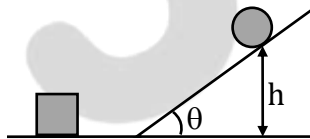


# KVPY-SX 2019 (PHYSICS)



## PART-I

- In a muonic atom, a muon of mass of 200 times of that of electron and same charge is bound to the proton. The wavelengths of its Balmer series are in the range of  
(a) X-rays                      (b) infrared                      (c)  $\gamma$  rays                      (d) microwave
- We consider the Thomson model of the hydrogen atom in which the proton charge is distributed uniformly over a spherical volume of radius 0.25 Angstrom. Applying the Bohr condition in this model the ground state energy (in eV) of the electron will be close to  
(a)  $-\frac{13.6}{4}$                       (b) -13.6                      (c)  $-\frac{13.6}{2}$                       (d)  $-2 \times 13.6$
- A spherical rigid ball is released from rest and starts rolling down and inclined plane from height  $h = 7$  m, as shown in the figure. It hits a block at rest on the horizontal plane (assume elastic collision). If the mass of both the ball and the block is  $m$  and the ball is rolling without sliding, then the speed of the block after collision is close to:

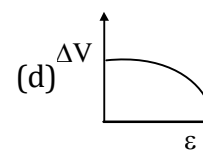
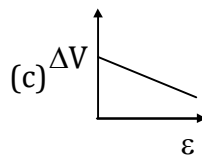
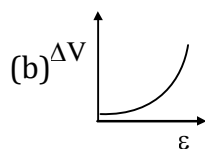
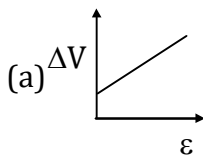
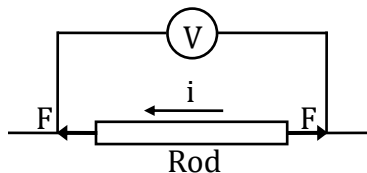


- (a) 6 m/s                      (b) 8 m/s                      (c) 10 m/s                      (d) 12 m/s
- A girl drops an apple from the window of a train which is moving on a straight track with speed increasing with a constant rate. The trajectory of the falling apple as seen by the girl is  
(a) Parabolic and in the direction of the moving train  
(b) Parabolic and opposite to the direction of the moving train  
(c) An inclined straight line pointing in the direction of the moving train  
(d) An inclined straight line pointing opposite to the direction of the moving train

# KVPY-SX 2019 (PHYSICS)



5. A train is moving slowly at 2 m/s next to a railway platform. A man, 1.5 m tall, alights from the train such that his feet are fixed on the ground. Taking him to be a rigid body, the instantaneous angular velocity (in rad/sec) is:  
(a) 1.5                      (b) 2.0                      (c) 2.5                      (d) 3.0
6. A point mass  $M$  moving with a certain velocity collides with a stationary point mass  $M/2$ . The collision is elastic and in one dimension. Let the ratio of the final velocity of  $M$  and  $M/2$  be  $x$ . The value of  $x$  is:  
(a) 2                      (b) 3                      (c)  $1/2$                       (d)  $1/4$
7. A particle of mass  $2/3$  kg with velocity  $v = -15$  m/s at  $t = -2$  s is acted upon by a force  $f = k - \beta t^2$ . Here  $k = 8$  N and  $\beta = 2$  N/s<sup>2</sup>. The motion is one dimensional. Then the speed at which the particle acceleration is zero again, is:  
(a) 1 m/s                      (b) 16 m/s                      (c) 17 m/s                      (d) 32 m/s
8. A certain stellar body has radius  $50 R_s$  and temperature  $2T_s$  and is at a distance of  $2 \times 10^{10}$  A.U. from the Earth. Here A.U. refers to the earth sun distance and  $R_s$  and  $T_s$  refer to the sun's radius and temperature respectively. Take both star and sun to be ideal black bodies. The ratio of the power received on earth from the stellar body as compared to that received from the sun is close to:  
(a)  $4 \times 10^{-20}$                       (b)  $2 \times 10^{-6}$                       (c)  $10^{-8}$                       (d)  $10^{-16}$
9. As shown in the schematic diagram below, a rod of uniform cross-sectional area  $A$  and length  $l$  is carrying a constant current  $i$  through it and voltage across the rod is measured using an ideal voltmeter. The rod is stretched by the application of a force  $F$ . Which of the following graphs would show the variation in the voltage across the rod as function of the strain,  $\epsilon$ , when the strain is small. Neglect Joule heating.



## KVPY-SX 2019 (PHYSICS)



10. Two identical coherent sound sources R and S with frequency  $f$  are 5m apart. An observer standing equidistant from the sources and at a perpendicular distance of 12 m from the line RS hears maximum sound intensity when he moves parallel to RS. The sound intensity varies and is a minimum when he comes directly in front of one of the two sources. Then a possible value of  $f$  is close to (the speed of sound is 330 m/s)
- (a) 495 Hz                      (b) 275 Hz                      (c) 660 Hz                      (d) 330 Hz
11. A photon falls through a height of 1 km through the earth's gravitational field. To calculate the change in its frequency, take its mass to be  $h\nu/c^2$ . The fractional change in frequency  $\nu$  is close to:
- (a)  $10^{-20}$                       (b)  $10^{-17}$                       (c)  $10^{-13}$                       (d)  $10^{-10}$
12. 0.02 moles of an ideal diatomic gas with initial temperature  $20^\circ\text{C}$  is compressed from  $1500\text{ cm}^3$  to  $500\text{ cm}^3$ . The thermodynamic process is such that  $PV^2 = \beta$  where  $\beta$  is a constant. Then the value of  $\beta$  is close to: (The gas constant,  $R = 8.31\text{ J/K/mol}$ )
- (a)  $7.5 \times 10^{-2}\text{ Pa}\cdot\text{m}^6$     (b)  $1.5 \times 10^2\text{ Pa}\cdot\text{m}^6$     (c)  $3 \times 10^{-2}\text{ Pa}\cdot\text{m}^6$     (d)  $2.2 \times 10^1\text{ Pa}\cdot\text{m}^6$
13. A heater supplying constant power  $P$  watts is switched on at time  $t = 0$  minutes to raise the temperature of a liquid kept in a calorimeter of negligible heat capacity. A student records the temperature of the liquid  $T(t)$  at equal time intervals. A graph is plotted with  $T(t)$  on the y-axis versus  $t$  on the x-axis. Assume that there is no heat loss to the surrounding heating. Then,
- (a) the graph is a straight line parallel to the time axis  
(b) the heat capacity of the liquid is inversely proportional to the slope of the graph  
(c) if some heat were lost at a constant rate to the surroundings during heating, the graph would be a straight line but with a larger slope  
(d) the internal energy of the liquid increases quadratic ally with time

