Senior School Certificate Examination-2020

Marking Scheme - Mathematics 65/C/1

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 80 has to be used. Please do nothesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/C/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No. **Marks**

1. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(A) $\frac{7}{15}$ **(B)** $\frac{8}{15}$ **(C)** $\frac{2}{15}$ **(D)** $\frac{14}{15}$

Ans: (A) $\frac{7}{15}$ **1**

2. The objective function of an LPP is

(A) a constant **(B)** a linear function to be optimised

(C) an inequality **(D)** a quadratic expression

Ans: (B) a linear function to be optimised **1**

3. If the two lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : x = 2, \frac{y}{-1} = \frac{z}{2-\alpha}$$

are perpendicular, then the value of α is

(A) $\frac{2}{3}$ **(B)** 3 **(C)** 4 **(D)** $\frac{7}{3}$

Ans: (D) $\frac{7}{3}$ **1**

4. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the points A(2, 3, –4), B(3, –4, –5) and C(3, 2, –3) respectively, then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is equal

(A) $\sqrt{113}$ **(B)** $\sqrt{185}$ **(C)** $\sqrt{203}$ **(D)** $\sqrt{209}$

Ans: (D) $\sqrt{209}$ **1**

5. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

(A) 1, 1 **(B)** 1, 2 **(C)** 2, 1 **(D)** 2, 2

Ans: (A) 1, 1 **1**

6. $\int_0^1 \tan\left(\sin^{-1} x\right) dx$ equals

(A) 2 **(B)** 0 **(C)** –1 **(D)** 1

Ans: (D) 1 **1**

7. $\int \frac{e^x}{x+1}\left[1+(x+1)\log(x+1)\right] dx$ equals

(A) $\frac{e^x}{x+1}+c$ **(B)** $e^x \frac{x}{x+1}+c$

(C) $e^x \log(x+1) + e^x + c$ **(D)** $e^x \log(x+1) + c$

Ans: (D) $e^x \log(x+1) + c$ **1**

8. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x-1}{y-1}$ **(B)** $\frac{x-1}{y+1}$ **(C)** $\frac{y-1}{x+1}$ **(D)** $\frac{y+1}{x-1}$

Ans: (C) $\frac{y-1}{x+1}$ **1**

9. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^2 equals

(A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & -2 \\ -2 & -2 \end{bmatrix}$ **(C)** $\begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix}$ **(D)** $\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

Ans: (A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ **1**

10. $\begin{vmatrix} 43 & 44 & 45 \\ 44 & 45 & 46 \\ 45 & 46 & 47 \end{vmatrix}$ equals

(A) 0 **(B)** –1 **(C)** 1 **(D)** 2

Ans: (A) 0 **1**

Fill in the blanks in questions numbers 11 to 15

11. Two angles of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$. The third angle of the triangle is __________.

Ans: $\frac{3\pi}{4}$ or $135°$ **1**

12. A square matrix A is said to be singular if ____________.

Ans: $|A| = 0$

OR

If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB| =$ ____________.

Ans: $|AB| = -100$

13. If $y = \log x$, then $\frac{d^2y}{dx^2} =$ ________.

Ans: $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$ **1**

14. The integrating factor of the differential equation $x\frac{dy}{dx} - y = \log x$ is __________

Ans: $\frac{1}{x}$ **1**

15. From a pack of 52 cards, 3 cards are drawn at random (without replacement). The probability that they are two red cards and one black card, is _________.

Ans: $\frac{13}{34}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. Find the distance of the point (a, b, c) from the x-axis.

Ans: $\sqrt{b^2 + c^2}$ **1**

17. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$.

Ans: $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}} = \frac{|\vec{a}|}{|\vec{b}|}$ **1/2**

$= \frac{3}{5\sqrt{2}}$ **1/2**

OR

Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

Ans: $\vec{c} \perp \vec{d} \Rightarrow \vec{c} \cdot \vec{d} = 0$ **1/2**

$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$

$\Rightarrow$ Angle between vectors $\hat{a}$ & $\hat{b} = \frac{\pi}{3}$ or $60°$ **1/2**

18. Solve the differential equation $\left(e^x + 1\right) y\,dy = e^x (y+1)\,dx$.

Ans: $\int \frac{e^x}{e^x + 1} dx = \int \frac{y}{y+1} dy$ **1/2**

$\Rightarrow \log(e^x + 1) = y - \log(y + 1) + c$ **1/2**

19. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then find the value of x.

Ans: $2x - 3 = x + 2$ **1/2**

$\Rightarrow x = 5$ **1/2**

OR

If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19A$.

Ans: $(2 + A)^3 - 19A = A^3 + 8 + 12A + 6A^2 - 19A$ **1/2**

$= 8$ **1/2**

20. If $f(x) = \frac{1-x}{1+x}$, then find $(f \circ f)(x)$.

Ans: $f \circ f(x) = f(f(x)) = \frac{1 - f(x)}{1 + f(x)}$ **1/2**

$= x$ **1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Let W denote the set of words in the English dictionary. Define the relation R by

R = {(x , y) $\in$ W $\times$ W such that x and y have at least one letter in common}

Show that this relation R is reflexive and symmetric, but not transitive.

Ans: For any word $x \in W$

x and x have atleast one (all) letter in common

$\therefore (x, x) \in R, \forall x \in W \quad \therefore$ R is reflexive **1**

Symmetric : Let $(x, y) \in R$, $x, y \in W$

$\Rightarrow$ x and y have atleast one letter in common

$\Rightarrow$ y and x have atleast one letter in common **1/2**

$\Rightarrow (y, x) \in R \therefore$ R is symmetric

Transitive : Taking example of three English dictionary words $x, y, z, \in W$ such that $(x, y), (y, z) \in R$ but $(x, z) \notin R$ **1/2**

$\therefore$ R is not transitive

OR

Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4}\right)$.

Ans: Let $y = f(x) = \frac{4x}{3x+4} \Rightarrow x = \frac{4y}{4-3y}$ **$1\frac{1}{2}$**

$\therefore \quad f^{-1}(y) = \frac{4y}{4-3y} \left(\text{or } f^{-1}(x) = \frac{4x}{4-3x}\right)$ **1/2**

22. For the matrix $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, verify the following:

$A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I$

Ans: $|A| = -12 + 12 = 0$, adj $A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$ **$\frac{1}{2}+1$**

$A \cdot (\text{adj } A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = (\text{adj } A) \cdot A = |A|\, I$ **$\frac{1}{2}$**

23. If $y = e^{x} + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$

Ans: $\frac{dy}{dx} = e^{x} - e^{-x}$ **1**

$= \sqrt{\left(e^{x} + e^{-x}\right)^2 - 4}$ **1/2**

$= \sqrt{y^2 - 4}$ **1/2**

24. Solve the following homogeneous differential equation: $x\frac{dy}{dx} = x + y$

Ans: Let $y = vx \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}+\frac{1}{2}$

$\therefore x\left(v + x\frac{dv}{dx}\right) = x + vx \Rightarrow x\frac{dv}{dx} = 1$ $\frac{1}{2}$

$\therefore \int dv = \int \frac{1}{x}dx \Rightarrow v = \log|x| + c$

$\Rightarrow y = x(\log|x| + c)$ $\frac{1}{2}$

25. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Ans: $\left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \cdot \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$

