 $\theta = \cos^{-1} [0 / 3 \cdot 4]$ $\theta = \cos^{-1} [0]$ $\theta = \pi / 2$

Question 18: The tangents at the end of any focal chord to the parabola $y^2 = 12x$ intersect on the line:

(1) y + 3 = 0 (2) y - 3 = 0 (3) x - 3 = 0 (4) x + 3 = 0

Solution: (4)

The tangent at the end of any focal chord to the parabola intersects on the directrix. The equation of the parabola is $y^2 = 12x$ $y^2 = 4$ (3) x => a = 3The equation of the directrix is x = -3

x + 3 = 0

Question 19: If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is:

(1)(1/k)) I ((2) kI	(3) $[1/k^2]$ I	(4)	$1 / k^{3}$
(1)(1)	/			· · ·	′ L * ′ ** J *

Solution: (1)

Question 20: The surface area of a sphere when the volume is increasing at the same rate as its radius, is:

(1) 4π (2) $4\pi/3$ (3) 1 (4) $1/2\pi$

Solution: (3)

Let S be the surface area, V be the volume, r be the radius of a sphere at time t Given dv / dt = dr / dt $V = (4 / 3) \pi r^3$ Surface area S = $4\pi r^2$ $V = (4 / 3) \pi r^3$ $dV / dt = (4 / 3) 3\pi^{2} (dr / dt)$ $dV / dt = 4\pi^{2} (dV / dt)$ 1 = S => S = 1Surface area = 1

PART - II

Answer any seven questions.

[7 * 2 = 14]

Question 21: To find the number of coins, in each category, write the suitable system of equations for the given situation:

"A bag contains 3 types of coins namely Re. 1, Rs. 2 and Rs. 5. There are 30 coins amounting to Rs. 100 in total."

Solution:

Let x, y and z be the number of coins respectively. x + y + z = 30x + 2y + 5z = 100

Question 22: If the two vectors 3i + 2j + 9k and i + mj + 3k are parallel, then prove that m = 2/3.

Solution:

a = 3i + 2j + 9k b = i + mj + 3kIt is given that the two vectors are parallel. $a = \lambda b$ $3i + 2j + 9k = \lambda (i + mj + 3k)$ $3i + 2j + 9k = \lambda i + \lambda mj + \lambda 3k$ Equating i, j, k vectors, $3i = \lambda i$ $\lambda = 3$ $m\lambda = 2$ m = 2 / 3 Question 23: Find the least positive integer n such that $[(1 + i) / (1 - i)]^n = 1$.

Solution:

$$\begin{split} & [(1+i) / (1-i)]^n \\ & 1 = [(1+i) / (1-i) * (1+i) / (1+i)]^n \\ & 1 = [(1-1+2i) / 2] \\ & i^n = 1 \\ & n = 4 \end{split}$$

Question 24: Draw the diagram for the given situation: "A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of π / 3 radians with the axis of the orbit."

Solution:



Question 25: Find the critical numbers of f(x) = sinx**.**

Solution:

f (x) = sinx f ' (x) = cosx f ' (x) = 0 cosx = 0 x = (2n + 1) $\pi / 2$, n $\in \mathbb{Z}$ x = $\pi / 2$, $3\pi / 2$, **Question 26:** Write the domain and extent of the function $f(x) = x^3 + 1$.

Solution:

Domain: $(-\infty, \infty)$ Extent: Vertical : $(-\infty, \infty)$ Horizontal : $(-\infty, \infty)$

Question 27: Prove that $\int_{\pi/6}^{\pi/3} dx / [1 + \sqrt{\cot x}] = \int_{\pi/6}^{\pi/3} dx / [1 + \sqrt{\cot x}]$ \sqrt{tanx}].

Solution:

LHS = $\int_{\pi/6} \pi/3 \, dx / [1 + \sqrt{\cot x}]$ $= \int_{\pi/6}^{\pi/3} dx / [1 + \sqrt{\cot [(\pi/6) + (\pi/3) - x]}]$ $= \int_{\pi/6} \pi/3 \, dx / [1 + \sqrt{\tan x}]$

Question 28: Show that the set of all non-zero rational numbers is not closed under addition.

Solution:

Let $G = Q - \{0\}$ \forall a, b \in G \Rightarrow a + b \notin G So, G is not closed under addition.

