KVPY-SX 2017 (PHYSICS)

PART-I

1. The magnitude of acceleration of the electron in the n^{th} orbit of hydrogen atom is a_H and that of singly ionized helium atom is a_{He}. The ratio $a_H : a_{He}$ is

(A) 1 : 8 (B) 1 : 4 (C) 1 : 2 (D) dependent on n

2. A carrot looks orange in colour because of the β carotene molecule in it. This means that the βcarotene molecule absorbs light of wavelengths

(A) longer than 550 nm (B) shorter than 550 nm

(C) longer than 700 nm (D) shorter than 700 nm

3. If some charge is given to a solid metallic sphere, the field inside remains zero and by Gauss's law all the charge resides on the surface. Suppose now that Colomb's force between two charges varies as $1/r^3$. Then, for a charged solid metallic sphere

(A) Field inside will be zero and charge density inside will be zero.

(B) Field inside will not be zero and charge density inside will not be zero.

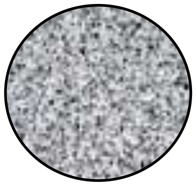
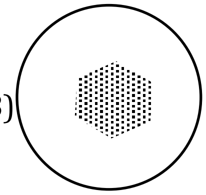
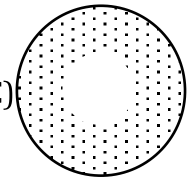
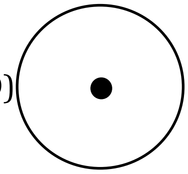
(C) Field inside will not be zero and charge density inside will be zero.

(D) Field inside will be zero and charge density inside will not be zero.

4. Using dimensional analysis the resistivity in terms of fundamental constants h, m_e, c, e, ε_0 can be expressed as

(A) $\dfrac{h}{\varepsilon_0 m_e c e^2}$ (B) $\dfrac{\varepsilon_0 m_e c e^2}{h}$ (C) $\dfrac{h^2}{m_e c e^2}$ (D) $\dfrac{m_e \varepsilon_0}{c e^2}$

5. Consider a bowl filled with water on which some black pepper powder have been sprinkled uniformly. Now a drop of liquid soap is added at the centre of the surface of water. The picture of the surface immediately after this will look like

(A)  (B)  (C)  (D) 

6. It was found that the refractive index of material of a certain prism varied as $1.5 + 0.004/\lambda^2$, where λ is the wavelength of light used to measure the refractive index. The same material was then used to construct a thin prism of apex angle 10°. Angles of minimum deviation (δ_m) of the prism were recorded for the sources with wavelengths λ_1 and λ_2 respectively. Then

(A) $\delta_m(\lambda_1) < \delta_m(\lambda_2)$ if $\lambda_1 < \lambda_2$
(B) $\delta_m(\lambda_1) > \delta_m(\lambda_2)$ if $\lambda_1 > \lambda_2$
(C) $\delta_m(\lambda_1) > \delta_m(\lambda_2)$ if $\lambda_1 < \lambda_2$
(D) δ_m is the same in both the cases

7. Two circularly shaped linear polarisers are placed coaxially. The transmission axis of the first polarizer is at 30° from the vertical while the second one is at 60°, both in the clockwise sense. If an unpolarised beam of light of intensity I = 20 W/m^2 is incident on this pair of polarisers, then the intensities I_1 and I_2 transmitted by the first and the second polarisers, respectively, will be close to

(A) I_1 = 10.0 W/m^2 and I_2 = 7.5 W/m^2
(B) I_1 = 20.0 W/m^2 and I_2 = 15 W/m^2
(C) I_1 = 10.0 W/m^2 and I_2 = 8.6 W/m^2
(D) I_1 = 15.0 W/m^2 and I_2 = 0.0 W/m^2

8. An electron in an electron microscope with initial velocity $v_0\hat{i}$ enters a region of a stray transverse electric field $E_0\hat{j}$. The time taken for the change in its de-Broglie wavelength from the initial value of λ to $\lambda/3$ is proportional to

(A) E_0
(B) $\frac{1}{E_0}$
(C) $\frac{1}{\sqrt{E_0}}$
(D) $\sqrt{E_0}$

9. A bird sitting on a single high tension wire does not get electrocuted because

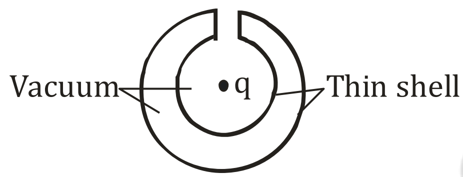
(A) The circuit is not complete.

(B) The bird feet has an insulating covering.

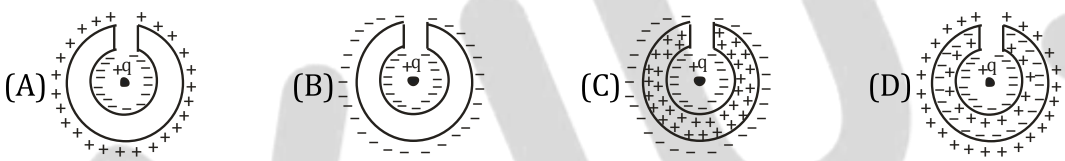
(C) Capacitance of the bird is too small and the line frequency is too small.

(D) Resistance of the bird is too high

10. A positive charge q is placed at the center of a neutral hollow cylindrical conducting shell with its cross section as shown in the figure below



Which one of the following figures correctly indicates the induced charge distribution on the conductor (ignore edge effects).

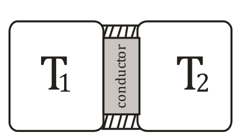


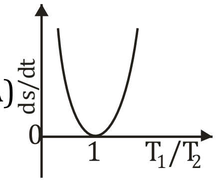
11. A transverse wave of frequency 500 Hz and speed 100 m/s is traveling in the positive x direction on a long string. At time t = 0 s the displacements at x = 0.0 m and at x = 0.25 m are 0.0 m and 0.02 m, respectively. The displacement at x = 0.2 m at $t = 5 \times 10^{-4}$ s is

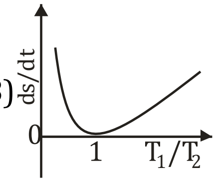
(A) –0.04 m (B) –0.02 m (C) 0.04 m (D) 0.02 m

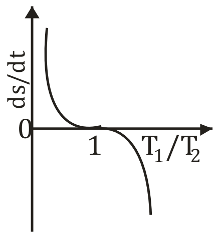
12. A thin piece of thermal conductor of constant thermal conductivity insulated on the lateral sides connects two reservoirs which are maintained at temperatures T_1 and T_2 as shown.