$= \left(|\vec{a}|\vec{b}\right)^2 - \left(|\vec{b}|\vec{a}\right)^2$ 1

$= |\vec{a}|^2 |\vec{b}|^2 - |\vec{b}|^2 |\vec{a}|^2 = 0$

$\therefore \left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \perp \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$ 1

26. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

Ans: Let X = No. of even tickets drawn

X	0	1	2
P(X)	$\frac{^{10}C_2}{^{19}C_2} = \frac{5}{19}$	$\frac{^{10}C_1 \cdot {^9}C_1}{^{19}C_2} = \frac{10}{19}$	$\frac{^9C_2}{^{19}C_2} = \frac{4}{19}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

OR

Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

Ans: X = No. of success = No. of times getting a number greater than 5

X	0	1	2
P(X)	$\frac{5}{5} \cdot \frac{5}{6} = \frac{25}{36}$	$2 \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{10}{36}$	$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

Ans: $\text{LHS} = 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7} \quad = \quad \tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ **2**

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right)=\tan^{-1}\frac{31}{17}$$ **2**

$= \text{RHS}$

28. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Ans. $\text{LHS} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$$\left.\begin{array}{l} (\text{Applying } C_1 \to C_1 - bC_3,\ C_2 \to C_2 + aC_3) \\ = \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b\left(1+a^2+b^2\right) & -a\left(1+a^2+b^2\right) & 1-a^2-b^2 \end{vmatrix} \end{array}\right\}$$ **2**

(Taking $\left(1+a^2+b^2\right)$ common from C_1 & C_2)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$ **1/2**

(Applying $C_3 \to C_3 + 2bC_1 - 2aC_2$)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & -a & 1+a^2+b^2 \end{vmatrix}$$ **1**

(Expand along C_3)

$= \left(1+a^2+b^2\right)^3 = \text{RHS}$ **1/2**

OR

Find the equation of the line joining A(1, 3) and B(0, 0), using determinants. Also, find k if D(k, 0) is a point such that the area of the ΔABD is 3 square units.

Ans: Equation of the line through A(1, 3) and B(0, 0) is

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow 3x - y = 0$$ **2**

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow k = \pm 2$$ **2**

29. Prove that three points A, B and C with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\left(\vec{b} \times \vec{c}\right) + \left(\vec{c} \times \vec{a}\right) + \left(\vec{a} \times \vec{b}\right) = \vec{0}$

Ans. Points $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are collinear

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \vec{0}$ **1**

$\Rightarrow \left(\vec{b} - \vec{a}\right) \times \left(\vec{c} - \vec{a}\right) = \vec{0}$ **1**

$\Rightarrow \vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{a} \times \vec{c} + \vec{a} \times \vec{a} = \vec{0}$ **1**

$\Rightarrow \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

Similarly, converse can be proved **1**

30. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y-1}{3} = z, \quad \frac{x+1}{5} = \frac{y-2}{1}, \quad z = 2$$

Ans. Let $\left.\begin{array}{l} \vec{a}_1 = \hat{i} - \hat{j} \;;\; \vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k} \\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k} \;;\; \vec{b}_2 = 5\hat{i} + \hat{j} \end{array}\right\}$ **1**

then, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 5\hat{j} - 13\hat{k}$ **1/2+1$\frac{1}{2}$**

$\therefore$ Shortest distance $= \left|\dfrac{\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \dfrac{9}{\sqrt{195}} \neq 0$ **1/2**

$\therefore$ lines are not intersecting **1/2**

OR

Find the equation of the plane through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j})+6=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0$, which is at a unit distance from the origin.

Ans: Equation of plane through the line of intersection of the two given planes is

$$\vec{r}\cdot\left[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}-4\lambda\hat{k}\right] = -6 \qquad 1\frac{1}{2}$$

As per the given condition

$$\left|\frac{-6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}\right|=1 \Rightarrow \lambda = \pm 1 \qquad 1\frac{1}{2}$$

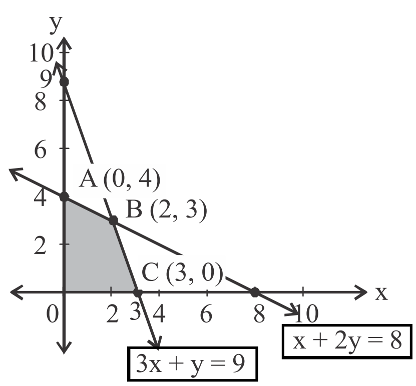
$\therefore$ Equation of plane is : $\vec{r}\cdot(4\hat{i}+2\hat{j}-4\hat{k})+6=0$ **1/2**

or $\vec{r}\cdot(-2\hat{i}+4\hat{j}+4\hat{k})+6=0$ **1/2**

31. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3g of silver and 1g of gold, while that of type B requires 1g of silver and 2g of gold. The company can use at the most 9g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 120 and that of type B ₹ 150, then find the number of units of each type that the company should produce to maximise profit.

Formulate the above LPP and solve it graphically. Also, find the maximum profit.

Ans.



Let No. of goods type A = x,

Number of goods type B = y.

Then the L.P. P. is:

Maximize (Profit) : $Z = 120x + 150y$ **1**

Subject to constraints :

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\} \qquad 1$$

Correct figure $1\frac{1}{2}$

Corner:	Value of Z (in ₹)
0(0, 0)	0
A(0, 4)	600
B(2, 3)	690 (Max)
C(3, 0)	360

1/2

$\therefore$ Max. profit = ₹ 690

when Good Type A = 2 units, Type B = 3 units

32. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

Ans. Let E_1 : Bag I is selected

E_2 : Bag II is selected

A : Two balls drawn at random both are red. **1/2**

$$P(E_1) = P(E_2) = \frac{1}{2},\ P\left(\frac{A}{E_1}\right) = \frac{{}^5C_2}{{}^9C_2} = \frac{5}{18},\ P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^9C_2} = \frac{1}{12}$$ **1/2+1+1**

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2}\cdot\frac{1}{12}}{\frac{1}{2}\cdot\frac{5}{18}+\frac{1}{2}\cdot\frac{1}{12}} = \frac{3}{13}$$ **1**

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. If $y = x^{\sin x} + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Ans. Let $u = x^{\sin x}$ $\therefore$ $y = u + \sin^{-1}\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{2\sqrt{x}\sqrt{1-x}} \quad \ldots \text{(i)}$$ **$1\frac{1}{2}$**

Also, $\log u = \sin x \cdot \log x$ **1**

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \cdot \log x + \frac{\sin x}{x}$$ **2**

$$\Rightarrow \frac{du}{dx} = x^{\sin x} \cdot \left(\cos x \cdot \log x + \frac{\sin x}{x}\right) \quad \ldots \text{(ii)}$$ **$\frac{1}{2}$**

Putting (ii) in (i) we get

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \log x + \frac{\sin x}{x}\right) + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$ **1**

34. Evaluate : $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx$

Ans. Let $\sin x - \cos x = t \quad \therefore \quad (\sin x + \cos x)\,dx = dt$ $\quad 1+\frac{1}{2}$

Squaring we get, $\sin 2x = 1 - t^2$ $\quad 1\frac{1}{2}$

$$\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx = \int \frac{1}{\sqrt{1-t^2}}\,dt = \sin^{-1} t$$

$$= \sin^{-1}(\sin x - \cos x) \qquad 1\frac{1}{2}$$

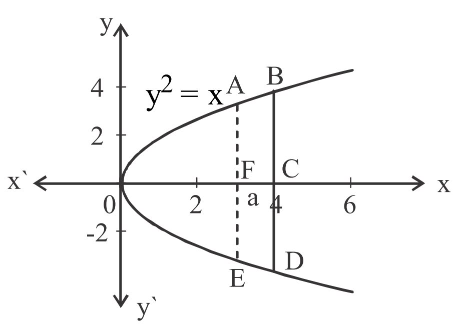
$$\therefore \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx = \sin^{-1}(\sin x - \cos x)\Big]_{\pi/6}^{\pi/3}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) \qquad 1\frac{1}{2}$$

$$= 2 \cdot \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

35. If the area between the curves $x = y^2$ and $x = 4$ divided into two equal parts by the line $x = a$, then find the value of a using integration.