Question 29: Prove that F (3) = $1 - e^{-9}$ if the probability density

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0\\ 0, & x \le 0 \end{cases}$$

function

Solution:

F (x) = **P** (X ≤ x) = $\int_{-\infty}^{x} f(t) dt$ **F** (3) = $\int_0^3 3e^{-3t} dt$

 $= 3 [e^{-3t} / -3]$ = [- e^{-3t}]_0^3 = - [e^{-9} - 1] = 1 - e^{-9}

Question 30: Verify Rolle's theorem for the function f(x) = |x - 2| + |x - 5| in [1, 6].

Solution:

f(x) = |x - 2| + |x - 5| in [1, 6]f(x) is continuous on [1, 6] f(x) is not differentiable on (1, 6) Rolle's theorem is not satisfied.

PART - III

Answer any seven questions.

[7 * 3 = 21]

Question 31: Prove that ρ (A) + ρ (B) $\neq \rho$ (A + B) by giving the suitable matrices A and B of order 3.

Solution:

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix} \Rightarrow \ell(A) = 2$$

$$B = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \Rightarrow \ell(B) = 2$$

$$A + B = \begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 8 & -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} \ell(A) + \ell(B) = 4 \\ -50 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 8 & -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 3 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 5 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 5 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 5 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 5 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 5 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 5 & 10 \end{pmatrix}$$

$$\ell(A + B) = 3 \rightarrow B$$

$$diom \quad D = O \quad \ell(A + B) \neq \ell(A) + \ell(B)$$

Question 32: Find the vectors of magnitude 6 which are perpendicular to both the vectors 4i - j + 3k and - 2i + j - 2k.

Solution:

a = 4i - j + 3k b = -2i + j - 2kRequired vector = $\pm \mu (a \times b) / |a \times b|$



Question 33: If n is a positive integer, prove that $[(1 + \sin\theta - i \cos\theta) / (1 + \sin\theta + i \cos\theta)]^n = \cos n (\pi / 2 - \theta) - i \sin n (\pi / 2 - \theta).$

Solution:

Let $Z = \sin\theta - i \cos\theta$ |Z| = 1 $(1 / Z) = (\tan Z)$ $(1 / Z) = \sin\theta + i \cos\theta$ LHS = $[(1 + \sin\theta - i \cos\theta) / (1 + \sin\theta + i \cos\theta)]^n$ = $[(1 + Z) / (1 + (1 / Z))]^n$ = $[((1 + Z) / (Z + 1) / Z)]^n$ = Z^n = $(\sin\theta - i \cos\theta)^n$ = $[\cos(\pi / 2 - \theta) - i \sin(\pi / 2 - \theta)]^n$ = $\cos n(\pi / 2 - \theta) - i \sin n(\pi / 2 - \theta)$ = RHS

Question 34: Show that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact.

Solution:



Equation of the tangent is $x + yt^2 = 2ct$ Put x = 0, y = 2c / t, B (0, 2c / t) Put x = 0, x = 2ct, A (2ct, 0) Midpoint of AB = (2ct / 2), (2c / t / 2) = (ct, c / t) (ct, c / t) is the point of contact.

Question 35: Show that the function $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is strictly increasing in the interval $(0, \pi / 4)$.

Solution:

 $f(x) = \tan^{-1} (\sin x + \cos x)$ $f'(x) = \{1 / [1 + (\sin x + \cos x)^2]\} * (\cos x - \sin x)$ $= (\cos x - \sin x) / [1 + \sin^2 x + \cos^2 x + 2 \sin x \cos x]$ $= (\cos x - \sin x) / [2 + 2 \sin x \cos x]$ $= (\cos x - \sin x) / [2 + \sin 2x] \forall x \in (0, \pi / 4)$ f'(x) > 0 $f(x) \text{ is strictly increasing in the interval } (0, \pi / 4).$

Question 36: If f (x, y) = 1 / $\sqrt{x^2 + y^2}$ then, prove that x $\partial f / \partial x + y \partial f / \partial y = -f$.