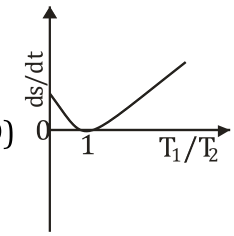
Assuming that the system is in steady state, which of the following plots best represents thedependence of the rate of change of entropy of the ratio of temperatures T_1/T_2



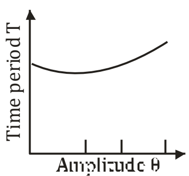
(A) 

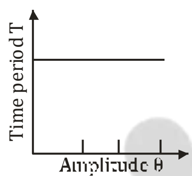
(B) 

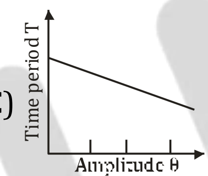
(C) 

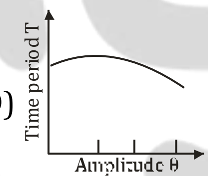
(D) 

13. Which of the following plots represents schematically the dependence of the timeperiod of a pendulum if measured and plotted as a function of its oscillations? (Note: Amplitude need not be small)

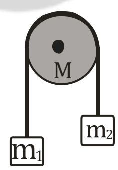
(A) 

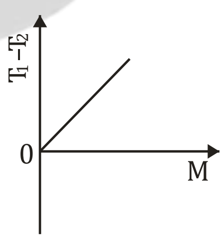
(B) 

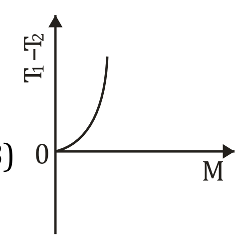
(C) 

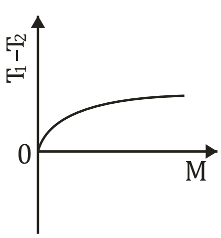
(D) 

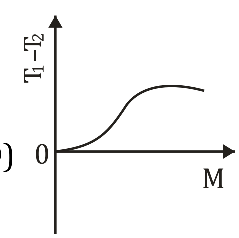
14. On a pulley of mass M hangs a rope with two masses m1 and m_2 ($m_1 > m_2$) tied at the ends as shown in the figure. The pulley rotates without any friction, whereas the friction between the rope and the pulley is large enough to prevent any slipping. Which of the following plots best represents the difference between the tensions in the rope on the two sides of the pulley as a function of the mass of the pulley?



(A) 

(B) 

(C) 

(D) 

15. Two satellites S_1 and S_2 are revolving around a planet in the opposite sense in coplanar circular concentric orbits. At time t = 0, the satellites are farthest apart. The periods of revolution of S_1 and S_2 are 3 h and 24 h respectively. The radius of the orbit of S_1 is 3×10^4 km. Then the orbital speed of S_2 as observed from

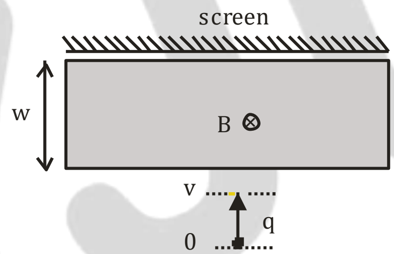
(A) the planet is $4\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1.

(B) the planet is $2\pi \times 10^4$ km h^{-1} when S_2 is farthest from S_1.

(C) S_1 is $\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1.

(D) S_1 is $3\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1.

16. A rectangular region of dimensions w × *l* (w <<*l*) has a constant magnetic field into the plane of the paper as shown. On one side the region is bounded by a screen. On the other side positive ions of mass m and charge q are accelerated from rest and towards the screen by a parallel plate capacitor at constant potential difference v< 0, and come out through a small hole in the upper plate. Which one of the following statements is correct regarding the charge on the ions that hit the screen?



(A) Ions with $q > \frac{2|v|m}{B^2w^2}$ will hit the screen.

(B) Ions with $q < \frac{2|v|m}{B^2w^2}$ will hit the screen.

(C) All ions will hit the screen.

(D) Only ions with $q = \frac{2|v|m}{B^2w^2}$ will hit the screen.

17. Force $\vec{F}$ applied on a body is written as $\vec{F} = (\hat{n}.\hat{F})\hat{n} + \vec{G}$, where $\hat{n}$ is a unit vector. The vector $\vec{G}$ is equal to

(A) $\hat{n} \times \vec{F}$ (B) $\hat{n} \times (\hat{n} \times \vec{F})$ (C) $(\hat{n} \times \vec{F}) \times \vec{F}/|\vec{F}|$ (D) $(\hat{n} \times \vec{F}) \times \hat{n}$

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18. A particle of mass m moves around the origin in a potential $\frac{1}{2}m\omega^2 r^2$, where r is the distance from the origin. Applying the Bohr model in this case, the radius of the particle in its nth orbit in terms of $a = \sqrt{h/(2\pi m\omega)}$ is:

(A) $a\sqrt{n}$ (B) an (C) an^2 (D) $an\sqrt{n}$

19. Two bottles A and B have radii R_A and R_B and heights h_A and h_B respectively with $R_B = 2R_A$ and $h_B = 2h_A$. These are filled with hot water at 60°C. Consider that heat loss for the bottles takes place only from side surfaces. If the time the water to cool down to 50°C is t_A and t_B for the bottles A and B, respectively, then t_A and t_B are best related as

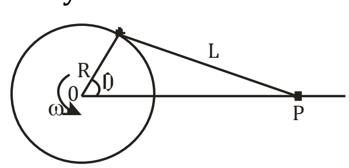
(A) $t_A = t_B$ (B) $t_B = 2t_A$ (C) $t_B = 4t_A$ (D) $t_A = t_A/2$

20. The number of gas molecules striking per second per square meter of the top surface of a table placed in a room at 20°C and 1 atmospheric pressure is of the order of ($k_B = 1.4 \times 10^{-23}$ J/K, and the average mass of an air molecules is 5×10^{-27} kg)

(A) 10^{27} (B) 10^{23} (C) 10^{25} (D) 10^{29}

PART-II

21. One end of a rod of length L=1 m is fixed to a point on the circumference of a wheel of radius $R = 1/\sqrt{3}$. The other end is sliding freely along a straight channel passing through the center O of the wheel as shown in the figure below. The wheel is rotating with a constant angular velocity ω about O.



The speed of the sliding end P when $\theta = 60°$ is

(A) $\frac{2\omega}{3}$ (B) $\frac{\omega}{3}$ (C) $\frac{2\omega}{\sqrt{3}}$ (D) $\frac{\omega}{\sqrt{3}}$

22. One mole of an ideal monatomic gas undergoes the following four reversible processes:

Step-1: It is first compressed adiabatically from volume V_1 to $1m^3$.

Step-2: then expanded isothermally to volume $10\ m^3$.

Step-3: then expanded adiabatically to volume V^3.

Step-4: then compressed isothermally to volume V_1.