Ans. Correct graph $\quad 1\frac{1}{2}$



$\text{ar}(OAEO) = \text{ar}(ABDEA)$

$\Rightarrow 2 \cdot \text{ar}(OAFO) = 2 \cdot \text{ar}(ABCFA)$

$$\int_0^a \sqrt{x}\,dx = \int_a^4 \sqrt{x}\,dx \qquad 1\frac{1}{2}$$

$$\frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right) \qquad 1\frac{1}{2}$$

$$\Rightarrow \frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right) \qquad 1\frac{1}{2}$$

$$\Rightarrow a^{3/2} = 4, \quad \therefore \; a = 4^{2/3} \qquad 1\frac{1}{2}$$

OR

Find: $\int \frac{x}{(x-1)^2(x+2)}dx$

Ans: $\int \frac{x}{(x-1)^2(x+2)}dx \quad = \quad \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{4+2}dx$ **4**

$= \quad \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$ **2**

36. Find the intervals in which the function f defined as f(x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing or decreasing.

Ans. $f'(x) = \cos x - \sin x\,,\ 0 \le x \le 2\pi$ **1**

$\therefore\ f'(x) = 0 \ \Rightarrow\ x = \frac{\pi}{4}, \frac{5\pi}{4}$ **2**

Sign of $f'(x)$: $\quad$ $+ve \quad -ve \quad +ve$; $\quad 0 \quad \frac{\pi}{4} \quad \frac{5\pi}{4} \quad 2\pi$ **$1\frac{1}{2}$**

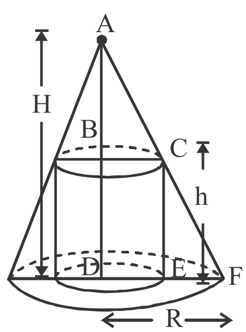
$\therefore$ f(x) is strictly increasing on $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ **1**

and f(x) is strictly decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ **$\frac{1}{2}$**

OR

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Ans.



Let h = Height of cylinder

r = Radius of cylinder

H = Height of cone

R = Radius of cone

where, H, R are constants

Correct figure **1**

$\frac{H-h}{H} = \frac{r}{R}\left(\because \Delta ABC \sim \Delta ADE\right) \quad \ldots(i)$ **$\frac{1}{2}$**

C = curved surface area = $2\pi rh$ **1**

$\therefore \quad C = 2\pi rh \cdot H \cdot \left(\frac{R - r}{R}\right)$ (Using (i))

$= \frac{2\pi H}{R}\left(rR - r^2\right)$ **1**

$C'(r) = \frac{2\pi H}{R}(R - 2r), \; C''(r) = \frac{-4\pi H}{R} < 0$ **1**

$C'(r) = 0 \; \Rightarrow \; r = \frac{R}{2}, \; C''\left(r = \frac{R}{2}\right) < 0$ **1**

$\therefore$ Curved surface area of cylinder is Max. iff $r = \frac{R}{2}$ $\mathbf{\frac{1}{2}}$

QUESTION PAPER CODE 65/C/2

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No. **Marks**

1. If $\vec{a} = \hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\left|\vec{a} + \vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$, then the value of λ is

(A) 1 **(B)** -1 **(C)** 2 **(D)** -2

Ans: (A) 1 — **1**

2. The order and the degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^3y}{dx3}\right)^3 + 5x = 0$ are

(A) 3 ; 6 **(B)** 3 ; 3 **(C)** 3 ; 9 **(D)** 6 ; 3

Ans: (B) 3 ; 3 — **1**

3. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x-1}{y-1}$ **(B)** $\frac{x-1}{y+1}$ **(C)** $\frac{y-1}{x+1}$ **(D)** $\frac{y+1}{x-1}$

Ans: (C) $\frac{y-1}{x+1}$ — **1**

4. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^2 equals

(A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & -2 \\ -2 & -2 \end{bmatrix}$ **(C)** $\begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix}$ **(D)** $\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

Ans: (A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ — **1**

5. $\begin{vmatrix} 43 & 44 & 45 \\ 44 & 45 & 46 \\ 45 & 46 & 47 \end{vmatrix}$ equals

(A) 0 **(B)** -1 **(C)** 1 **(D)** 2

Ans: (A) 0 — **1**

6. $\int \frac{e^x}{x+1}[1+(x+1)\log(x+1)]\,dx$ equals

(A) $\frac{e^x}{x+1}+c$ **(B)** $e^x \frac{x}{x+1}+c$

(C) $e^x \log(x+1)+e^x+c$ **(D)** $e^x \log(x+1)+c$

Ans: (D) $e^x \log(x+1)+c$ 1

7. $\int_0^{\pi/2}\left(\sin^{100}x-\cos^{100}x\right)dx$ equals

(A) $\frac{\pi}{100}$ **(B)** 0 **(C)** $\frac{1}{100}$ **(D)** $\frac{100}{(100)^{100}}$

Ans: (B) 0 1

8. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(A) $\frac{7}{15}$ **(B)** $\frac{8}{15}$ **(C)** $\frac{2}{15}$ **(D)** $\frac{14}{15}$

Ans: (A) $\frac{7}{15}$ 1

9. The objective function of an LPP is

(A) a constant **(B)** a linear function to be optimised

(C) an inequality **(D)** a quadratic expression

Ans: (B) a linear function to be optimised 1

10. If the two lines

$$L_1 : x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$$

$$L_2 : x=2, \frac{y}{-1}=\frac{z}{2-\alpha}$$

are perpendicular, then the value of α is

(A) $\frac{2}{3}$ **(B)** 3 **(C)** 4 **(D)** $\frac{7}{3}$

Ans: (D) $\frac{7}{3}$ 1

Fill in the blanks in questions numbers 11 to 15

11. The integrating factor of the differential equation $x\frac{dy}{dx} - y = \log x$ is ________

$\boxed{\textbf{Ans: } \frac{1}{x}}$ **1**

12. Two angles of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$. The third angle of the triangle is __________.

$\boxed{\textbf{Ans: } \frac{3\pi}{4} \text{ or } 135°}$ **1**

13. From a pack of 52 cards, 3 cards are drawn at random (without replacement). The probability that they are two red cards and one black card, is ________.

$\boxed{\textbf{Ans: } \frac{13}{34}}$ **1**

14. A square matrix A is said to be singular if ___________.

$\boxed{\textbf{Ans: } |A| = 0}$ **1**

OR

If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB| =$ ____________.