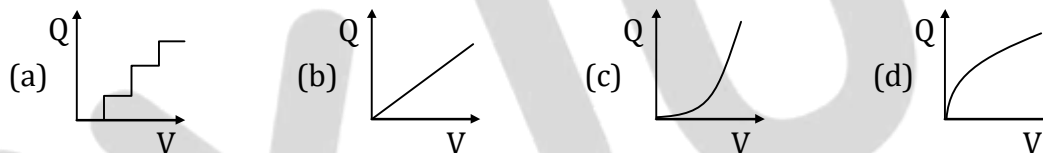
# KVPY-SX 2019 (PHYSICS)



14. Unpolarized red light is incident on the surface of a lake at incident angle  $\theta_R$ . An observer seeing the light reflected from the water surface through a polarizer notices that on rotating the polarizer, the intensity of light drops to zero at a certain orientation. The red light is replaced by unpolarized blue light. The observer sees the same with reflected blue light at incident angle  $\theta_B$ . then:

- (a)  $\theta_B < \theta_R < 45^\circ$       (b)  $\theta_B = \theta_R$       (c)  $\theta_B > \theta_R > 45^\circ$       (d)  $\theta_B > \theta_R > 45^\circ$

15. A neutral spherical copper particle has a radius of 10 nm ( $1 \text{ nm} = 10^{-9} \text{ m}$ ). It gets charged by applying the voltage slowly adding one electron at a time. Then the graph of the total charge on the particle vs the applied voltage would look like:



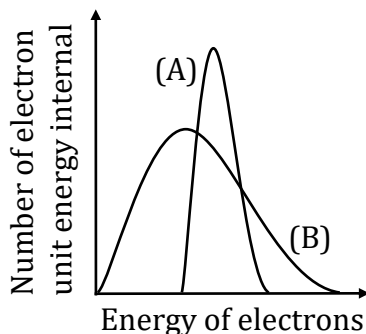
16. A charge  $+q$  is distributed over a thin ring of radius  $r$  with line charge density  $\lambda = \theta \sin^2 \theta / (\pi r)$ . Note that the ring is in the  $x$ - $y$  plane and  $\theta$  is the angle made by  $\vec{r}$  with the  $x$ -axis. The work done by the electric force in displacing a point charge  $+Q$  from the centre of the ring to infinity is:

- (a) Equal to  $qQ/2\pi\epsilon_0 r$   
(b) Equal to  $qQ/4\pi\epsilon_0 r$   
(c) Equal to zero only if the path is a straight line perpendicular to the plane of the ring  
(d) Equal to  $qQ/8\pi\epsilon_0 r$

# KVPY-SX 2019 (PHYSICS)



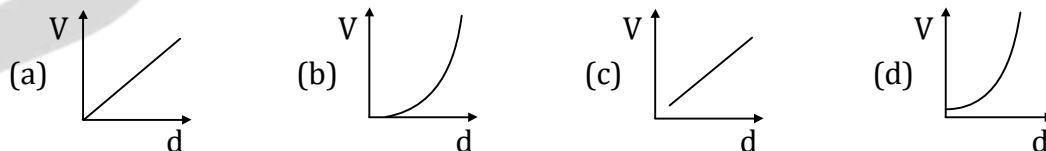
17. Originally the radioactive beta decay was thought as a decay of a nucleus with the emission of electrons only (Case I). However, in addition to the electron, another (nearly) massless and electrically neutral particle is also emitted (Case II). Based on the figure below, which of the following is correct:



- (a)(A) in both cases I and II                      (b)(A) in case I and (B) in case II  
(c)(A) in case II and (B) in case I                (d)(B) in both cases I and II
18. One gram mole of an ideal gas A with the ratio of constant pressure and constant volume specific heats,  $\gamma_A = 5/3$  is mixed with  $n$  gram-moles of another ideal gas B with  $\gamma_B = 7/5$ . If the  $\gamma$  for the mixture is  $19/13$  what will be the value of  $n$ ?

- (a) 0.75                      (b) 2                      (c) 1                      (d) 3

19. How will the voltage ( $V$ ) between the two plates of a parallel plate capacitor depend on the distance ( $D$ ) between the plates, if the charge on the capacitor remains the same?



20. Three large identical plates are kept close and parallel to each other. The outer two plates are maintained at temperatures  $T$  and  $2T$ , respectively. The temperature of the middle plate in steady state will be close to:

- (a) 1.1  $T$                       (b) 1.3  $T$                       (c) 1.7  $T$                       (d) 1.9  $T$

# KVPY-SX 2019 (PHYSICS)



## PART-II

21. A metal rod of cross-sectional area  $10^{-4} \text{ m}^2$  is hanging in a chamber kept at  $20^\circ\text{C}$  with a weight attached to its free end. The coefficient of thermal expansion of the rod is  $2.5 \times 10^{-6} \text{ K}^{-1}$  and its Young's modulus is  $4 \times 10^{12} \text{ N/m}^2$ . When the temperature of the chamber is lowered to  $T$  then a weight of  $5000 \text{ N}$  needs to be attached to the rod so that its length is unchanged. Then  $T$  is
- (a)  $15^\circ\text{C}$                       (b)  $12^\circ\text{C}$                       (c)  $5^\circ\text{C}$                       (d)  $0^\circ\text{C}$
22. A short solenoid (length  $l$  and radius  $r$ , with  $n$  turns per unit length) lies well inside and on the axis of a very long, coaxial solenoid (length  $L$ , radius  $R$  and  $N$  turns per unit length, with  $R > r$ ). Current  $I$  flow in the short solenoid. Choose the correct statement.
- (a) There is uniform magnetic field  $\mu_0 n I$  in the long solenoid.
- (b) Mutual inductance of the solenoids is  $\pi \mu_0 r^2 n N l$ .
- (c) Flux through outer solenoid due to current  $I$  in the inner solenoid is proportional to the ratio  $R/r$ .
- (d) Mutual inductance of the solenoids is  $\pi \mu_0 r R n N l (rR)^{1/2}$ .
23. Consider the wall of a dam to be straight with height  $H$  and length  $L$ . It holds a lake of water of height  $h$  ( $h < H$ ) on one side. Let the density of water be  $\rho_w$ . Denote the torque about the axis along the bottom length of the wall by  $\tau_1$ . Denote also a similar torque due to the water up to height  $h/2$  and wall length  $L/2$  by  $\tau_2$ . Then  $\tau_1/\tau_2$  (ignore atmospheric pressure) is
- (a) 2                      (b) 4                      (c) 8                      (d) 16