If the efficiency of the above cycle is 3/4 then V_1 is,

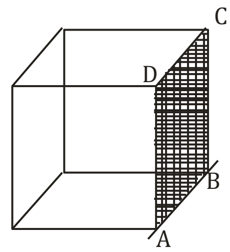
(A) $2m^3$ (B) $4m^3$ (C) $6\ m^3$ (D) $8\ m^3$

23. A neutron star with magnetic moment of magnitude m is spinning with angular velocity ω about its magnetic axis. The electromagnetic power P radiated by it is given by $\mu_0^x m^y \omega^z c^u$ where μ_0 and c are the permeability and speed of light in free space, respectively. Then

(A) x = 1, y = 2, z = 4 and u = –3 (B) x = 1, y = 2, z = 4 and u = 3

(C) x = –1, y = 2, z = 4 and u = –3 (D) x = –1, y = 2, z = 4 and u = 3

24. A solid cube of wood of side 2a and mass M is resting on a horizontal surface as shown in the figure. The cube is free to rotate about a fixed axis AB. A bullet of mass m (<<M) and speedv is shot horizontally at the face opposite to ABCD at a height 4a/3 from the surface to impart the cube an angular speed ω. It strike the face and embeds in the cube. Then ω_c is close to (Note: The moment of inertia of the cube about an axis perpendicular to the face and passing through the center of mass is $2Ma^2/3$)



(A) Mv/ma (B) Mv/2ma (C) mv/Ma (D) mv/2Ma

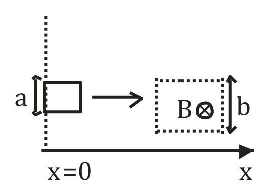
25. A gas obeying the equation of state PV = RT undergoes a hypothetical reversible process described by the equation, $PV^{5/3}\exp\left(-\frac{PV}{E_0}\right)=c_1$ where c_1 and E_0 are dimensioned constants. Then, for this process, the thermal compressibility at high temperature

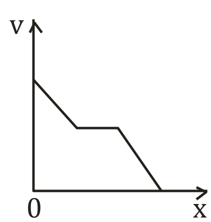
(A) Approaches a constant value
(B) is proportional to T
(C) is proportional to $T^{1/2}$
(D) is proportional to T^2

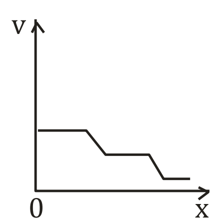
26. To calculate the size of a hydrogen anion using the Bohr model, we assume that its two electrons move in an orbit such that they are always on diametrically opposite sides of the nucleus. With each electron having the angular momentum h = h/2π, and taking electron interaction into account the radius of the orbit in terms of the Bohr radius of hydrogen atom $a_B = \frac{4\pi\varepsilon_0 h^2}{me^2}$ is:

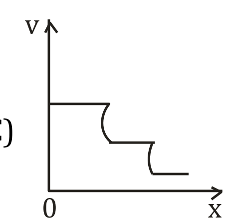
(A) a_B
(B) $\frac{4}{3}a_B$
(C) $\frac{2}{3}a_B$
(D) $\frac{3}{2}a_B$

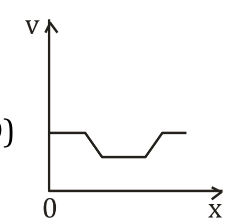
27. A square-shaped conducting wire loop of dimension *a* moving parallel to the x-axis approaches a square region of size *b* (*a* < *b*) where a uniform magnetic field B exists pointing into the plane of the paper (see figure). As the loop passes through this region, the plot correctly depicting its speed (v) as a function of x is



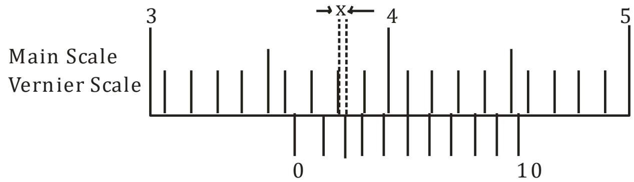
(A) 

(B) 

(C) 

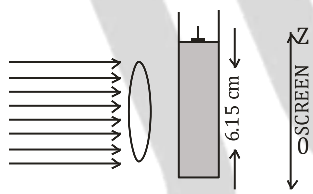
(D) 

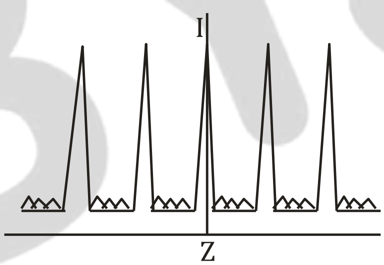
28. The figure of a centimeter scale below shows a particular position of the vernier calipers. In this position the value of x shown in the figure is (figure is not to scale)

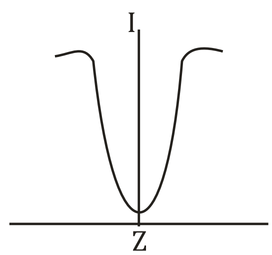


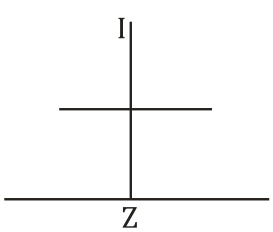
(A) 0.02 cm (B) 3.65 cm (C) 4.15 cm (D) 0.03 cm

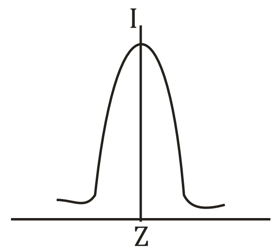
29. A parallel beam of light is incident on a tank filled with water up to a height of 61.5 mm as shown in the figure below. Ultrasonic waves of frequency 0.5 MHz are sent along the length of the water column using a transducer placed at the top, and they form longitudinal standing waves in the water. Which of the schematic plots below best describes the intensity distribution of the light as seen on the screen?
Take the speed of sound in water to be 1,500 m/s.



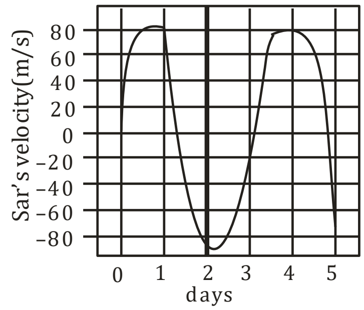
(A)


(B)


(C)


(D)


30. A star of mass M (equal to the solar mass) with a planet (much smaller than the star) revolves around the star in a circular orbit. The velocity of the star with respect to the center of mass of the star-planet system is shown below:



The radius of the planet's orbit is closest to (1 A. U. = Earth-Sun distance)

(A) 0.004 A. U. (B) 0.008 A.U. (C) 0.004 A.U. (D) 0.12 A.U.