$\boxed{\textbf{Ans: } |AB| = -100}$ **1**

15. If $x = e^t \sin t$, $y = e^t \cos t$, then the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ is ____________.

$\boxed{\textbf{Ans: } \left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = 0}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. If $f(x) = \frac{1-x}{1+x}$, then find $(f o f)\,(x)$.

Ans: $f o f(x) = f(f(x)) = \frac{1-f(x)}{1+f(x)}$ **1/2**

$= x$ **1/2**

17. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then find the value of x.

Ans: $2x - 3 = x + 2$ **1/2**

$\Rightarrow \quad x = 5$ **1/2**

OR

If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19A$.

Ans: $(2 + A)^3 - 19A = A^3 + 8 + 12A + 6A^2 - 19A$ **1/2**

$= 8$ **1/2**

18. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{2y+4}{7} = \frac{6-z}{2}$. Write its vector equation.

Ans: $\vec{r} = 5\hat{i} - 2\hat{j} + 6\hat{k} + \lambda\left(3\hat{i} + \frac{7}{2}\hat{j} - 2\hat{k}\right)$ **1**

19. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$.

Ans: $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}} = \frac{|\vec{a}|}{|\vec{b}|}$ **1/2**

$= \frac{3}{5\sqrt{2}}$ **1/2**

OR

Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

Ans: $\vec{c} \perp \vec{d} \Rightarrow \vec{c} \cdot \vec{d} = 0$ **1/2**

$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$

$\Rightarrow$ Angle between vectors $\hat{a}$ & $\hat{b} = \frac{\pi}{3}$ or $60°$ **1/2**

20. Find the particular solution of the differential equation $\frac{dy}{dx} = y \tan x$,

when $y(0) = 1$.

Ans: $\frac{dy}{dx} = y \cdot \tan x \Rightarrow \int \frac{1}{y} dy = \int \tan x dx \Rightarrow y = c \cdot \sec x$ **1/2**

$y(0) = 1 \Rightarrow c = 1 \therefore$ particular solution is $y = \sec x$ **1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew-symmetric matrix.

Ans: (i) Let $P = A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}, P' = P \ \therefore\ P$ is symmetric matrix. $\frac{1}{2} + \frac{1}{2}$

(ii) Let $Q = A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, Q' = -Q \ \therefore\ Q$ is skew symmetric matrix. $\frac{1}{2} + \frac{1}{2}$

22. Solve: $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

Ans: Pulling $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1/2

$\therefore$ Given differential can be written as $x\frac{dv}{dx} = -\cos^2 v$

$\Rightarrow \int \sec^2 v dv = -\int \frac{1}{x}dx$ 1/2

$\Rightarrow \tan v = -\log|x| + c$ 1/2

$\Rightarrow \tan\left(\frac{y}{x}\right) = -\log|x| + c$ 1/2

23. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Ans: $\left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \cdot \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$

$= \left(|\vec{a}|\vec{b}\right)^2 - \left(|\vec{b}|\vec{a}\right)^2$ 1

$\left.\begin{array}{l} = |\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2 = 0 \\ \therefore \left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \perp \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right) \end{array}\right\}$ 1

24. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$

Ans: $\frac{dy}{dx} = e^x - e^{-x}$ **1**

$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4}$ **1/2**

$= \sqrt{y^2 - 4}$ **1/2**

25. Let W denote the set of words in the English dictionary. Define the relation R by

R = {(x , y) $\in$ W $\times$ W such that x and y have at least one letter in common}

Show that this relation R is reflexive and symmetric, but not transitive.

Ans: For any word x $\in$ W

x and x have atleast one (all) letter in common

$\therefore$ (x , x) $\in$ R, $\forall$ x $\in$ W $\therefore$ R is reflexive **1**

Symmetric : Let (x, y) $\in$ R, x, y $\in$ W

$\Rightarrow$ x and y have atleast one letter in common

$\Rightarrow$ y and x have atleast one letter in common

$\Rightarrow$ (y, x) $\in$ R $\therefore$ R is symmetric **1/2**

Transitive : Taking example of three English dictionary words x, y, z, $\in$ W such that (x , y), (y, z) $\in$ R but (x, z) $\notin$ R **1/2**

$\therefore$ R is not transitive

OR

Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4}\right)$.

Ans: Let $y = f(x) = \frac{4x}{3x+4} \Rightarrow x = \frac{4y}{4-3y}$ **$1\frac{1}{2}$**

$\therefore \quad f^{-1}(y) = \frac{4y}{4-3y} \left(\text{or } f^{-1}(x) = \frac{4x}{4-3x}\right)$ **1/2**

26. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

Ans: Let X = No. of even tickets drawn

X	0	1	2
P(X)	$\frac{{}^{10}C_2}{{}^{19}C_2}=\frac{5}{19}$	$\frac{{}^{10}C_1 \cdot {}^{9}C_1}{{}^{19}C_2}=\frac{10}{19}$	$\frac{{}^{9}C_2}{{}^{19}C_2}=\frac{4}{19}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

OR

Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

Ans: X = No. of success = No. of times getting a number greater than 5

X	0	1	2
P(X)	$\frac{5}{5}\cdot\frac{5}{6}=\frac{25}{36}$	$2\cdot\frac{1}{6}\cdot\frac{5}{6}=\frac{10}{36}$	$\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in[0,1]$.

Ans: LHS $=\tan^{-1}(\sqrt{x})=\frac{1}{2}.2\tan^{-1}(\sqrt{x})$ **2**

$$=\frac{1}{2}.\cos^{-1}\left(\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right)=\frac{1}{2}.\cos^{-1}\left(\frac{1-x}{1+x}\right)$$ **2**

$= \text{RHS}$

28. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that

$\sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$.

Ans: $|\hat{a}-\hat{b}|^2=(\hat{a}-\hat{b})^2=|\hat{a}|^2+|\hat{b}|^2-2\hat{a}.\hat{b}$ **1+1**

$=1+1-2|\hat{a}||\hat{b}|\cos\theta$ **1**

$$\left.\begin{aligned}&=2(1-\cos\theta)=4\sin^2\frac{\theta}{2}\\ &\therefore \sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|\end{aligned}\right\}$$ **1**

29. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Ans. LHS $= \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

(Applying $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$)

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b\left(1+a^2+b^2\right) & -a\left(1+a^2+b^2\right) & 1-a^2-b^2 \end{vmatrix}$$ **2**

(Taking $\left(1+a^2+b^2\right)$ common from C_1 & C_2)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$ **1/2**

(Applying $C_3 \to C_3 + 2bC_1 - 2aC_2$)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & -a & 1+a^2+b^2 \end{vmatrix}$$ **1**

(Expand along C_3)

$= \left(1+a^2+b^2\right)^3 = \text{RHS}$ **1/2**

OR

Find the equation of the line joining A(1, 3) and B(0, 0), using determinants. Also, find k if D(k, 0) is a point such that the area of the ΔABD is 3 square units.