## KVPY-SX 2019 (PHYSICS)



24. Two containers C1 and C2 of volumes  $V$  and  $4V$  respectively hold the same ideal gas and are connected by a thin horizontal tube of negligible volume with a valve which is initially closed. The initial pressures of the gas in C1 and C2 are  $P$  and  $5P$ , respectively. Heat baths are employed to maintain the temperatures in the containers at  $300\text{ K}$  and  $400\text{ K}$  respectively. The valve is now opened. Select the correct statement:
- (a) The gas will flow from the hot container to the cold one and the process is reversible.
  - (b) The gas will flow from one container to the other till the number of moles in two containers are equal.
  - (c) A long time after the valve is opened, the pressure in both the containers will be  $3P$ .
  - (d) A long time after the valve is opened, number of moles of gas in the hot container will be thrice that of the cold one.
25. Four electrons, each of mass  $m_e$  are in a one dimensional box of size  $L$ . Assume that the electrons are non-interacting, obey the Pauli exclusion principle and are described by standing de Broglie waves confined within the box. Define  $\alpha = h^2/8m_eL^2$  and  $U_0$  to be the ground state energy. Then
- (a) The energy of the highest occupied state is  $16\alpha$
  - (b)  $U_0 = 30\alpha$
  - (c) The total energy of the first excited state is  $U_0 + 9\alpha$
  - (d) The total energy of the second excited state is  $U_0 + 8\alpha$
26. A rope of length  $L$  and uniform linear density is hanging from the ceiling. A transverse wave pulse, generated close to the free end of the rope, travels upwards through the rope. Select the correct option.
- (a) The speed of the pulse decreases as it moves up
  - (b) The time taken by the pulse to travel the length of the rope is proportional to  $\sqrt{L}$
  - (c) The tension will be constant along the length of the rope
  - (d) The speed of the pulse will be constant along the length of the rope.

# KVPY-SX 2019 (PHYSICS)



27. A circuit consists of a coil with inductance  $L$  and an uncharged capacitor of capacitance  $C$ . The coil is in a constant uniform magnetic field such that the flux through the coil is  $\Phi$ . At time  $t = 0$ , the magnetic field is abruptly switched off. Let  $\omega_0 = 1/\sqrt{LC}$  and ignore the resistance of the circuit. Then,

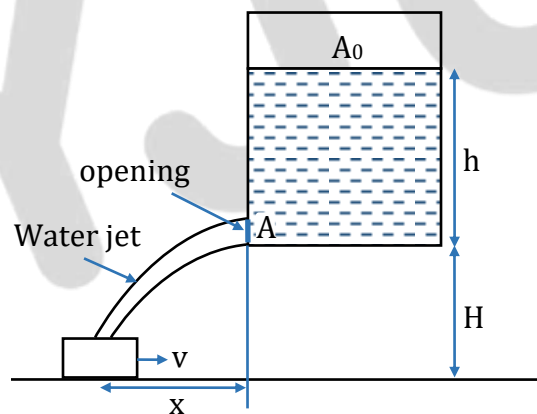
(a) Current in the circuit is  $I(t) = (\Phi/L) \cos\omega_0 t$

(b) Magnitude of the charge on the capacitor is  $|Q(t)| = 2C\omega_0 |\sin\omega_0 t|$

(c) Initial current in the circuit is infinite

(d) Initial charge on the capacitor is  $C\omega_0 \Phi$

28. Consider the configuration of a stationary water tank of cross section area  $A_0$ , and a small bucket as shown in figure below:



What should be the speed,  $v$ , of the bucket so that the water leaking out of a hole of cross-section area  $A$  (as shown) from the water tank does not fall outside the bucket?

Take  $h = 5$  m,  $H = 5$  m,  $g = 10$  m/s<sup>2</sup>,  $A = 5$  cm<sup>2</sup> and  $A_0 = 500$  cm<sup>2</sup>.

(a) 1 m/s

(b) 0.5 m/s

(c) 0.1 m/s

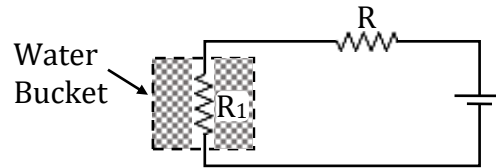
(d) 0.05 m/s



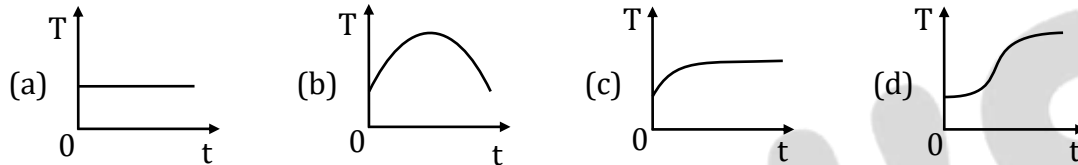
# KVPY-SX 2019 (PHYSICS)



29. The circuit below is used to heat water kept in a bucket



Assuming heat loss only by Newton's law of cooling, the variation in the temperature of the water in the bucket as a function of time is depicted by:



30. A bubble of radius  $R$  in water of density  $\rho$  is expanding uniformly at speed  $v$ . Given that water is incompressible, the kinetic energy of water being pushed is

- (a) Zero      (b)  $2\pi\rho R^3 v^2$       (c)  $2\pi\rho R^3 v^2/3$       (d)  $4\pi\rho R^3 v^2/3$

# KVPY-SX 2019 (PHYSICS)



## ANSWER KEY

1. (a)	2. (b)	3. (c)	4. (d)	5. (b)
6. (d)	7. (c)	8. (d)	9. (a)	10. (a)
11. (c)	12. (a)	13. (b)	14. (c)	15. (a)
16. (b)	17. (b)	18. (b)	19. (c)	20. (c)
21. (a)	22. (b)	23. (d)	24. (d)	25. (d)
26. (b)	27. (a)	28. (c)	29. (c)	30. (b)

## SOLUTIONS PART-I

1. (a)

As we know that the, energy of an orbit is given by,

$$E = \frac{-me^4 Z^2}{8\epsilon_0^2 h^2 n^2}$$

m = mass of electron

z = number protons/ Atomic no.

h = planks constant

n = no. of series / integer value

$\therefore$  Mass = 200 times of that of electron i.e.  $m_\mu = 200 m_e$

Energy directly proportional to mass

$\Rightarrow$  Energy increases by 200 times.

$$\therefore E = \frac{hc}{\lambda} \Rightarrow E \propto \frac{1}{\lambda} \Rightarrow \frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1}$$

Let  $E_1 = E$  So,  $E_2 = 200 E$

$$\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{E}{200E}$$

$$\Rightarrow \lambda_2 = \frac{1}{200} \lambda_1$$

Where  $\lambda_2$  = wavelength for corresponding Balmer series.

$\lambda_1$  = wavelength for corresponding hydrogen atom.

$\Rightarrow$  Wavelength will decrease by 200 times.

# KVPY-SX 2019 (PHYSICS)



So, the wavelength  $\lambda$  for corresponding Balmer series will be  $\frac{1}{200}$  times that of hydrogen atom, so it will be in range of X-rays.

Hence the correct option is (a).