ANSWER KEY

1. (A)	2. (B)	3. (D)	4. (C)	5. (C)
6. (C)	7. (A)	8. (B)	9. (C)	10. (A)
11. (B)	12. (B)	13. (A)	14. (C)	15. (D)
16. (B)	17. (D)	18. (A)	19. (B)	20. (A)
21. (A)	22. (D)	23. (A)	24. (D)	25. (A)
26. (B)	27. (B)	28. (D)	29. (A)	30. (C)

SOLUTIONS

PART-I

1. (A)

Centripetal acceleration of electron in n^{th} orbit,

$$a_n = \frac{v_n^2}{r_n}$$

a_n = acceleration of electron in n^{th} orbit

n = integer, z = atomic number

Since, $v_n \propto \frac{z}{n}$ and $r_n \propto \frac{n^2}{z}$

$\Rightarrow$ $a_n \propto \frac{z^3}{n^4}$

So, we get $\frac{a_H}{a_{He}} = \frac{z_H^3}{z_{He}^3}$

z_H = atomic number of hydrogen

z_{He} = atomic number of helium

Given: $z_{He} = 2$ and $z_H = 1$

$\Rightarrow$ $\frac{a_H}{a_{He}} = \frac{1^3}{2^3} = \frac{1}{8}$

2. (B)

By the help of VIBGYOR

VIBG	|	YOR
400 nm	|	700 nm
Absorbed		Reflected

As we know in VIBGYOR, the wavelength of a different colour is different in a specific region. So, O stood for orange and observed the wavelength shorts than 550 nm. The concept behind it is that the colour that we see in the reflected specific wavelength and rest all the complementary colours are absorbed. As we can see that in the question it is given that we are perceiving the orange colour then this means that only the orange wavelength is the reflected one. And the wavelength of yellow, orange, and red lies above 700 nm. So the complementary absorbed wavelengths are violet, indigo, blue, green and all these wavelengths lie below 550 nm.

3. (D)

According to questions,

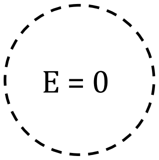
Coulomb's force is

$$F \propto \frac{1}{r^3}$$

Then Gauss law will not follow.

$\therefore$ $\phi \neq \frac{q_{in}}{\varepsilon_o}$

If condition is static then



Where, $\phi \rightarrow$ field lines per unit area.

If electric field is zero then field lines will also be zero.

$$\phi = 0$$

$$\Rightarrow \phi \neq \frac{q_{in}}{\varepsilon_o}$$

$$\therefore \frac{q_{in}}{\varepsilon_o} \neq 0$$

So, charge inside will not be zero and charge density also will not be zero. But electric field is zero inside.

4. (C)

$\because$ Electric field = Resistivity × Current density

$$E = \rho \times J$$

$$\Rightarrow \rho = \frac{E}{J} \qquad \text{...(1)}$$

Dimension of E and J are

$\because$ $[E] = [MLT^{-3}I^{-1}]$ and $J = [IL^{-2}]$

Put these dimension in eq.(1)

$$\therefore [\rho] = \frac{[MLT^{-3}I^{-1}]}{[IL^{-2}]} = [ML^3T^{-3}I^{-2}]$$

$\because$ $h = [ML^2T^{-1}]$

$m_e = [M]$

$C = [LT^{-1}]$

$\varepsilon_o = [M^{-1}L^{-3}T^4I^2]$

$e = [IT]$

From question

$$\rho = h^a M_e^b C^c \varepsilon_o^d e^e$$

Now, put all the dimensions in equation

$$[ML^3T^{-3}I^{-2}] = [ML^2T^{-1}]^a[M]^b[LT^{-1}]^c[M^{-1}L^{-3}T^4I^2]^d[IT]^e$$

Now compare

$$[M^1L^3T^{-3}I^{-2}] = [M^{a+b-d}][L^{2a+c-3d}][T^{-a-c+4d+e}][I^{2d+e}]$$

Now compare powers of LHS and RHS,

$1 = a + b - d$ ...(1)

$3 = 2a + c - 3d$ ...(2)

$-3 = -a - c + 4d + e$ ...(3)

$-2 = 2d + e$ ...(4)

By solving we get, $d = 0, a = 2, b = -1, c = -1, e = -2$

$\therefore \quad \rho = \dfrac{h^2}{m_e C e^2}$

5. (C)

The water surface exerts a surface tension when the pepper powder is sprinkled uniformly on the water surface, the powder particles remain at their position. But as soon as a drop of liquid soap is added at the center of water surface, surface tension of water breaks at the center, so the remaining part of water exerts a net surface tension outwards and the water spreads outwards taking the powder particles outwards along with it as shown in the figure C.

6. (C)

Angle of minimum deviation,

$\delta_m = (\mu - 1)A$

$\delta_m \propto \mu$

μ = refractive index

So, more is the refractive index of the material,

Lesser is the deviation.

$\delta_m \propto \frac{1}{\lambda^2}$ (λ = wavelength)

Also, more is the wavelength of light, lesser is the refractive index of material.

$\Rightarrow$ $\delta_m(\lambda_1) > \delta_m(\lambda_2)$ if $\lambda_1 < \lambda_2$

$\lambda_2 > \lambda_1$ if $(\delta_m)_1 > (\delta_m)_2$

7. (A)

Intensity of unpolarized light, I = 20 w/m^2

Intensity of light passing through first polarizer,

$I_1 = \frac{I}{2} = \frac{20}{2} = 10$ w/m^2

Angle between transmission axis of two polarizer, $\theta = 60° - 30° = 30°$

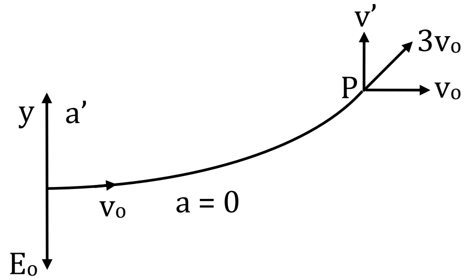
Intensity of light passing through second polarizer,

$I_2 = I_1 \cos^2\theta$

$I_2 = 10 \times \frac{3}{4}$

$I_2 = 7.5$ w/m^2

8. (B)



Path traced by the electron is shown in the figure such that electron reaches the point P when its de-Broglie wavelength becomes $\frac{\lambda}{3}$.

Acceleration of electron is zero in x-direction.