Ans: Equation of the line through A(1, 3) and B(0, 0) is

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow 3x - y = 0$$ **2**

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow k = \pm 2$$ **2**

30. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

Ans. Let E_1 : Bag I is selected

E_2 : Bag II is selected

A : Two balls drawn at random both are red. **1/2**

$$P(E_1) = P(E_2) = \frac{1}{2},\ P\left(\frac{A}{E_1}\right) = \frac{{}^5C_2}{{}^9C_2} = \frac{5}{18},\ P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^9C_2} = \frac{1}{12}$$ **1/2+1+1**

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2}\cdot\frac{1}{12}}{\frac{1}{2}\cdot\frac{5}{18} + \frac{1}{2}\cdot\frac{1}{12}} = \frac{3}{13}$$ **1**

31. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y-1}{3} = z,\quad \frac{x+1}{5} = \frac{y-2}{1},\ z = 2$$

Ans. Let $\vec{a}_1 = \hat{i} - \hat{j}$; $\vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$; $\vec{b}_2 = 5\hat{i} + \hat{j}$ **1**

then, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 5\hat{j} - 13\hat{k}$ **$\mathbf{1/2+1\frac{1}{2}}$**

$$\therefore \text{ Shortest distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{9}{\sqrt{195}} \neq 0$$ **1/2**

$\therefore$ lines are not intersecting **1/2**

OR

Find the equation of the plane through the line of intersection of the planes $\vec{r}\cdot(\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r}\cdot(3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which is at a unit distance from the origin.

Ans: Equation of plane through the line of intersection of the two given planes is

$$\vec{r}\cdot\left[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}-4\lambda\hat{k}\right]=-6 \qquad 1\frac{1}{2}$$

As per the given condition

$$\left|\frac{-6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}\right|=1 \Rightarrow \lambda=\pm 1 \qquad 1\frac{1}{2}$$

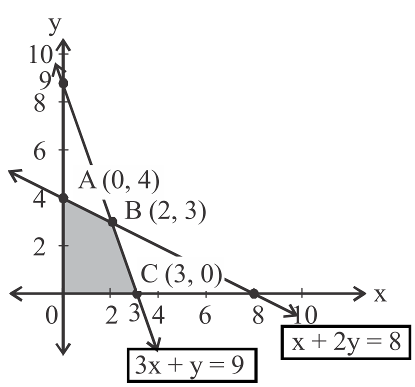
$\therefore$ Equation of plane is : $\vec{r}\cdot\left(4\hat{i}+2\hat{j}-4\hat{k}\right)+6=0$ **1/2**

or $\vec{r}\cdot\left(-2\hat{i}+4\hat{j}+4\hat{k}\right)+6=0$ **1/2**

32. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3g of silver and 1g of gold, while that of type B requires 1g of silver and 2g of gold. The company can use at the most 9g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 120 and that of type B ₹ 150, then find the number of units of each type that the company should produce to maximise profit.

Formulate the above LPP and solve it graphically. Also, find the maximum profit.

Ans.



Let No. of goods type A = x,

Number of goods type B = y.

Then the L.P. P. is:

Maximize (Profit) : Z = 120x + 150y **1**

Subject to constraints :

$$\left.\begin{aligned} 3x+y &\le 9\\ x+2y &\le 8\\ x, y &\ge 0\end{aligned}\right\} \quad 1$$

Correct figure $1\frac{1}{2}$

Corner:	Value of Z (in ₹)
0(0, 0)	0
A(0, 4)	600
B(2, 3)	690 (Max)
C(3, 0)	360

1/2

$\therefore$ Max. profit = ₹ 690

when Good Type A = 2 units, Type B = 3 units

33. If y = 3 cos (log x) + 4 sin (log x), then show that

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

Ans. $y=3\cos(\log x)+4\sin(\log x)\Rightarrow\dfrac{dy}{dx}=\dfrac{-3\sin(\log x)}{x}+\dfrac{4\cos(\log x)}{x}$.... **2**

$\Rightarrow\quad x\dfrac{dy}{dx}=-3\sin(\log x)+4\cos(\log x)$ **1/2**

Differentiate w.r.t. x

$\Rightarrow\quad x^2\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=\dfrac{-2\cos(\log x)}{x}-\dfrac{-\sin(\log x)}{x}$.... **2**

$\Rightarrow\quad x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$ **$1\frac{1}{2}$**

34. Find the intervals in which the function f defined as f(x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing or decreasing.

Ans. $f'(x)=\cos x-\sin x\,,\ 0\le x\le 2\pi$ **1**

$\therefore\ f'(x)=0\ \Rightarrow\ x=\dfrac{\pi}{4},\dfrac{5\pi}{4}$ **2**

Sign of $f'(x)$:

$+$ve	$-$ve	$+$ve
0 to $\frac{\pi}{4}$	$\frac{\pi}{4}$ to $\frac{5\pi}{4}$	$\frac{5\pi}{4}$ to 2π

$1\frac{1}{2}$

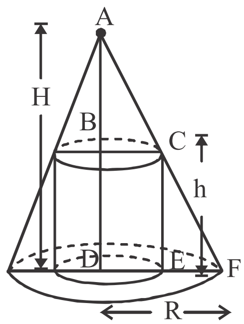
$\therefore$ f(x) is strictly increasing on $\left[0,\dfrac{\pi}{4}\right)\cup\left(\dfrac{5\pi}{4},2\pi\right]$ **1**

and f(x) is strictly decreasing on $\left(\dfrac{\pi}{4},\dfrac{5\pi}{4}\right)$ **$\frac{1}{2}$**

OR

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Ans.



Let h = Height of cylinder

r = Radius of cylinder

H = Height of cone

R = Radius of cone

where, H, R are constants

Correct figure **1**

$$\frac{H-h}{H}=\frac{r}{R}\left(\because \Delta ABC \sim \Delta ADE\right) \quad \ldots(i)$$ $\frac{1}{2}$

C = curved surface area = $2\pi rh$ 1

$\therefore \quad C = 2\pi rh \cdot H \cdot \left(\frac{R-r}{R}\right)$ (Using (i))

$$= \frac{2\pi H}{R}\left(rR - r^2\right)$$ 1

$$C'(r)=\frac{2\pi H}{R}(R-2r),\ C''(r)=\frac{-4\pi H}{R}<0$$ 1

$$C'(r)=0 \Rightarrow r=\frac{R}{2},\ C''\left(r=\frac{R}{2}\right)<0$$ 1

$\therefore$ Curved surface area of cylinder is Max. iff $r=\frac{R}{2}$ $\frac{1}{2}$

35. Evaluate : $\displaystyle\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$

Ans. Let $\sin x - \cos x = t \quad \therefore \quad (\sin x + \cos x)\,dx = dt$ $1+\frac{1}{2}$

Squaring we get, $\sin 2x = 1 - t^2$ $1\frac{1}{2}$

$$\int\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx = \int\frac{1}{\sqrt{1-t^2}}dt = \sin^{-1} t$$

$$= \sin^{-1}(\sin x - \cos x)$$ $1\frac{1}{2}$

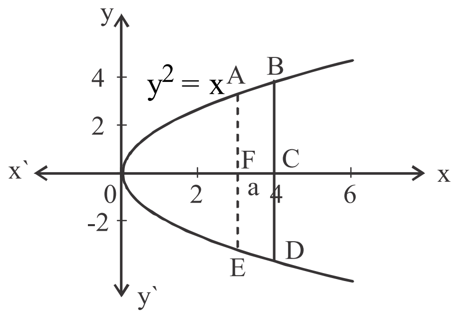
$$\therefore \int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx = \sin^{-1}(\sin x-\cos x)\Big]_{\pi/6}^{\pi/3}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)$$

$$= 2\cdot\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$ $1\frac{1}{2}$

36. If the area between the curves $x = y^2$ and $x = 4$ divided into two equal parts by the line $x = a$, then find the value of a using integration.