2. (b)

For electron orbiting around a proton

By Bohr's condition

$$R_H = 0.529 \text{ \AA} \quad [R_H = \text{radius of 1st Bohr orbit}]$$

$$\text{Given, } r = 0.25 \text{ \AA}$$

Which is less than  $0.529 \text{ \AA}$

And we can say that if electron is orbiting a proton or a positive sphere having charge equal to proton then its Kinetic energy & Potential energy will be  $-13.6 \text{ eV}$

$$\therefore \text{K.E.} = \text{P.E. of electron}$$

$$\therefore \text{TE} = -13.6 \text{ eV}$$

So, total energy is same as Bohr's model =  $-13.6 \text{ eV}$ .

3. (c)

Using energy conservation for ball

Kinetic energy + rotational energy = Potential energy.

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$m$  = mass of ball

$v$  = linear velocity of ball

$\omega$  = angular velocity of ball

$h$  = height of ball from ground

$I$  = moment of inertia of ball (sphere)

Ball is a sphere and moment of inertia of sphere =  $\left(\frac{2}{5}mR^2\right)$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} = mgh$$

{ $\therefore$  Relation between linear velocity and angular velocity is  $v = \omega R \Rightarrow \omega = \frac{v}{R}$ }

# KVPY-SX 2019 (PHYSICS)



$$\frac{1}{2}v^2 + \frac{1}{5}v^2 = gh \Rightarrow \left(\frac{5+2}{10}\right)v^2 = gh$$

$$\Rightarrow v = \sqrt{\frac{10gh}{7}}$$

$$\therefore v = \sqrt{\frac{10 \times 10 \times 7}{7}} \quad \{\because g = \text{acceleration due to gravity} = 10 \text{ m/s}^2\}$$

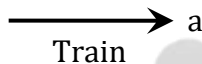
$$\therefore v = 10 \text{ m/s}$$

As the collision is elastic and mass of block & ball are same, so after collision it will transfer whole velocity to the block.

$$\Rightarrow v_{\text{block}} = 10 \text{ m/s}$$

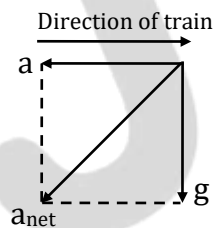
4. (d)

Let the acceleration of train is 'a'



In the frame of train, for apple when it is dropped; a pseudo force will act on it due to which it will get an acceleration same as that of train but in opposite direction to that of train; also, acceleration due to gravity will act downwards.

$\Rightarrow$  For apple



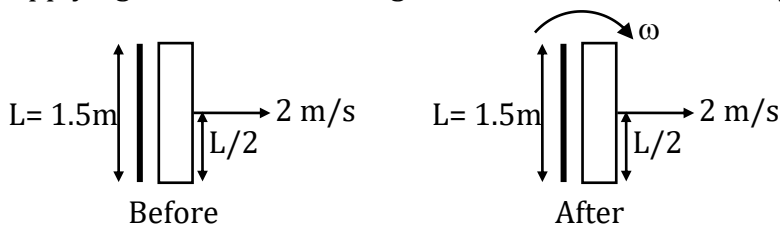
$$\therefore a_{\text{net}} = \sqrt{a^2 + g^2}$$

$\therefore$  The trajectory of the falling apple as seen by the girl is an incline straight line pointing opposite to the direction of the moving train, as relative initial velocity is zero.

5. (b)

Man is considered a rigid body.

Applying conservation of angular momentum about feet just before and after landing.



$$mv \frac{L}{2} = I\omega$$

# KVPY-SX 2019 (PHYSICS)



Where  $I$  = moment of inertia and here moment of inertia about a fixed point or on end is  $\frac{ML^2}{3}$  (considering like uniform thin rod)

$$m \times v \times \frac{L}{2} = \frac{mL^2}{3} \times \omega$$

$$\Rightarrow \omega = \frac{3v}{2L}$$

Substituting the values  $L = 1.5 \text{ m}$ ,  $v = 2 \text{ m/s}$

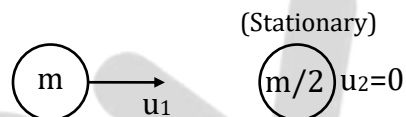
$$\Rightarrow \omega = \frac{3 \times 2}{2 \times 1.5}$$

$$\Rightarrow \omega = 2 \text{ rad/sec}$$

6. (d)

Let before collision velocity of mass  $M$  is  $u_1$  and mass  $M/2$  is 0 because it is stationary.

Before collision



After collision let the velocity of mass  $M$  is  $v_1$  and velocity of mass  $M/2$  is  $v_2$ .



By linear momentum conservation

$$p_i = p_f$$

$$mu_1 = mv_1 + \frac{mv_2}{2}$$

$$u_1 = v_1 + \frac{v_2}{2} \quad \dots (1)$$

Now, as collision is elastic so,

$$\text{Coefficient of restitution } e = \frac{\text{Velocity of separation (after collision)}}{\text{Velocity of approach (before collision)}}$$

# KVPY-SX 2019 (PHYSICS)



$$e = \frac{v_2 - v_1}{u_1} = 1$$

$$\Rightarrow u_1 = v_2 - v_1 \quad \dots (2)$$

By equation (1) & equation (2)

$$v_2 = \frac{4}{3} u_1 \quad \& \quad v_1 = \frac{u_1}{3}$$

$$\text{So, } \frac{v_1}{v_2} = \frac{u_1/3}{4u_1/3} = \frac{1}{4}$$

7. (c)

$\therefore$  From the definition of force  $F$  (Force) = mass  $\times$  acceleration

$$F = ma \quad \left\{ \because \text{acceleration } a = \frac{dv}{dt} \right\}$$

$$F = m \frac{dv}{dt}$$

$$\text{Given } f = k - \beta t^2, k = 8\text{N}, \beta = 2\text{N/s}^2, m = \frac{2}{3}\text{kg}$$

Substituting the values

$$8 - 2t^2 = m \frac{dv}{dt} = \frac{2}{3} \frac{dv}{dt} \Rightarrow dv = \frac{3}{2} (8 - 2t^2) dt$$

Integrating both sides

$$\int dv = \frac{3}{2} \int (8 - 2t^2) dt$$

$$v = \frac{3}{2} \left[ 8t - \frac{2t^3}{3} + C \right] \quad \dots (1)$$

On substituting  $t = -2$  &  $v = -15$

$$\Rightarrow C = \frac{2}{3}$$

Force will be zero again at

$$F = 0$$

$$k - \beta t^2 = 0$$

$$\Rightarrow 8 - 2t^2 = 0 \Rightarrow 2t^2 = 8 \Rightarrow t^2 = 4 \Rightarrow t = \pm 2 \text{ sec.}$$

$\therefore$  Time cannot be negative therefore

# KVPY-SX 2019 (PHYSICS)



$\Rightarrow t = 2 \text{ sec}$  will be considered

Putting  $t = 2$  & value of  $c$  in equation (1)

$$v' = \frac{3}{2} \left[ 8(2) - \frac{2(2)^3}{3} + \frac{2}{3} \right]$$

$$v' = \frac{3}{2} \left[ 16 - \frac{16}{3} + \frac{2}{3} \right]$$

$$v' = \frac{3}{2} \left[ \frac{48 - 16 + 2}{3} \right]$$

$$v' = \frac{3}{2} \times \frac{34}{3}$$

$$v' = 17 \text{ m/s}$$

8. (d)

From kirchoff's law power of stellar body i.e. Energy per unit time.

$$P_{\text{Body}} = e\sigma AT^4$$

$P$  = Power

$A$  = surface area

$T$  = temperature

$\sigma$  = Stefan Boltzmann const.