Acceleration of electron in y-direction a' = $\frac{eE_o}{m}$

Velocity of electron initially, $v_o = \frac{h}{\lambda}$

At P, $v_p = \frac{h}{(\lambda/3)} = 3v_o$

So, velocity is in y direction,

$v' = \sqrt{(v')^2 - v_o^2} = \sqrt{9v_o^2 - v_o^2}$

$v' = 2\sqrt{2}$

Using, v' = a't

$\therefore \quad 2\sqrt{2}\, v_o = \frac{eE_o}{m} t \qquad (\therefore\ a = \frac{F}{m}, F = qE = eE, \text{so } a = \frac{eE}{m})$

We get, $t = \frac{2\sqrt{2} v_o m}{eE_o}$

$\Rightarrow \ t \propto \frac{1}{E_o}$

9. (C)

The answer to this question has to be explained with the context of how electricity flows. It will always move along the path of least resistance and flows best through conductors. The copper in electrical wires is a great conductor, birds are not good conductors. That's one reason they don't get shocked when they sit on electrical wires. The energy bypasses the birds and keeps flowing along the wire instead.

There's another reason why birds can sit on a wire without getting shocked. There's no voltage difference in a single wire. For electrons to move, there must be what scientists call a difference in electrical potential. For example, energy flows from area of high voltage to areas of low voltage. If it flows through a single power line at 35,000 volts, it will continue along the path of least resistance. That means it'll bypass birds because there's no difference in electrical potential.

10. (A)

When a positive charge is placed at the centre of hollow cylindrical conducting shell, an equal amount of negative charge is induced on the inner surface of shell due to induction process and so an equal amount of positive charge is induced on the outer surface of shell as shown in option A.

11. (B)

Wavelength is $\lambda = \frac{v}{f} = \frac{100}{500} = 0.2$ m

Let the wave equation be,

$y = A\sin(kx - \omega t + \phi)$ ...(1)

Where, $\omega t + \phi$ = phase

Given,

$y = 0$ at $t = 0$, $x = 0$

$0 = A\sin(\phi)$

$\Rightarrow \phi = 0$

Also, $y = 0.02$ at $t = 0$, $x = 0.25$

Put the values in eq.(1)

$$0.02 = A\sin\left(\frac{2\pi x}{\lambda}\right)$$

$$0.02 = A\sin\left(\frac{2\pi\times 0.25}{0.2}\right)$$

$\Rightarrow A = 0.02$ m

At $t = 5 \times 10^{-4}$ sec and $x = 0.2$ m

Again put these values

$$y = 0.02\sin\left(\frac{2\pi\times 0.2}{0.2} - 2\pi\times 5\times 10^{-4}\times 500\right)$$

$$= 0.02\sin\left(2\pi - \frac{\pi}{2}\right)$$

$y = -0.02$ m

12. (B)

Let $T_1 > T_2$ and R is thermal resistance and apply entropy change in the process,

$$\Delta S = -(\Delta S_{reservoir} - \Delta S_{sink})$$

$$= -\left(\frac{Q}{T_1} - \frac{Q}{T_2}\right)$$

∵ Q is same since the system is at steady state.

$$\Delta S = -Q\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\frac{dS}{dt} = -\frac{dQ}{dt}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

∵ V = IR

$$(T_1 - T_2) = QR$$

$$Q = \frac{T_1 - T_2}{R}$$

Put the values of Q in ΔS

$$\therefore \quad \frac{dS}{dt} = -\left(\frac{T_1 - T_2}{R}\right)\left(\frac{T_2 - T_1}{T_1 T_2}\right)$$

$$\frac{dS}{dt} = \frac{(T_2 - T_1)^2}{T_1 T_2} \cdot \frac{1}{R}$$

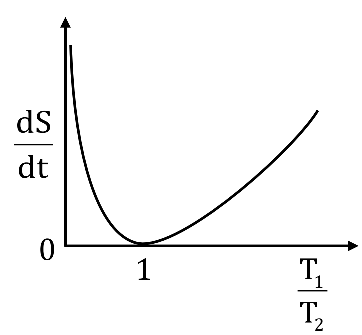
$$\frac{dS}{dt} = \left(\frac{T_1^2 + T_2^2 - 2T_1T_2}{T_1 T_2}\right) \cdot \frac{1}{R}$$

$$\frac{dS}{dt} = \left(\frac{T_1}{T_2} + \frac{T_2}{T_1} - 2\right) \cdot \frac{1}{R}$$

Let $\frac{T_1}{T_2} = x$

$$\frac{dS}{dt} = \left(x + \frac{1}{x} - 2\right) \cdot \frac{1}{R}$$

So, the graph between rate of change of entropy and ratio of temperatures $\frac{T_1}{T_2}$ is,



And it is the graph of $\left(x + \frac{1}{x} - 2\right)$.

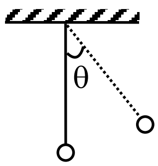
13. (A)

Time period versus amplitude (θ),

Generally time period is

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

ℓ = length of pendulum, g = acceleration due to gravity



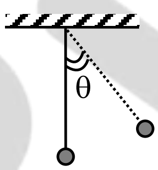
(Valid only when θ is very very small)

But the original formula when θ is large

Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}\left(1+\frac{1}{16}\theta_0^2+\frac{11}{3072}\theta_0^4+......\right)$

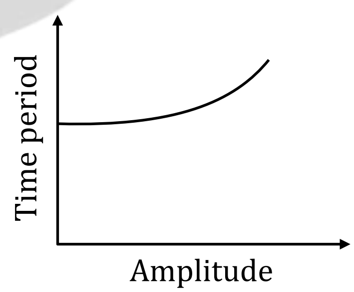
∴ Time period of a simple pendulum when θ_0 (amplitude) is large is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1+\frac{\theta_0^2}{16}\right)$$

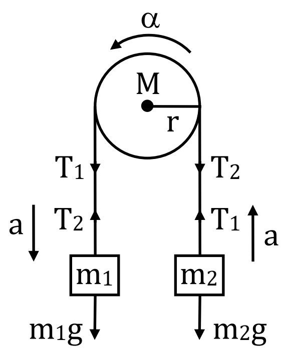


So, according to this relation time period will increase when amplitude increase.

$$T \propto \left(1+\frac{\theta_0^2}{16}\right)$$



14. (C)



Let the tension in both ropes be T_1 and T_2 respectively and acceleration of masses be a as shown in the figure.