Ans.



Correct graph $\quad 1\frac{1}{2}$

$\text{ar}(OAEO) = \text{ar}(ABDEA)$

$\Rightarrow 2 \cdot \text{ar}(OAFO) = 2 \cdot \text{ar}(ABCFA)$

$\int_0^a \sqrt{x}\,dx = \int_a^4 \sqrt{x}\,dx \quad 1\frac{1}{2}$

$\frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right) \quad 1\frac{1}{2}$

$\Rightarrow \frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right) \quad 1\frac{1}{2}$

$\Rightarrow a^{3/2} = 4, \quad \therefore\ a = 4^{2/3} \quad 1\frac{1}{2}$

OR

Find: $\int \frac{x}{(x-1)^2(x+2)}dx$

Ans: $\int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{4+2}dx \quad 4$

$= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C \quad 2$

QUESTION PAPER CODE 65/C/3

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions.
Select the correct option.

Q.No. **Marks**

1. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the points A(2, 3, –4), B(3, –4, –5) and C(3, 2, –3) respectively, then $|\vec{a}+\vec{b}+\vec{c}|$ is equal

(A) $\sqrt{113}$ **(B)** $\sqrt{185}$ **(C)** $\sqrt{203}$ **(D)** $\sqrt{209}$

Ans: (D) $\sqrt{209}$ **1**

2. If $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, then A^2 equals

(A) $\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ **(B)** $\begin{bmatrix}2 & -2\\ -2 & -2\end{bmatrix}$ **(C)** $\begin{bmatrix}-2 & -2\\ -2 & 2\end{bmatrix}$ **(D)** $\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$

Ans: (A) $\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ **1**

3. The distance between the planes $4x - 4y + 2z + 5 = 0$ and $2x - 2y + z + 6 = 0$

(A) $\frac{1}{6}$ **(B)** $\frac{7}{6}$ **(C)** $\frac{11}{6}$ **(D)** $\frac{16}{6}$

Ans: (B) $\frac{7}{6}$ **1**

4. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(A) $\frac{7}{15}$ **(B)** $\frac{8}{15}$ **(C)** $\frac{2}{15}$ **(D)** $\frac{14}{15}$

Ans: (A) $\frac{7}{15}$ **1**

5. The objective function of an LPP is

(A) a constant **(B)** a linear function to be optimised

(C) an inequality **(D)** a quadratic expression

Ans: (B) a linear function to be optimised **1**

6. If $\sec^{-1}\left(\frac{1+x}{1-y}\right)=a$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x-1}{y-1}$ **(B)** $\frac{x-1}{y+1}$ **(C)** $\frac{y-1}{x+1}$ **(D)** $\frac{y+1}{x-1}$

Ans: (C) $\frac{y-1}{x+1}$ **1**

7. If $\int\frac{\cos 8x+1}{\tan 2x-\cot 2x}dx=\lambda\cos 8x+c$, then the value of λ is

(A) $\frac{1}{16}$ **(B)** $\frac{1}{8}$ **(C)** $-\frac{1}{16}$ **(D)** $-\frac{1}{8}$

Ans: (A) $\frac{1}{16}$ **1**

8. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

(A) 1, 1 **(B)** 1, 2 **(C)** 2, 1 **(D)** 2, 2

Ans: (A) 1, 1 **1**

9. $\int_0^1 \tan\left(\sin^{-1}x\right)dx$ equals

(A) 2 **(B)** 0 **(C)** –1 **(D)** 1

Ans: (D) 1 **1**

10. The roots of the equation $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix}=0$ are

(A) –4, 4 **(B)** 2, –4 **(C)** 2, 4 **(D)** 2, 8

Ans: (A) –4, 4 **1**

Fill in the blanks in questions numbers 11 to 15

11. A square matrix A is said to be singular if ____________.

Ans: $|A| = 0$

OR

If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB| =$ ____________.

Ans: $|AB| = -100$ **1**

12. Two angles of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$. The third angle of the triangle is __________.

Ans: $\frac{3\pi}{4}$ or $135°$ **1**

13. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is _________

Ans: $\frac{1}{4}$ **1**

14. The integrating factor of the differential equation $x\frac{dy}{dx} - y = \log x$ is _________

Ans: $\frac{1}{x}$ **1**

15. If $y = \log x$, then $\frac{d^2y}{dx^2}$ = ________.

Ans: $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then find the value of x.

Ans: $2x - 3 = x + 2$ **1/2**

$\Rightarrow \quad x = 5$ **1/2**

OR

If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19A$.

Ans: $(2 + A)^3 - 19A = A^3 + 8 + 12A + 6A^2 - 19A$ **1/2**

$= 8$ **1/2**

17. Find the solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$

Ans: Separating the variables and integrating as :

$$\int e^{-by} dy = \int e^{ax} dx$$ **1/2**

$$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c \quad \left(\text{or } be^{ax} + ae^{-by} + k = 0\right)$$ **1/2**

18. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$.

Ans: $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}} = \frac{|\vec{a}|}{|\vec{b}|}$ **1/2**

$$= \frac{3}{5\sqrt{2}}$$ **1/2**

OR

Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

Ans: $\vec{c} \perp \vec{d} \Rightarrow \vec{c} \cdot \vec{d} = 0$ **1/2**

$$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$$

$$\Rightarrow \text{Angle between vectors } \hat{a} \;\&\; \hat{b} = \frac{\pi}{3} \text{ or } 60^\circ$$ **1/2**

19. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{1/3}$, then find $(f o f)(x)$.

Ans: $fof(x) = f(f(x)) = (3 - (f(x))^3)^{1/3}$ **1/2**

$$= (3 - ((3 - x^3)^{1/3})^3)^{1/3} = (x^3)^{1/3} = x$$ **1/2**

20. Find the distance of the point (a, b, c) from the x-axis.

Ans: $\sqrt{b^2 + c^2}$ **1**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. If $y=\sqrt{a+\sqrt{a+x}}$, then find $\frac{dy}{dx}$.