$e$  = emissivity of block body ( $e = 1$ )

Given radius of stellar =  $50R_s$ , where  $R_s$  sun's radius,

Temperature of staller body =  $2T_s$

Where  $T_s$  = Temperature of sun Now, Substituting the values.

$$P_{\text{Body}} = 1 \cdot \sigma \cdot (4\pi(50R_s)^2)(2T_s)^4$$

{As the bodies are ideal black bodies, so emissivity  $e = 1$ }

$$P_{\text{Body}} = 50^2 \times 2^4 \times P_{\text{sun}} \quad \dots\dots\dots (1)$$

$\therefore$  Intensity is given by Energy per unit area per unit time i.e.  $I = \frac{E}{At} = \frac{P_{\text{body}}}{A}$

$$\text{Intensity at earth due to body} = \frac{P_{\text{Body}}}{4\pi(2 \times 10^4 \text{ AV})^2} \quad \dots\dots\dots (2)$$

From equation (1) & (2)

$$\begin{aligned} &= \frac{50^2 \times 2^4 \times P_{\text{sun}}}{4 \times 10^{20} \times (4\pi \text{ AV})^2} \\ &= \frac{50^2 \times 2^4}{4 \times 10^{20}} \times I_{\text{sun}} \end{aligned}$$

$$\Rightarrow I_{\text{Body}} = 10^{-6} I_{\text{sun}}$$

# KVPY-SX 2019 (PHYSICS)



9. (a)  
From ohm's law  
 $V = iR$   
 $V = \text{voltage}$   
 $i = \text{current}$   
 $R = \text{Resistance}$   
 $i \rightarrow \text{constant}$

$$V = i(R_0 + \Delta R) \dots\dots\dots (1)$$

When current  $i$  flows through the conducting wire the resistance is given by

$$R = \frac{\rho \ell}{A} \dots\dots\dots (2)$$

Where,  $\rho =$  resistivity of wire (Rod),  $A =$  Area of cross section,  $\ell =$  length of wire (Rod).

Let change in resistance is  $\Delta R$ ,

Let change in length is  $\Delta \ell$ ,

Let change in Area is  $\Delta A$ ,

$\therefore$  Equation (2) becomes.

$$\frac{\Delta R}{R} = \rho \left( \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A} \right)$$

$$\frac{\Delta A}{A} = -\frac{\Delta \ell}{\ell} \quad \& \quad \rho \rightarrow \text{is constant because there is no joule heating}$$

$$\Delta R = \rho R \left( \frac{2\Delta \ell}{\ell} \right)$$

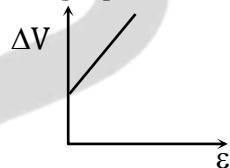
$$= 2\rho R \epsilon$$

Substituting it in equation (1) we get

$$V = i(R + 2\rho R \epsilon)$$

$\therefore V$  will increase linearly with strain from an initial value.

So, graph will look like.

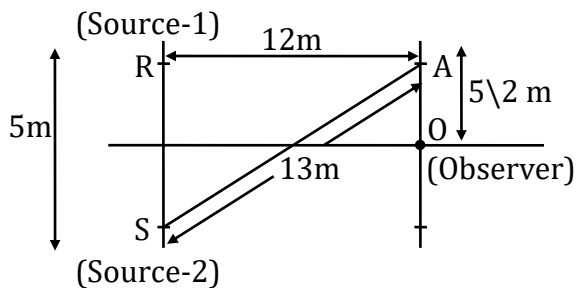




# KVPY-SX 2019 (PHYSICS)



10. (a)



Here when observer hears minimum sound intensity the path difference will be odd multiple of  $\frac{\lambda}{2}$ .

$$AS - AR = (2n + 1) \frac{\lambda}{2} = (2n + 1) \frac{330}{2f}$$

$$13 - 12 = (2n + 1) \frac{330}{2f}$$

$$f = 165(2n + 1)$$

To find a possible of f

Put  $n = 1$

$$f = 495 \text{ Hz.}$$

11. (c)

Total energy of photon will be,

Total energy of photon = Kinetic energy + Potential energy of photon

$$hv' = hv + mgh$$

$$\therefore h(v' - v) = mgh \dots\dots\dots (1)$$

Given that; mass  $(m) = \frac{hv}{c^2}$

Equation (1) becomes

$$\therefore h(v' - v) = \frac{hv}{c^2} \times gh$$

To find the fractional change in frequency, above equation divided by v

$$\frac{v' - v}{v} = \frac{gh}{c^2}$$

So, substituting the values of acceleration due to gravity

$G = 9.8 \text{ m/s}^2$ , Height  $(h) = 1 \text{ km} = 1 \times 10^3 \text{ m}$ , speed of light  $c = (3 \times 10^8) \text{ m/s}$ .

$$\frac{v' - v}{v} = \frac{9.8 \times 1 \times 10^3}{(3 \times 10^8)^2} = 1.1 \times 10^{-13}$$

# KVPY-SX 2019 (PHYSICS)



12. (a)  
Given that,  $PV^2 = \beta$  .....(1)  
(Number of moles)  $n = 0.02$ ,  $f = 5$ ,  $T_1 = 20^\circ\text{C}$   
 $V_1 = 1500 \times 10^{-6}$ ,  $V_2 = 500 \times 10^{-6}$

By ideal gas equation

$$PV = nRT$$

$$\Rightarrow P = \frac{nRT}{V}$$

( $n$  = number of moles,  $R$  = gas constant,  $V$  = volume,  $T$  = Temperature,  $P$  = pressure)

Substituting the value of  $P$  in equation (1)

$$\frac{nRT}{V} \cdot V^2 = \beta$$

$$\Rightarrow nRTV = \beta$$

Substituting the value of  $n$ ,  $R$ ,  $T$  and  $V$

$$\Rightarrow 0.02 \times 8.31 \times (273 + 20) \times 1500 \times 10^{-6} = \beta$$

$$\Rightarrow \beta = 0.073$$

$$\Rightarrow \beta \cong 7.5 \times 10^{-2} \text{ Pa.m}^6$$

13. (b)

From the definition of specific heat  $s = \frac{Q}{mdT} \Rightarrow Q = msdT$  ..... (1)

Where  $Q$  = amount of heat,  $m$  = mass of the substance,  $dT$  = change in temperature.