For block m_1:

$m_1a = m_1g - T_1$ ...(1)

For block m_2:

$m_2a = T_2 - m_2g$ ...(2)

Adding eq.(1) & (2), we get

$(m_1 + m_2)a = (m_1 - m_2)g - (T_1 - T_2)$ ...(a)

$\because$ Torque, $\tau = I\alpha$

I = moment of inertia, α = angular acceleration

$\therefore$ $(T_1 - T_2)r = \frac{1}{2}Mr^2\alpha$ $\left(\because \text{Moment of inertia of disc } I = \frac{MR^2}{2}\right)$

$(T_1 - T_2) = \frac{1}{2}Mr\alpha = \frac{1}{2}Ma$ $(\because a = r\alpha)$ ...(b)

From eq.(a) & (b),

$(m_1 + m_2) \times \frac{2(T_1 - T_2)}{M} = (m_1 - m_2)g - (T_1 - T_2)$

$\Rightarrow T_1 - T_2 = \frac{(m_1 - m_2)g}{1 + \frac{2(m_1 + m_2)}{M}}$

15. (D)

From Kepler's law, $T^2 \propto r^3$

$$\frac{R_1^3}{R_2^3} = \frac{T_1^2}{T_2^2} = \frac{3\times3}{24\times24}$$

$$\Rightarrow \left(\frac{3\times10^4}{R_2}\right)^3 = \frac{1}{64}$$

$$\Rightarrow \frac{3\times10^4}{R_2} = \frac{1}{4}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{1}{4}$$

$R_2 = 12 \times 10^4$ km

Relative to the planet,

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi\times3\times10^4}{3} = 2\pi \times 10^4 \text{ km/hr}$$

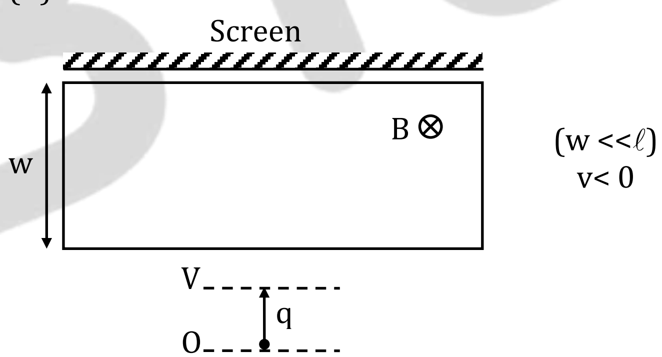
$$v_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi\times12\times10^4}{24} = \pi \times 10^4 \text{ km/hr}$$

At closest approach distance between $S_1$and S_2, relative orbital speed of S_2 as screen from S_1 will be $v_1 + v_2$ as they are moving in the opposite directions.

Hence, the above orbital speed is,

$v = v_1 + v_2 = 3\pi \times 10^4$ km/hr

16. (B)



Let an ion enters with velocity = v

In the presence of magnetic field it will describe circular motion with $r = \frac{mv}{qB}$

If $r < w$,

It will make a 180° turn and come out of the rectangular region having velocity downward.

17. (D)

$(\vec{n}\cdot\vec{F})\vec{n} = 0$

Because in scalar tripe product if two vectors are equal, value of scalar tripe product is zero.

$\therefore \quad \vec{F} = \vec{G}$

But $\quad F = (\vec{n} \times \vec{F}) \times \vec{n}$

Because $\vec{n}$ is unit vector.

$\therefore \quad \vec{G} = (\vec{n} \times \vec{F}) \times \vec{n}$

18. (A)

Relation between force and potential energy is given by

$\because \quad F = -\dfrac{dU}{dr}$

$$F = -\frac{d\left(\frac{1}{2}mv^2\right)}{dr}$$

$\because \quad v = r\omega$

But if r >ω, it hits the screen.

$\therefore \quad \dfrac{mv}{qB} > \omega \qquad$ (hits the screen)

$\Rightarrow \quad q < \dfrac{mv}{qB} \qquad ...(1)$

Since, charge q accelerated by a potential difference of V volt

$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}}$$

Put the value of v in eq.(1)

$\therefore \quad q < m\sqrt{\dfrac{2qV}{m}}\cdot\dfrac{1}{\omega B}$

$\Rightarrow \quad \sqrt{q} < \dfrac{\sqrt{2mV}}{\omega B}$

$\Rightarrow \quad q < \dfrac{2mV}{\omega^2 B^2} \qquad$ (hit the screen)

19. (B)

From Newton's law of cooling

$$\frac{dQ}{dt} = hA\Delta T$$

Assuming, constant rate of cooling,

$$\Delta Q = hAT\Delta t$$

$$\frac{\Delta Q_A}{\Delta Q_B} = \frac{A_A t_A}{A_B t_B}$$

$\therefore$ $Q = mS\Delta T$

Here, Q = heat, m = mass, S = specific heat andΔT = change in temperature

$$\frac{m_A S\Delta T}{m_B S\Delta T} = \frac{A_A t_A}{A_B t_B} \quad \text{(because density = mass/volume)}$$

$$\frac{\rho V_A}{\rho V_B} = \frac{A_A t_A}{A_B t_B} \qquad (\because V = Al)$$

$$\frac{h_A A_A}{h_B A_B} = \frac{A_A t_A}{A_B t_B}$$

$$\frac{t_A}{t_B} = \frac{1}{2}$$

$$t_B = 2t_A$$

20. (A)

Force exerted by the gas molecules on unit area of table, $F = P_o A = 10^5 \times 1 = 10^5$ N

Average velocity of gas molecules, $v = \sqrt{\frac{8K_B T}{\pi m}}$

Where, T = 20 + 273 = 293 K

$$\therefore \quad V = \sqrt{\frac{8 \times 1.4 \times 10^{-23} \times 293}{\pi \times 5 \times 10^{-27}}} = 1.4 \times 10^3 \text{ m/s}$$

Rate of change in momentum $\frac{\Delta P}{\Delta t}$ = nm(v – (–v)) = 2 mnv

Using $F = \frac{\Delta P}{\Delta t} = 2\ mnv$

$$10^5 = 2 \times 5 \times 10^{-27} \times n \times 1.4 \times 10^3$$

$$\Rightarrow n = 7 \times 10^{27}$$

$$\therefore F = \frac{-d\left(\frac{1}{2}m\omega^2 r^2\right)}{dr} = -\frac{1}{2}m\omega^2\frac{d(r^2)}{dr}$$

$$\Rightarrow F = -\frac{1}{2}m\omega^2(2r)$$

$$F = -m\omega^2 r$$

Now, $\frac{mv^2}{r} = m\omega^2 r$ ...(1)

And by using Bohr's quantization,

$$mvr = \frac{nh}{2\pi} \quad ...(2)$$

Where, h = Planck's constant, r = radius, m = mass, n = integer andω = angular speed

From equation (1) and (2)

$$\frac{m}{r}\left(\frac{nh}{2\pi mr}\right)^2 = m\omega^2 r$$

$$\frac{n^2h^2}{(2\pi m)^2\omega^2} = r^4$$

$$r = \sqrt{\frac{nh}{2\pi m\omega}}$$

$$\because a = \sqrt{\frac{h}{2\pi n\omega}}$$

$$\therefore r = a\sqrt{n}$$

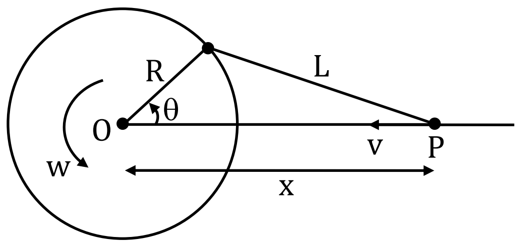
KVPY-SX 2017 (PHYSICS)

PART-II

21. (A)

Given: $L = 1$ m

$R = \frac{1}{\sqrt{3}}$ m



$\because$ It is slider crank mechanism and in it $v = -\frac{dx}{dt}$

From cosine rule ($\because$ L, R $\rightarrow$ constant)

$\cos\theta = \frac{R^2 + x^2 - L^2}{2Rx}$

$\Rightarrow R^2 + x^2 - L^2 = 2Rx\cos\theta$

Differentiating with respect to time in both the sides.