Ans: $y=\sqrt{a+\sqrt{a+x}}$, let $u=a+\sqrt{a+x}$

$$y=\sqrt{u} \Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{u}}.\frac{du}{dx}, \frac{du}{dx}=\frac{1}{2\sqrt{a+x}}$$ **1+½**

$$=\frac{1}{2\sqrt{a+\sqrt{a+x}}}.\frac{1}{2\sqrt{a+x}}$$

$$=\frac{1}{4\sqrt{a+\sqrt{a+x}}\sqrt{a+x}}$$ **1**

22. Show that the three vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right-angled triangle.

Ans: Let $A(2\hat{i}-\hat{j}+\hat{k}), B(\hat{i}-3\hat{j}-5\hat{k}), C(3\hat{i}-4\hat{j}-4\hat{k})$

then $\overrightarrow{AB}=-\hat{i}-2\hat{j}-6\hat{k}, \overrightarrow{BC}=2\hat{i}-\hat{j}+\hat{k}, \overrightarrow{CA}=-\hat{i}+3\hat{j}+5\hat{k}$ **1**

$|\overrightarrow{AB}|=\sqrt{41}, |\overrightarrow{BC}|=\sqrt{6}$, $|\overrightarrow{CA}|=\sqrt{35}$, $|\overrightarrow{CA}|^2+|\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2$

$\therefore$ A,B,C are vertices of a righy angle. **1**

23. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

Ans: Let X = No. of even tickets drawn

X	0	1	2	
P(X)	$\frac{^{10}C_2}{^{19}C_2}=\frac{5}{19}$	$\frac{^{10}C_1 \cdot {^9}C_1}{^{19}C_2}=\frac{10}{19}$	$\frac{^9C_2}{^{19}C_2}=\frac{4}{19}$	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

$\frac{1}{2}$ (for the X row)

OR

Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

Ans: X = No. of success = No. of times getting a number greater than 5

X	0	1	2
P(X)	$\frac{5}{5}\cdot\frac{5}{6}=\frac{25}{36}$	$2\cdot\frac{1}{6}\cdot\frac{5}{6}=\frac{10}{36}$	$\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$

$\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

23. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx}=\sqrt{y^2-4}$

Ans: $\frac{dy}{dx} = e^x - e^{-x}$ **1**

$= \sqrt{(e^x+e^{-x})^2-4}$ **1/2**

$= \sqrt{y^2-4}$ **1/2**

24. Let W denote the set of words in the English dictionary. Define the relation R by

R = {(x , y) ∈ W × W such that x and y have at least one letter in common}

Show that this relation R is reflexive and symmetric, but not transitive.

Ans: For any word $x \in W$

x and x have atleast one (all) letter in common

$\therefore (x, x) \in R, \forall x \in W \therefore$ R is reflexive **1**

Symmetric : Let $(x, y) \in R$, $x, y \in W$

$\Rightarrow$ x and y have atleast one letter in common

$\Rightarrow$ y and x have atleast one letter in common

$\Rightarrow (y, x) \in R \therefore$ R is symmetric **1/2**

Transitive : Taking example of three English dictionary words
$x, y, z, \in W$ such that $(x , y), (y, z) \in R$ but $(x, z) \notin R$ **1/2**

$\therefore$ R is not transitive

OR

Find the inverse of the function $f(x)=\left(\frac{4x}{3x+4}\right)$.

Ans: Let $y = f(x) = \frac{4x}{3x+4} \Rightarrow x = \frac{4y}{4-3y}$ **$1\frac{1}{2}$**

$\therefore\ f^{-1}(y)=\frac{4y}{4-3y}\left(\text{or } f^{-1}(x)=\frac{4x}{4-3x}\right)$ **1/2**

25. For the matrix $A=\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}$, verify the following:

A(adj A) = (adj A) A = |A| I

Ans: $|A| = -12 + 12 = 0$, adj $A=\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}$ $\frac{1}{2}+1$

$A \cdot (\text{adj } A) = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = (\text{adj } A) \cdot A = |A|\, I$ $\frac{1}{2}$

26. Solve the following homogeneous differential equation: $x\frac{dy}{dx} = x + y$

Ans: Let $y = vx \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}+\frac{1}{2}$

$\therefore x\left(v + x\frac{dv}{dx}\right) = x + vx \Rightarrow x\frac{dv}{dx} = 1$ $\frac{1}{2}$

$\therefore \int dv = \int \frac{1}{x}dx \Rightarrow v = \log|x| + c$

$\Rightarrow y = x(\log|x| + c)$ $\frac{1}{2}$

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Solve for x: $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

Ans: $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2} \Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1} x$ $\frac{1}{2}$

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} + 2\sin^{-1} x\right)$ 1

$\Rightarrow 1 - x = \cos(2\sin^{-1} x) = 1 - 2\sin^2(\sin^2 x)$ $1\frac{1}{2}$

$\Rightarrow 1 - x = 1 - 2x^2 \quad \therefore x = 0, \frac{1}{2}$ $\frac{1}{2}$

$\Rightarrow x = \frac{1}{2}$ does not satisfy the given equation $\therefore x \neq \frac{1}{2}, x = 0$ … $\frac{1}{2}$

28. The random variable X has a probability P(x) as defined below, where k is some number:

$$P(x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}.$$

Find :

(i) The value of k.

(ii) $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$

Ans: (i) $P(x = 0) + P(x = 1) + P(x = 2) + P(otherwise) = 1 \Rightarrow k = \frac{1}{6}$ **1**

(ii) $P(x < 2) = P(x = 0) + P(x = 1) = 3k = \frac{1}{2}$ **1**

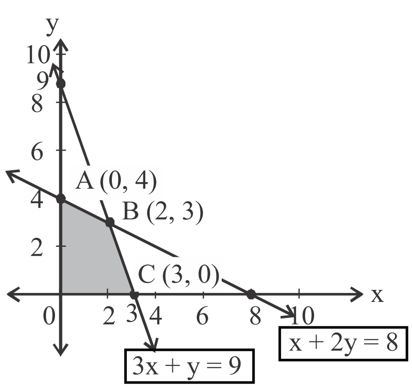
(iii) $P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2) = 6k = 1$ **1**

(iv) $P(x \geq 2) = P(x = 2) + P(otherwise) = 3k = = \frac{1}{2}$ **1**

29. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3g of silver and 1g of gold, while that of type B requires 1g of silver and 2g of gold. The company can use at the most 9g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 120 and that of type B ₹ 150, then find the number of units of each type that the company should produce to maximise profit.

Formulate the above LPP and solve it graphically. Also, find the maximum profit.

Ans.



Let No. of goods type A = x,

Number of goods type B = y.

Then the L.P. P. is:

Maximize (Profit) : Z = 120x + 150y **1**

Subject to constraints :

$3x + y \leq 9$

$x + 2y \leq 8$ **1**

$x , y \geq 0$

Correct figure **$1\frac{1}{2}$**

Corner:	Value of Z (in ₹)
0(0, 0)	0
A(0, 4)	600
B(2, 3)	690 (Max)
C(3, 0)	360

1/2

$\therefore$ Max. profit = ₹ 690

when Good Type A = 2 units, Type B = 3 units

30. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y-1}{3} = z, \quad \frac{x+1}{5} = \frac{y-2}{1}, \; z = 2$$

Ans. Let $\left.\begin{array}{l}\vec{a}_1 = \hat{i} - \hat{j} \; ; \; \vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k} \\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k} \; ; \; \vec{b}_2 = 5\hat{i} + \hat{j}\end{array}\right\}$ **1**

then, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 5\hat{j} - 13\hat{k}$ **1/2+1$\frac{1}{2}$**

$\therefore$ Shortest distance $= \left| \dfrac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \dfrac{9}{\sqrt{195}} \neq 0$ **1/2**

$\therefore$ lines are not intersecting **1/2**

OR

Find the equation of the plane through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which is at a unit distance from the origin.