Solution:

f (x, y) = 1 / $\sqrt{x^2 + y^2}$ f (tx, ty) = 1 / $\sqrt{t^2x^2 + t^2y^2}$ = 1 / t $\sqrt{x^2 + y^2}$ = $t^{-1} f(x, y)$ f is a homogeneous function of degree 1. By Euler's theorem, x $\partial f / \partial x + y \partial f / \partial y = - f$

Question 37: Derive the formula for the volume of a cylinder with radius 'r' and height 'h' by using integration.

Solution:



Consider a triangle AOB with vertices O (0, 0), A (h, 0), B (h, r). Equation of OB, y = (r / h) x $V = \pi \int_0^h y^2 dx$ $= \pi \int_0^h (r^2 / h^2) x^2$ $= \pi r^2 / h^2 [x^3 / 3]_0^h$ $= \pi r^2 / h^2 [h^3 / 3]$ $= (1 / 3) \pi r^2h$ cubic units

Question 38: Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.

Solution:

р	q	p∧q	p∨q	$(\mathtt{p} \land \mathtt{q}) \rightarrow$ $(\mathtt{p} \lor \mathtt{q})$
Т	Т	Т	Т	Т
Т	F	F	Т	Т

F	Т	F	Т	Т
F	F	F	F	Т

It is a tautology.

Question 39: A die is thrown 120 times and getting 1 or 5 is considered a success. Find the mean and variance of the number of successes.

Solution:

n = 120 p = 2 / 6 = 1 / 3 q = 1 - p = 1 - (1 / 3) = 2 / 3Mean = np = 120 * (1 / 3) = 40 Variance = npq = (120) * (1 / 3) * (2 / 3) = 80 / 3

Question 40: Show that the solution of the differential equation $yx^3 dx + e^{-x} dy = 0$ is $(x^3 - 3x^2 + 6x - 6) e^x + \log y = c$.

Solution:

 $yx^{3} dx + e^{-x} dy = 0$ $e^{-x} dy = -yx^{3} dx$ $dy / y = -x^{3} e^{x} dx$ $\log y = -[x^{3} e^{x} - 3 x^{2} e^{x} + 6x e^{x} - 6e^{x}] + c$ $\log y + e^{x} [x^{3} - 3 x^{2} + 6x - e^{x}] = c$

PART - IV

Answer all the questions.

[7 * 5 = 35]

Question 41: (a) For what values of μ the system of homogeneous equations x + y + 3z = 0; 4x + 3y + μ z = 0; 2x + y + 2z = 0 have:

(i) only trivial solution

(ii) infinitely many solutions

OR

(b) Prove by vector method that sin (A + B) = sinA cosB + cosA sinB.

Solution:

[a] The system of equations can be written as AX = B.

$$\begin{bmatrix} 1 & 1 & 3 \\ 4 & 3 & \mu \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A, B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix} \begin{bmatrix} R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - 2R_1 \\ R_3 \rightarrow R_3 - 2R_1 \\ - \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & 0 & 8 - \mu & 0 \end{bmatrix} R_3 \rightarrow R_3 - R_2$$

Case [i] Let $\mu \neq 8$

ρ [A, B] = 3 and ρ [A] = 3

The given system is consistent and has a trivial solution.

x = 0, y = 0, z = 0

Case [ii]: Let $\mu = 8$

 ρ [A, B] = 2 and ρ [A] = 2 < 3 [is equal to number of unknowns]

The corresponding equations are x + y + 3z = 0; y + 4z = 0

Take z = k, y = -4k and x = k

The solutions set is (x, y, z) = (k, -4k, k), which are non-trivial.

The given system is consistent and has infinitely many non-trivial solutions.

OR

[b] Let α and β be two unit vectors, and A and B be the angles made by them respectively with the x-axis.

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\alpha = \cos Ai + \sin Aj \text{ and } \beta = \cos Bi + \sin Bj
Now, \alpha \cdot \beta = (\cos Ai + \sin Aj) (\cos Bi + \sin Bj)
\Rightarrow \alpha\beta \cos (A - B) = \cos A \cos B + \sin A \sin B
\Rightarrow \cos (A - B) = \cos A \cos B + \sin A \sin B \quad [\because \alpha = 1, \beta = 1] ------(1)
Putting -B in place of B in (1):-
\cos (A - (-B)) = \cos A \cos (-B) + \sin A \sin (-B)
\Rightarrow \cos (A + B) = \cos A \cos B - \sin A \sin B
Similarly,
\alpha \times \beta = (\cos Ai + \sin Aj) \times (\cos Bi + \sin Bj)
\Rightarrow \alpha \times \beta = \cos A \sin Bk - \sin A \cos Bk
\Rightarrow \alpha\beta \sin (A - B) (-k) = (\sin A \cos B - \cos A \sin B) (-k)
\Rightarrow \sin(A - B) = \sin A \cos B - \cos A \sin B -------(2)
Putting B = - B in (2),
\sin (A - (-B)) = \sin A \cos (-B) - \cos A \sin (-B)
\Rightarrow \sin (A + B) = \sin A \cos B + \cos A \sin B
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Question 42: [a] Find the cartesian equation of the plane containing the line x - 2/2 = y - 2/3 = z - 1/-2 and passing through the point (-1, 1, -1).