$\Rightarrow 0 + 2x.\frac{d}{dt}(x) - 0 = (2R).\left\{\frac{d}{dt}(x\cos\theta)\right\}$

$\Rightarrow 2x\frac{d}{dt}(x) = 2R\left\{\frac{d}{dt}(x).\cos\theta + x.(-\sin\theta).\frac{d(\theta)}{dt}\right\}$

$\because \frac{d\theta}{dt} = \omega$

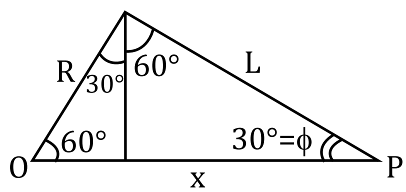
$\Rightarrow x.\frac{dx}{dt} = R\left\{\frac{dx}{dt}.\cos\theta - x.\sin\theta.\omega\right\}$

$\Rightarrow x.(-v) = R\{-v.\cos\theta - x.\sin\theta.\omega\}$

$\Rightarrow -xv = -Rv\cos\theta - Rx\omega\sin\theta$

$\Rightarrow$ $(x - R\cos\theta).v = Rx\omega\sin\theta$

$\Rightarrow$ $v = \dfrac{Rx\omega\sin\theta}{(x - R\cos\theta)}$ ...(1)



$\because$ $R\sin 60°$

$= \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{2} = \dfrac{1}{2}$

$\sin\phi = \dfrac{1/2}{1}$

$\Rightarrow$ $\phi = 30°$

By Pythagoras theorem,

$x = \sqrt{L^2 + R^2}$

$x = \sqrt{1^2 + \left(\dfrac{1}{\sqrt{3}}\right)^2}$

$x = \dfrac{2}{\sqrt{3}}$ Put all values in eq.(1)

$\therefore$ $v = \dfrac{\left(\dfrac{1}{\sqrt{3}}\right) \times \left(\dfrac{2}{\sqrt{3}}\right) \times (\omega) \times \left(\dfrac{\sqrt{3}}{2}\right)}{\left(\dfrac{2}{\sqrt{3}} - \dfrac{1}{\sqrt{3}} \times \dfrac{1}{2}\right)}$

$v = \dfrac{2\omega}{3}$

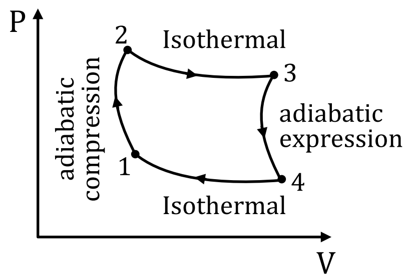
22. (D)

Here, $v_1 = v_1$

$v_2 = 1\ m^3$

$v_3 = 10\ m^3$

$v_4 = v_4$



Since, efficiency

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \frac{T_2}{T_1} = 1 - \eta$$

$$\frac{T_2}{T_1} = 1 - \frac{3}{4} = \frac{1}{4}$$

For the path, $1 \to 2$ (adiabatic process)

$$\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \text{ (for monoatomic } \gamma = \frac{5}{3}\text{)}$$

$4 = (v_1)^{2/3}$

$v_1 = 8\ m^3$

23. (A)

According to question,

$p = \mu_0^x m^y \omega^z c^u$ ...(1)

Where,

Power, $p = \frac{\text{work}}{\text{time}} = \frac{M^1L^2T^{-2}}{T^1} = [M^1L^2T^{-3}]$

Angular velocity, $\omega = \frac{\text{rad.}}{\text{sec.}} = [T^{-1}]$

Permeability of free space, $\mu_0 = [MLT^{-2}I^{-2}]$

$m = [IL^2]$

Speed of light, $c = [LT^{-1}]$

Put all the dimension is equation (1),

$[M^1L^2T^{-3}] = [MLT^{-2}I^{-2}]^x[IL^2]^y[T^{-1}]^z[LT^{-1}]^u$

$[M^1L^2T^{-3}I^0] = M^x.L^{x+2y+u}.T^{-2x-z-u}.I^{-2x+y}$

Now compare all the powers,

$x = 1$ ...(2)

$x + 2y + u = 2$ ...(3)

$-2x - z - u = -3$ ...(4)

$-2x + y = 0$ ...(5)

Put value of x in eq.(5)

$-2(1) + y = 0$

$\Rightarrow y = 2$

Put value of x and y in eq.(3)

$(1) + 2(2) + u = 2$

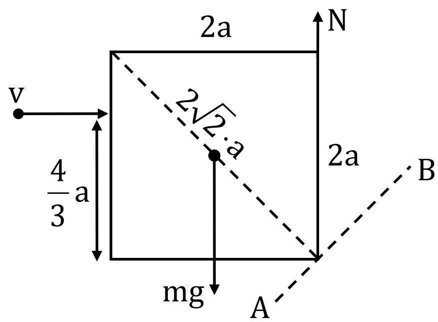
$\Rightarrow u = -3$

Put value of x andu in eq.(4)

$-2(1) - z - (-3) = -3$

$\Rightarrow z = 4$

24. (D)



Impulsive force is of large magnitude for very-very small interval of time.

We can neglect mg for very short interval of time and also the reason of neglecting mg is that impulsive force is of large magnitude as compared to mg.