Ans: Equation of plane through the line of intersection of the two given planes is

$\vec{r} \cdot \left[(1+3\lambda)\hat{i} + (3-\lambda)\hat{j} - 4\lambda\hat{k}\right] = -6$ **1$\frac{1}{2}$**

As per the given condition

$\left| \dfrac{-6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}} \right| = 1 \Rightarrow \lambda = \pm 1$ **1$\frac{1}{2}$**

$\therefore$ Equation of plane is : $\vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) + 6 = 0$ **1/2**

or $\vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) + 6 = 0$ **1/2**

31. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \; \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Ans: $\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$; $\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$ **1/2+1/2**

Let $\left.\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2\hat{i} + 4\hat{j} - 2\hat{k}\right\}$ [1]

Unit vector perpendicular to each of the vector

$\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is $\hat{c} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ **1**

32. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Ans. $\text{LHS} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

(Applying $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$)

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b\left(1+a^2+b^2\right) & -a\left(1+a^2+b^2\right) & 1-a^2-b^2 \end{vmatrix}$$ **2**

(Taking $\left(1+a^2+b^2\right)$ common from C_1 & C_2)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$ **1/2**

(Applying $C_3 \to C_3 + 2bC_1 - 2aC_2$)

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & -a & 1+a^2+b^2 \end{vmatrix}$$ **1**

(Expand along C_3)

$= \left(1+a^2+b^2\right)^3 = \text{RHS}$ **1/2**

OR

Find the equation of the line joining A(1, 3) and B(0, 0), using determinants. Also, find k if D(k, 0) is a point such that the area of the ΔABD is 3 square units.

Ans: Equation of the line through A(1, 3) and B(0, 0) is

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow 3x - y = 0$$ **2**

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow k = \pm 2$$ **2**

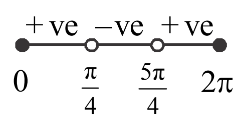
SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. Find the intervals in which the function f defined as f(x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing or decreasing.

Ans. $f'(x) = \cos x - \sin x,\ 0 \le x \le 2\pi$ **1**

$\therefore\ f'(x) = 0 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$ **2**

Sign of $f'(x)$:  **$1\frac{1}{2}$**

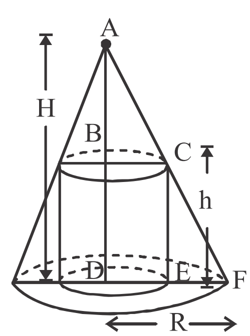
$\therefore$ f(x) is strictly increasing on $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ **1**

and f(x) is strictly decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ **$\frac{1}{2}$**

OR

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Ans.



Let h = Height of cylinder

r = Radius of cylinder

H = Height of cone

R = Radius of cone

where, H, R are constants

Correct figure **1**

$$\frac{H-h}{H} = \frac{r}{R} \quad (\because \Delta ABC \sim \Delta ADE) \quad \ldots(i)$$ **$\frac{1}{2}$**

C = curved surface area = $2\pi rh$ **1**

$$\therefore\ C = 2\pi rh \cdot H \cdot \left(\frac{R-r}{R}\right) \qquad \text{(Using (i))}$$

$$= \frac{2\pi H}{R}\left(rR - r^2\right) \qquad \mathbf{1}$$

$$C'(r) = \frac{2\pi H}{R}(R-2r),\ C''(r) = \frac{-4\pi H}{R} < 0 \qquad \mathbf{1}$$

$$C'(r) = 0 \ \Rightarrow\ r = \frac{R}{2},\ C''\left(r = \frac{R}{2}\right) < 0 \qquad \mathbf{1}$$

$\therefore$ Curved surface area of cylinder is Max. iff $r = \frac{R}{2}$ **$\frac{1}{2}$**

34. Evaluate $\int_1^3 \left(x^2 + 1 + e^x\right)dx$ as the limit of sums.

Ans. Let $a = 1$, $b = 3$, $nh = 2$, $f(x) = x^2 + 1 + e^x$ **1**

$$f(a) + f(a+h) + f(a+2h) + \dots + f(a+\overline{n-1}h)$$

$$= \underbrace{(2+2+\dots+2)}_{\leftarrow n\ terms \rightarrow} + 2h(1+2+3+\dots+(n-1)) + h^2(1^2+2^2+3^2+\dots+(n-1)^2)$$

$$+(e + e^{1+h} + e^{1+2h} + \dots + e^{1+(n-1)h}) \qquad \mathbf{1\frac{1}{2}}$$

$$= 2h + h(n-1)n + \frac{h^2}{6}(n-1)(n)(2n-1) + \frac{e(e^{nh}-1)}{e^h - 1} \qquad \mathbf{1\frac{1}{2}}$$

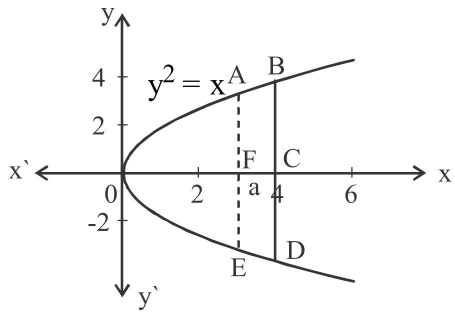
$$\int_1^3 (x^2+1+e^x)dx = \lim_{h\to 0}\left[2(nh) + (nh-h)(nh) + \frac{(nh-h)(nh)(2nh-h)}{6} + e.\frac{h}{e^h-1}(e^{nh}-1)\right]$$

$$= \lim_{h\to 0}\left[4 + (2-h)(2) + \frac{(2-h).2.(4-h)}{6} + e.\frac{h}{e^h-1}(e^2-1)\right] \qquad \mathbf{1}$$

$$= \frac{32}{3} + e(e^2-1) \qquad \mathbf{1}$$

35. If the area between the curves $x = y^2$ and $x = 4$ divided into two equal parts by the line $x = a$, then find the value of a using integration.

Ans.



Correct graph $1\frac{1}{2}$

$\text{ar}(OAEO) = \text{ar}(ABDEA)$

$\Rightarrow 2 \cdot \text{ar}(OAFO) = 2 \cdot \text{ar}(ABCFA)$

$\int_0^a \sqrt{x}\, dx = \int_a^4 \sqrt{x}\, dx$ $1\frac{1}{2}$

$\frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right)$ $1\frac{1}{2}$

$\Rightarrow \frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right)$ $1\frac{1}{2}$

$\Rightarrow a^{3/2} = 4, \ \therefore\ a = 4^{2/3}$ $1\frac{1}{2}$

OR

Find: $\int \frac{x}{(x-1)^2 (x+2)} dx$

Ans: $\int \frac{x}{(x-1)^2(x+2)} dx = \frac{2}{9}\int \frac{1}{(x-1)} dx + \frac{1}{3}\int \frac{1}{(x-1)^2} dx - \frac{2}{9}\int \frac{1}{4+2} dx$ 4

$= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$ 2

36. If $y = x^{\sin x} + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Ans. Let $u = x^{\sin x}$ $\therefore\ y = u + \sin^{-1}\sqrt{x}$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{2\sqrt{x}\,\sqrt{1-x}}$ ... (i) $1\frac{1}{2}$

Also, $\log u = \sin x \cdot \log x$ 1

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \cdot \log x + \frac{\sin x}{x}$ 2

$\Rightarrow \frac{du}{dx} = x^{\sin x} \cdot \left(\cos x \cdot \log x + \frac{\sin x}{x}\right)$ ... (ii) $\frac{1}{2}$

Putting (ii) in (i) we get

$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \log x + \frac{\sin x}{x}\right) + \frac{1}{2\sqrt{x}\,\sqrt{1-x}}$ 1