OR

(b) Solve: $x^{11} - x^6 + x^5 - 1 = 0$.

Solution:

[a] $(x_1, y_1, z_1) = (-1, 1, -1)$ $(x_2, y_2, z_2) = (2, 2, 1)$ $(l_1, m_1, n_1) = (2, 3, -2)$ The equation of the plane is $\begin{vmatrix} x+1 & y-1 & z+1 \\ 3 & 1 & 2 \\ 2 & 3 & -2 \end{vmatrix} = 0$ 0 = (x + 1) (-2 - 6) - (y - 1) (-6 - 4) + (z + 1) (9 - 2)0 = (x + 1) (-8) - (y - 1) (-10) + (z + 1) (7) 0 = -8x - 8 + 10y - 10 + 7z + 78x + 8 - 10y + 10 - 7z - 7 = 0 8x - 10y - 7z + 11 = 0

OR

 $[b] x^{11} - x^{6} + x^{5} - 1 = 0$ $x^{6} [x^{5} - 1] [x^{5} - 1] = 0$ $x = (-1)^{\frac{1}{6}} = (\operatorname{cis} \pi)^{\frac{1}{6}}$ $= (\operatorname{cis} (2k\pi + \pi))^{\frac{1}{6}} \quad k = 0, 1, 2, 3, 4, 5$ $x = \operatorname{cis} \pi / 6, \operatorname{cis} 3\pi / 6, \operatorname{cis} 5\pi / 6, \operatorname{cis} 7\pi / 6, \operatorname{cis} 9\pi / 6, \operatorname{cis} 11\pi / 6$ $x = (\operatorname{cis} 0)^{\frac{1}{5}}$ $= (\operatorname{cis} (2k\pi))^{\frac{1}{5}}$ $= \operatorname{cis} 2k\pi / 5, k = 0, 1, 2, 3, 4, 5$ $x = 0, \operatorname{cis} 2\pi / 5, \operatorname{cis} 4\pi / 5, \operatorname{cis} 6\pi / 5, \operatorname{cis} 8\pi / 5.$

Question 43: [a] Show that the sum of the focal distances of any point on an ellipse is equal to the length of the major axis and also prove that the locus of a point which moves so that the sum of its distances from (3, 0) and (-3, 0) is 9, is $x^2/(81/4) + y^2/(45/4) = 1$.

OR

(b) Prove that the area of the largest rectangle that can be inscribed in a circle of radius 'r' is $2r^2$.

Solution:

[a]



 $F_1P + F_2P = 2a$

x = a / e, x = -a / e $F_1P / PM = e$ $F_2P / PM = e$ $F_1P = e PM$ $F_2P = e PM$ $F_1P + F_2P = e (PM + PM)$ = e (MM')= e (2a / e)= 2a= length of major axis 2a = 9a = 9 / 2ae = 3 $b^2 = a^2 - (ae)^2$ =(81/4)-9= 45 / 4The equation is $x^2 / (81 / 4) + y^2 / (45 / 4) = 1$.

OR

[b] $x = 2r \cos \theta$ $y = 2r \sin \theta$ Area of rectangle = 2x.2y A (θ) = 2r $\cos\theta$. 2r $\sin\theta$ d (A(θ)) / d θ = 4r² $\cos 2\theta$ d (A(θ)) / d θ = 0 then $\theta = \pi / 4$ A '' (θ) = - 8r² $\sin 2\theta < 0$ for $\theta = \pi / 4$ So A is maximum for $\theta = \pi / 4$ X = y = $\sqrt{2r}$, Required area = x.y = 2r. 2r = 2r² Question 44: [a] A missile fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - (25 / 2)t^2$.