Let power supplied by the heater is 'P' and  $dT$  be the change in temperature.

$$\therefore P = ms \frac{dT}{dt}$$

$$\therefore dT = \frac{P}{ms} \times dt$$

Integrating both sides

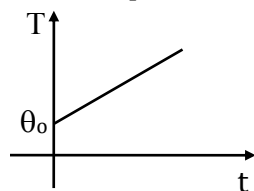
$$\int_0^T dT = \int_0^T \frac{P}{ms} \times dt$$

$$\Rightarrow T(t) = \frac{P}{ms} T + \theta_0$$

Comparing it with straight line equation  $y = mx + C$

$$\text{(Slope)} = m = \frac{P}{ms}, C = \theta_0$$

$\theta_0 \rightarrow$  temperature at  $t = 0$



# KVPY-SX 2019 (PHYSICS)



14. (c)

By Cauchy's theorem, we know that

$$\mu = \mu_0 + \frac{A}{\lambda^2}$$

$\mu_0$ ,  $A$  are coefficients that can be determined for a material by fitting the equation to measured refractive indices at known wavelengths. The coefficients are usually quoted for  $\lambda$  as the vacuum wavelength in micrometers.

$$\Rightarrow \mu_{\text{red}} < \mu_{\text{blue}}$$

As the reflected light is polarized, so incidence angle must be equal to Brewster's angle.

by Brewster law

$$\mu = \tan i_B, \mu = \text{refractive index}, i_B = \text{angle of incidence of blue light.}$$

$$\Rightarrow i_B = \tan^{-1}(\mu)$$

$$\Rightarrow \theta_R < \theta_B$$

15. (a)

For spherical surface, potential

$$V = \frac{kQ}{r}$$

$K = \text{Constant } (9 \times 10^9 \text{ N-m}^2/\text{C}^2)$

$Q = \text{Magnitude of charge}$

$r = \text{Distance of the charge}$

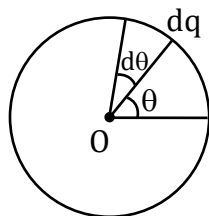
Since charge is increasing in discrete manner, so the potential will remain same till the time charge is same on sphere.



16. (b)

$$\text{Given radius of the ring} = r, \lambda = \frac{\sin^2 \theta}{\pi r}$$

From the definition of linear charge density  $\lambda = \frac{q}{\ell}$ , here  $\ell = r$



Let  $d\theta$  be the small angle made by small charge element  $dq$  so, the potential at the

$$\text{Centre of the ring i.e. } O = \int \frac{k \cdot dq}{r}$$

# KVPY-SX 2019 (PHYSICS)

B

$$\Rightarrow V_0 = \int \frac{k}{r} \cdot \lambda \cdot R \cdot d\theta$$

$$\Rightarrow V_0 = \int \frac{k}{r} \frac{q \sin^2 \theta}{\pi r} \cdot r \cdot d\theta$$

$$\Rightarrow V_0 = \frac{kq}{r\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta \quad \{\cos 2\theta = 1 - 2\sin^2 \theta\}$$

$$\Rightarrow V_0 = \frac{kq}{r\pi} \int_0^{2\pi} \left( \frac{1 - \cos 2\theta}{2} \right) d\theta$$

$$\Rightarrow V_0 = \frac{kq}{r\pi} \left[ \frac{1}{2} \cdot 2\pi - \frac{1}{2} \cdot \frac{1}{2} [\sin 2\theta]_0^{2\pi} \right]$$

$$\Rightarrow V_0 = \frac{kq}{r\pi} \times \pi \quad \Rightarrow \quad V_0 = \frac{kq}{r}$$

So, work done by external agent.

$$W = Q[V_i - V_f] = \frac{kqQ}{r} = \frac{qQ}{4\pi\epsilon_0 r}$$

17. (b)

In case-I energy of all the electrons will be same as no neutrino or anti-neutrino are coming out but in case-II as neutrino or anti neutrino are coming out so energy of electron will be different.

18. (b)

During the mixing process internal energy remains conserved

By energy conservation

$$\Rightarrow \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT = \frac{f_{\text{mix}}}{2} (n_1 + n_2) RT$$

Where  $f_1$  = degree of freedom for ideal gas A,

T = temperature

R = gas constant

$f_2$  = degree of freedom for ideal gas B,

$n_1$  = Number of moles of ideal gas A,

$n_2$  = Number of moles of ideal gas B,

$f_{\text{mix}}$  = degree of freedom of mixture of gases A & B

$$f_{\text{mix}} = \frac{f_1 n_1 + f_2 n_2}{n_1 + n_2}$$

As,  $\gamma = 1 + \frac{2}{f}$  { $\gamma$  = Ratio of constant pressure and constant volume specific heats}

$$\Rightarrow f = \frac{2}{\gamma - 1}$$

# KVPY-SX 2019 (PHYSICS)



$$\Rightarrow \frac{2}{\gamma_{\text{mix}} - 1} = \left( \frac{n_1}{n_1 + n_2} \right) \left( \frac{2}{\gamma_1 - 1} \right) + \left( \frac{n_2}{n_1 + n_2} \right) \left( \frac{2}{\gamma_2 - 1} \right)$$

As,  $\gamma_{\text{mix}} = \frac{19}{13}$ ;  $\gamma_1 = \frac{5}{3}$ ,  $\gamma_2 = \frac{7}{3}$ ,  $n_1 = 1$  and  $n_2 = n$

$\therefore n_2 = 2$

19. (c)

The charge Q on the plates is proportional to the potential difference V across the two plates

i.e,  $Q \propto V$

$\Rightarrow Q = CV$  ..... (1)

Where C = Capacitance

We know that as,  $C = \frac{\epsilon_0 A}{d}$  ..... (2)

$\epsilon_0$  = Permittivity of free space

A = Area of plate

D = distance between the plates.

So, from equation (1) & (2)

$\Rightarrow Q = \frac{\epsilon_0 A}{d} \times V$

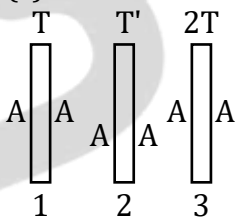
$\Rightarrow V = \frac{Qd}{\epsilon_0 A}$

$\therefore V \propto d$

But as d cannot be zero.

$\therefore$  Graph in option (c) is correct.

20. (c)



A → Area of cross section

Let the temperature of middle plate is 'T' and the area of all the plates is same i.e, A.

Let us assume that all the plates are perfectly black bodies.

From Stefan Boltzmann law

In steady state.