Angular momentum will be conserved about AB axis,

$$mv.\frac{4a}{3} = Iw \qquad ...(1)$$

Let coordinates of COM will not change and using parallel axis theorem,

$$I_{AB} = I_{COM} + Mr^2$$

Moment of inertia of the cube = $\frac{2Ma^2}{3}$ (Given)

$$\Rightarrow I_{AB} = \frac{2Ma^2}{3} + M\left(\frac{2\sqrt{2}.a}{2}\right)^2$$

$$\Rightarrow I_{AB} = \frac{2Ma^2}{3} + 2Ma^2$$

$$\Rightarrow I_{AB} = \frac{8Ma^2}{3}$$

Put the value in equation (1),

$$mv.\frac{4a}{3} = \frac{8Ma^2}{3}.\omega$$

$$\Rightarrow mv = 2Ma.\omega$$

$$\Rightarrow \omega = \frac{mv}{2Ma}$$

25. (A)

$\because$ Given equation is,

$$PV^{5/3}.\exp\left(\frac{-PV}{E_0}\right) = C_1$$

Here, P = pressure, V = volume

Where, C_1 and E_0 are dimensional constants.

$$PV^{5/3}.\,e^{\left(\frac{-PV}{E_0}\right)} = C_1$$

$$\Rightarrow PV^{5/3}.\frac{1}{e^{\left(\frac{PV}{E_0}\right)}} = C_1$$

$$\Rightarrow PV^{5/3} = C_1\,e^{\left(\frac{PV}{E_0}\right)}$$

By taking $\log_e$ both the sides,

$$\ln(PV^{5/3}) = \ln[C_1 \cdot e^{PV/E_0}]$$

$\because$ $\ln(AB) = \ln A + \ln B$

and $\ln A^B = B \ln A$

$\therefore$ $\ln P + \ln V^{5/3} = \ln C_1 + \ln e^{PV/E_0}$

$$\ln P + \frac{5}{3}\ln V = \ln C_1 + \frac{PV}{E_0}\ln e$$

$$\ln P + \frac{5}{3}\ln V = \ln C_1 + \frac{PV}{E_0}$$

$\because$ $\log_e e = 1$

Differentiating both the sides with respect to t,

$$\frac{1}{P}.\frac{dP}{dt} + \frac{5}{3}.\frac{1}{V}.\frac{dV}{dt} = 0 + \frac{1}{E_0}\left(P\frac{dV}{dt} + V\frac{dP}{dt}\right)$$

$$\frac{dP}{P} + \frac{5}{3}\frac{dV}{V} = \frac{(PdV + VdP)}{E_0}$$

$$dP\left[\frac{1}{P} - \frac{V}{E_0}\right] = dV\left[P - \frac{5}{3V}\right]$$

$$\frac{dV}{dP} = \frac{\left[\frac{1}{P} - \frac{V}{E_0}\right]}{\left[P - \frac{5}{3V}\right]}$$

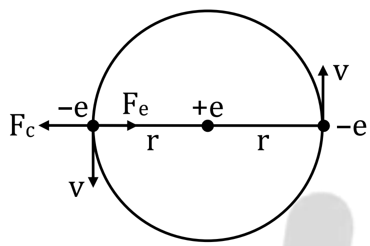
$\therefore$ The thermal compressibility at high temperature,

$$C = -\frac{1}{V}.\frac{dV}{dP} = -\frac{1}{V}.\frac{\left[\frac{1}{P}-\frac{V}{E_0}\right]}{\left[P-\frac{5}{3V}\right]}$$

$$C = \frac{\left[\frac{1}{E_0}-\frac{1}{PV}\right]}{\left[P-\frac{5}{3V}\right]}$$

At very high temperature $C = \frac{1}{E_0}$

26. (B)



Let the velocity of the electron be v.

Angular momentum of electron,

$mvr = h$ (given)

Net electrostatic force acting on the electron,

$$F_e = \frac{e^2}{4\pi\varepsilon_o r^2} - \frac{e^2}{4\pi\varepsilon_o (2r^2)}$$

$$= \frac{3}{4}\cdot\frac{e^2}{4\pi\varepsilon_o r^2}$$

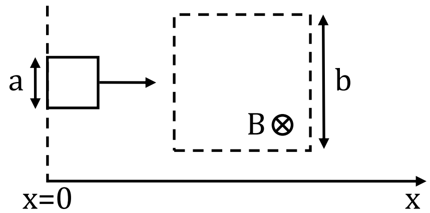
Centrifugal force, $F_c = \frac{mv^2}{r} = \frac{(mvr)^2}{r \times mr^2} = \frac{h^2}{mr^3}$

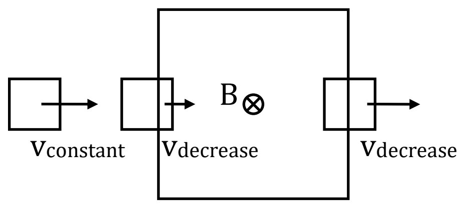
But, $F_c = F_e$

$$\therefore \quad \frac{h^2}{mr^3} = \frac{3}{4}\cdot\frac{e^2}{4\pi\varepsilon_o r^2}$$

$$\Rightarrow \quad r = \frac{4}{3}\cdot\frac{4\pi\varepsilon_o h^2}{me^2} = \frac{4}{3}a_B$$

27. (B)





Initially, the velocity remains the same. Then when it is about to enter the loop, by Faraday's law the velocity will decrease and then will remain same until it comes out of the loop, when it exits the loop again by Faraday's law the velocity will decrease.

28. (D)

At the 5^{th} reading ofvernier's scale, both readings of the scales coincide.

At the 4^{th} reading of vernier's scale, the length between the two scales is minimum and will be the least count = 0.01 cm

So difference at 3^{rd} reading of vernier scale is 2 × 0.01 cm

$\Rightarrow$ Difference at 2^{nd} reading of vernier scale will be

= 3 × 0.01 cm = 0.03 cm

29. (A)

As the rays pass through water, there will be a phase difference between the two consecutive emerging rays thereby forming a pattern similar to YDSE.

$\because$ We know that,

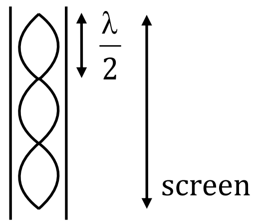
Velocity = frequency × wavelength

$v = f\lambda$

$$\Rightarrow \lambda = \frac{1500}{0.5\times10^{6}} = 3000 \times 10^{-6}$$

$\Rightarrow \lambda = 3 \times 10^{-3}$ m

$\Rightarrow \lambda = 3$ mm



Intensity vary according to graph 'A'.

30. (C)

From Kepler's-3rd law, we have square of time period is directly proportional to cube of radius of orbit.

Let T_1 = time period of planet

from graph,

T_1 = 3 days

T_2 = Time period of earth = 365 days

r_1 = radius of planet orbit

r_2 = radius of earth orbit

r_2 = 1 A.U. (given)

By Kepler's law, we have

$$\frac{(T_1)^2}{(T_2)^2} = \frac{(r_1)^3}{(1\text{ A.U.})^3}$$

$$\Rightarrow \left(\frac{3}{365}\right)^2 = \frac{(r_1)^3}{(1\text{ A.U.})^3}$$

$$\Rightarrow \left(\frac{3}{365}\right)^{2/3} = \frac{r_1}{1\text{ A.U.}}$$

$$\Rightarrow 0.04 = \frac{r_1}{1\text{ A.U.}}$$

$\Rightarrow r_1 = 0.04$ A.U.