Find :

(i) the initial velocity of the missile

(ii) the time when the height of the missile is a maximum

(iii) the maximum height reached

(iv) the velocity with which the missile strikes the ground

OR

(b) Find the centre, foci and vertices of the hyperbola $16x^2 - 9y^2 - 32x - 18y + 151 = 0$ and draw the diagram.

Solution:

[a] $x = 100t - (25 / 2)t^2$

(i) To find the initial velocity of the missile. So, t = 0v = dx / dt = 100 - (25 / 2) 2tv = 100 - 0=100

(ii) The time when the height of the missile is a maximum is given by v = 0. 100 - 25t = 0 t = 4 seconds

(iii) The maximum height reached when t = 4 seconds x = 100 (4) - (25 / 2)16x = 200 meters

(iv) velocity if t = 4 + 4 = 8 seconds v = 100 - 25(8) = -100 m/sec

[b] equation $(y + 1)^2 / 16 - (x - 1)^2 / 9 = 1$ $a^2 = 16$ b² = 9 e = 5 / 4 Centre: (1, - 1) Foci: (1, 4), (1, - 6) Vertices : (1, 3), (1, - 4)



Question 45: [a] The mean score of 1000 students for an examination is 34 and the standard deviation is 16. Determine the limit of the marks of the central 70% of the candidates by assuming the distribution is normal. P [0 < Z < 1.04] = 0.35

OR

(b) Compute the area between the curve y = sinx and y = cosx and the lines x = 0 and $x = \pi$.

Solution:

[a] a) P ($Z_1 < Z < Z_2$) = 70% P ($Z_1 < Z < Z_2$) = 0.70 P($Z_1 < Z < 0$) = 0.35 P (0 < Z < Z_2) = 0.35 Z₁ and Z₂ lie on the left and right side of the normal curve. Z₁ = -1.04 and Z₂ = 1.04 Z = [$x - \mu$] / σ = [x - 34] / 16 = -1.04 X = 17.36 Z = x - 34 / 16 = 1.04 x = 50.64 70% of the students' score lies between 17.36 and 50.64.

[b] $y = \sin x$ $y = \cos x$ Then $\sin x = \cos x$ $\rightarrow x = \pi / 4 \in (0, \pi)$ Required area = $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$ = 2 $\sqrt{2}$ square units

Question 46: [a] If $w = x + 2y + z^2$ and $x = \cos t$; $y = \sin t$; z = t find dw / dt by using chain rule. Also find dw / dt by substitution of x, y and z in w and hence verify the result.

OR

(b) A cup of tea at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

Solution:

[a] $dw / dt = (\partial w / \partial x) (dx / dt) + (\partial w / \partial y) (dy / dt) + (\partial w / \partial z) (dz / dt)$ = - sint + 2 cost + 2t (1) w = cost + 2 sint + t² dw / dt = - sint + 2 cost + 2t (2)

b) dT / dt = k (T - s), s = 15 $T = s + ce^{kt}$(1) t = 0 then T = 100(1) => c = 85 t = 5, T = 60 then $e^{5k} = 45 / 85$ t = 10, then T = ? $T = 15 + 83 e^{10k} = 38.82$

Question 47: [a] State all the five properties of groups.

(b) Prove that the solution of the differential equation : $(5D^2 - 8D - 4)y = 5e^{(-2/5x)} + 2e^x + 3$ is $y = Ae^{2x} + Be^{(-2/5x)} - (5/12) xe^{(-2/5x)} - (2/7) e^x - (3/4)$.

Solution:

[a] (i) Identity element of group is unique (ii) Inverse of each element of group is unique (iii) Reversal law (iv) Cancellation law (v) $[a^{-1}] - 1 = a$ b) C.F = $Ae^{2x} + Be^{(-2/5)x}$ P.I₁ = (-5 / 12) $xe^{(-2/5)x}$ P.I₂ = (-2 / 7) e^x P.I₃ = -3 / 4 General solution y = C.F + P.I₁ + P.I₂ + P.I₃ = $Ae^{2x} + Be^{(-2/5)x} + (-5 / 12) xe^{(-2/5)x} + (-2 / 7) e^x (-3 / 4)$