Rate of Heat loss = Rate of heat gain

$$\sigma(2A)(T')^4 = \sigma AT^4 + \sigma A(2T)^4$$

$\sigma$  = Stefan constant

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

$\Rightarrow 2(T')^4 = \sigma AT^4 + \sigma A(2T)^4$

# KVPY-SX 2019 (PHYSICS)



$$\Rightarrow 2T'^4 = T^4 + 16T^4$$

$$(T')^4 = \frac{17}{2} T^4$$

$$\Rightarrow T' = \left(\frac{17}{2}\right)^{1/4} \cdot T$$

$$\Rightarrow T' = 1.7 T$$

## PART-II

21. (a)

Given (area)  $A = 10^{-4} \text{ m}^2$

$\theta_1 = 20^\circ\text{C}$ ,  $\theta_2 = T$ ,  $\alpha = 2.5 \times 10^{-6} \text{ K}^{-1}$ ,  $Y = 4 \times 10^{12} \text{ N/m}^2$

$\therefore$  we know that coefficient of linear expansion

$$\alpha = \frac{\text{change in length}}{\text{original length} \times \text{rise in temperature}}$$

$$\alpha = \frac{\Delta \ell}{\ell \times \Delta \theta} \quad \dots (1)$$

And Young's modulus  $Y = \frac{F\ell}{A\Delta\ell}$ , where  $F$  = force,  $A$  = Area

Substituting the values

$$4 \times 10^{12} = \frac{5000 \times \ell}{10^{-4} \times \Delta \ell}$$
$$\Rightarrow \frac{\Delta \ell}{\ell} = \frac{5000}{4 \times 10^{12} \times 10^{-4}} = 1250 \times 10^{-8} \quad \dots (2)$$

Now from equation (1) & (2)

$$\Rightarrow \Delta \theta = \frac{\Delta \ell}{\ell \times \alpha} = \frac{1250 \times 10^{-8}}{2.5 \times 10^{-6}} = 500 \times 10^{-2} = 5$$

$$20 - T = 5$$

$$\Rightarrow T = 15^\circ\text{C}$$

22. (b)

The magnetic flux linked with long solenoid of radius  $R$  due to short solenoid of radius  $r$  having current  $I$  is,

$$\Phi = MI \quad \dots (1)$$

$I$  = current

Where  $M$  = mutual inductance

The magnetic field due to the short solenoid,

$$B = \mu_0 n \ell I$$

Magnetic flux linked is,

$$\Phi = B.A$$

$B$  = magnetic field

A = area

$$\Phi = \mu_0 n \ell I (\pi r^2) \quad \dots (2)$$

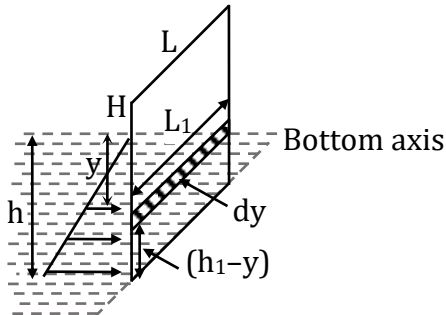
From eq.(1) and (2), we get

$$M = \pi \mu_0 r^2 n \ell$$

For N number of turns

$$M = \pi \mu_0 r^2 N n \ell$$

23. (d)



Pressure at any height h:

$$P = P_0 + \rho gh$$

$$P = \rho gh \quad \{ \because \text{given ignore atmospheric pressure} \}$$

$P \propto h$ ,  $p_0$  = atmosphere pressure

Force on elemental area dA:

$$dF = P_y \cdot dA$$

$$dF = \rho_w g y dA$$

$$dF = \rho_w g y L_1 dy$$

Torque about bottom length when height of water is  $h_1$  & length of dam is  $L_1$  and  $y$  = height of water:

$$d\tau = dF(h_1 - y)$$

$$d\tau = \rho_w g y L_1 dy [h_1 - y]$$

$$\tau = \int_0^{h_1} \rho_w g y L_1 [h_1 - y] dy$$

$$\tau = \rho_w g L_1 \int_0^{h_1} [h_1 y - y^2] dy$$

$$\tau = \rho_w g L_1 \left[ h_1 \frac{y^2}{2} - \frac{y^3}{3} \right]_0^{h_1}$$

# KVPY-SX 2019 (PHYSICS)



$$\tau = \rho_w g L_1 \left[ \frac{h_1^3}{2} - \frac{h_1^3}{3} \right]$$

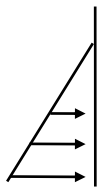
$$\tau = \rho_w g L_1 \frac{h_1^3}{6}$$

Case-I:

$$L_1 = L, h_1 = h$$

$$\tau_1 = \rho_w g L \frac{h^3}{6}$$

Pressure value linearly



Case-II:

$$L_1 = \frac{L}{2}, h_1 = \frac{h}{2}$$

$$\tau_2 = \rho_w g \frac{L}{2} \frac{h^3}{48} = \rho_w g L \frac{h^3}{96}$$

$$\therefore \frac{\tau_1}{\tau_2} = \frac{1}{6} \times \frac{96}{1}$$

$$\frac{\tau_1}{\tau_2} = 16$$

24. (d)

When valve is opened pressure will be same in both the containers after a long time.

So,  $P_1 = P_2$

$\therefore$  From ideal gas equation  $PV = nRT \Rightarrow P = \frac{nRT}{V}$

$$\frac{n_1 RT_1}{V_1} = \frac{n_2 RT_2}{V_2}$$

$n_1$  = number of moles in container  $C_1$

$n_2$  = number of moles in container  $C_2$

$V_1 = V$  = volume of gas in container  $C_1$

$V_2 = 4V$  = volume of gas in container  $C_2$

$T_1 = 300$  K = temperature of gas in container  $C_1$

$T_2 = 400$  K = temperature of gas in container  $C_2$

So,



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$$\frac{n_1}{n_2} = \frac{T_2 V_1}{T_1 V_2}$$
$$\frac{n_1}{n_2} = \frac{400 \times V}{300 \times 4V}$$
$$\frac{n_1}{n_2} = \frac{1}{3}$$

25. (d)

∴ Electrons are described by standing de Broglie waves confined within the box.

So length will be  $L = \frac{n\lambda}{2}$  ... (1)

∴ De Broglie wavelength  $\lambda = \frac{h}{p}$  ... (2)

From equation (1) & (2)

$$p = \frac{hn}{2L}$$

$$\therefore \text{Energy } E = \frac{p^2}{2m_e} = \frac{n^2 h^2}{4L^2 \cdot 2m_e} = \frac{n^2 h^2}{8L^2 m_e}$$

$$\therefore \text{Given } \alpha = \frac{h^2}{8m_e L^2}$$

$$\therefore E = n^2 \alpha,$$

For ground state  $E = \alpha = u_0$

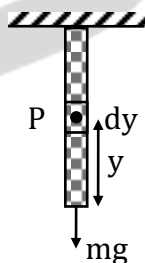
For first excited state  $E = 2^2 \alpha = 4u_0$

$$= u_0 + 3\alpha$$

For second excited state  $E = 3^2 \alpha = 9\alpha$

$$= u_0 + 8\alpha$$

26. (b)



Considering the small element of rope  $dy$  which is at height  $y$  from the bottom or free end of the rope.

So, mass of the total length is =  $m$

$$\text{Mass of the 1 length is} = \frac{m}{L}$$

# KVPY-SX 2019 (PHYSICS)



$$\therefore \text{Mass of the } y \text{ length is } = \frac{m}{L} y \quad \dots (1)$$

$$\therefore \text{Mass per unit } \mu = \frac{m}{L}$$

$\therefore$  eq. (1) becomes  $\mu y$

Tension in the rope at point P will be  $T = \mu y g$

As we know that speed of transverse wave in rope is given by

$$v = \sqrt{\frac{T}{\mu}}$$

$$\therefore v = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{y g}$$

$$\frac{dy}{dt} = \sqrt{y g}$$

$$\frac{dy}{\sqrt{y}} = \sqrt{g} dt$$

Integrating

$$\int_0^L \frac{dy}{\sqrt{y}} = \sqrt{g} \int_0^t dt$$

$$\Rightarrow \int_0^L (y^{-1/2}) dy = \sqrt{g} \int_0^t dt$$

$$\Rightarrow [2\sqrt{y}]_0^L = \sqrt{g} [t]_0^t$$

$$\Rightarrow 2\sqrt{L} = \sqrt{g} t$$

$$\Rightarrow t = 2\sqrt{\frac{L}{g}}$$

$$\Rightarrow t \propto \sqrt{L}$$

27. (a)

From kirchoff's voltage law

$$-\frac{q}{C} = L \frac{di}{dt}$$

q = charge

C = capacitance

L = inductance

$\frac{di}{dt}$  = rote of change of current

$\phi$  = fluse

Antiquely  $Li_0 - 0 = \phi$

$$\Rightarrow i_0 = \frac{\phi}{L}$$

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$$\therefore I = \frac{dq}{dt}$$

So, from equation (1)

$$-\frac{q}{LC} = \frac{d}{dt} \left( \frac{dq}{dt} \right)$$

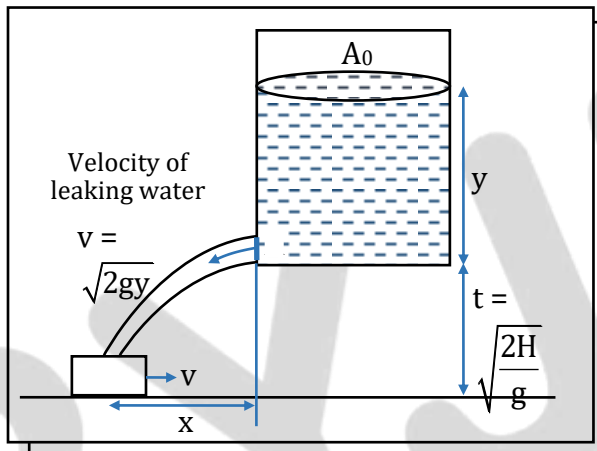
$$-\frac{q}{LC} = \frac{d^2q}{dt^2}$$

$$\Rightarrow q = q_0 \sin \omega_0 t$$

$$i = \frac{dq}{dt} = q_0 \omega_0 \cos \omega_0 t$$

$$i = \frac{\Phi}{L} \cos \omega_0 t$$

28. (c)



Distance (x) = velocity (v) × time (t)

$$= \sqrt{2gy} \times \sqrt{\frac{2H}{g}}$$

$$x = \sqrt{4yH} \quad \dots(1)$$

$$\text{Velocity } v = \frac{dx}{dt}$$

Differentiating equation (1) with respect to t

$$v = \frac{dx}{dt} = \frac{d}{dt} (\sqrt{4yH}) = \sqrt{4H} \frac{1}{2\sqrt{y}} \frac{dy}{dt}$$

$$v = \sqrt{\frac{H}{y}} \frac{dy}{dt}$$

$$\frac{dy}{dt} = \sqrt{2gy} \times \frac{A}{A_0}$$

$$\therefore v = \sqrt{\frac{H}{y}} \times \sqrt{2gy} \times \frac{A}{A_0} = \sqrt{2gH} \frac{A}{A_0}$$

Substituting the values

$$v = \sqrt{2 \times 10 \times 5} \times \frac{5}{500}$$

$$v = \sqrt{100} \times \frac{1}{100}$$

$$v = \frac{10}{100} = 0.1 \text{ m/s}$$

29. (c)

From Newton's law of cooling and Joule's heating effect.

Rate of heat gained by water

$$ms \left[ \frac{dT}{dt} \right] = i^2 R_1 - 4\sigma e A T_0^3 [T - T_0]$$

$$\frac{dT}{dt} = \frac{i^2 R_1}{ms} - \frac{4\sigma e A T_0^3}{ms} (T - T_0)$$

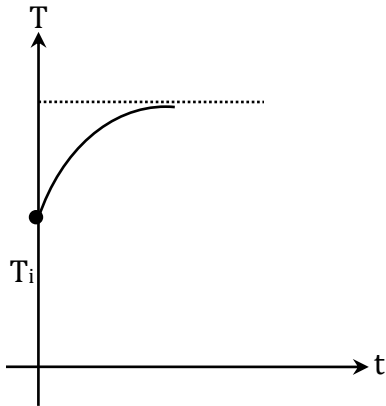
$$\frac{dT}{dt} = C_1 - C_2 T \quad (\text{Here } C_1 \text{ and } C_2 \text{ are positive constant})$$

$$\int_{T_i}^T \frac{dT}{C_1 - C_2 T} = \int dt$$

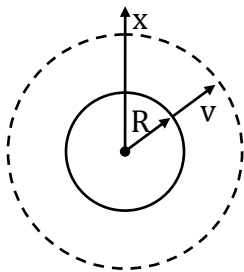
$$\frac{1}{-C_2} \ln \frac{C_1 - C_2 T}{C_1 - C_2 T_i} = t$$

$$C_1 - C_2 T = (C_1 - C_2 T_i) e^{-C_2 t}$$

$$C_2 T = C_1 - (C_1 - C_2 T_i) e^{-C_2 t}$$



30. (b)



As bubble increases its volume, due to surface tension work is to be done, velocity flux will be same

$$A_1V_1 = A_2V_2$$

$$4\pi R^2v = 4\pi x^2v_x$$

$$v_x = \frac{R^2v}{x^2}$$

So small kinetic energy at x for width dx

$$dK = \frac{1}{2}dmv_x^2 \left\{ \text{density}(\rho) \frac{\text{mass}(m)}{\text{Volume}(V)} \right\}$$

$$= \frac{1}{2} \rho \times 4\pi x^2 dx \times \frac{R^4v^2}{x^4}$$

Total kinetic energy

$$K = 2\pi\rho R^4v^2 \int_R^\infty \frac{dx}{x^2}$$

$$= 2\pi\rho R^4v^2 \left[ -\frac{1}{x} \right]_R^\infty$$

$$K = 2\pi\rho R^3v^2$$