Exercise 16C

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1.

Solution:

Let us consider

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{2\cos x}{\sin x + \cos x}\,dx$$

We can write it as

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos x + \cos x - \sin x + \sin x}{\sin x + \cos x}\,dx$$

So we get

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \frac{\cos x - \sin x}{\sin x + \cos x}\,dx$$

By integrating the first term

$$= \frac{1}{2}\left((x)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{\sin x + \cos x}\,dx \right)$$

Consider sin x + cos x = t

We get (cos x – sin x) dx = dt

Here if x = 0 we get t = 1

And if x = π/2 we get t = 1

So we get

$$y = \frac{1}{2}\left(\frac{\pi}{2} + \int_1^1 \frac{1}{t}\,dt \right)$$

By integrating w.r.t t

$$y = \frac{1}{2}\left(\frac{\pi}{2} + (\log t)_1^1\right)$$

By substituting the value of t

y = π/4

2.

Solution:

Let us take

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx \quad \ldots\ldots (1)$$

Using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

We get

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx \quad \ldots\ldots (2)$$

Now adding both the equations we get

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$$

It can be written as

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx$$

On further simplification

$$= \int_0^{\pi/2} 1\,dx$$

So we get

$= (x)_0^{\frac{\pi}{2}}$

$y = \pi/4$

3.

Solution:

(i)

$$\int_0^{\pi/2} \frac{\sin^3 x}{\left(\sin^3 x + \cos^3 x\right)} dx = \frac{\pi}{4}$$

It is given that

$$y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx \quad \ldots. (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{sin^3\left(\frac{\pi}{2} - x\right)}{sin^3\left(\frac{\pi}{2} - x\right) + cos^3\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx \quad \ldots. (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx + \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx$$

So we get

$$= \int_0^{\pi/2} \frac{sin^3 x + cos^3 x}{sin^3 x + cos^3 x} dx$$

It can be written as

$$= \int_0^{\pi/2} 1 \, dx$$

Here

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

(ii)

$$\int_0^{\pi/2} \frac{\cos^3 x \, dx}{(\sin^3 x + \cos^3 x)} = \frac{\pi}{4}$$

It is given that

$$y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a + b - x) \, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{cos^3\left(\frac{\pi}{2} - x\right)}{sin^3\left(\frac{\pi}{2} - x\right) + cos^3\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx + \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx$$

On further calculation

$$2y = \int_0^{\pi/2} \frac{cos^3 x + sin^3 x}{sin^3 x + cos^3 x} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

Here y = π/4

4.

Solution:

(i)

$$\int_0^{\pi/2} \frac{\sin^7 x}{\left(\sin^7 x + \cos^7 x\right)} dx = \frac{\pi}{4}$$

It is given that

$$y = \int_0^{\pi/2} \frac{sin^7 x}{sin^7 x + cos^7 x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{sin^7\left(\frac{\pi}{2} - x\right)}{sin^7\left(\frac{\pi}{2} - x\right) + cos^7\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{cos^7 x}{sin^7 x + cos^7 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{sin^7 x}{sin^7 x + cos^7 x} dx + \int_0^{\pi/2} \frac{cos^7 x}{sin^7 x + cos^7 x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{sin^7 x + cos^7 x}{sin^7 x + cos^7 x} dx$$

We get

$$2y = \int_0^{\pi/2} 1 \, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

$$(ii) \int_0^{\pi/2} \frac{sin^5 x}{(sin^5 x + cos^5 x)} dx = \frac{\pi}{4}$$

It is given that

$$y = \int_0^{\pi/2} \frac{sin^5 x}{sin^5 x + cos^5 x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{sin^5\left(\frac{\pi}{2} - x\right)}{sin^5\left(\frac{\pi}{2} - x\right) + cos^5\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{cos^5 x}{sin^5 x + cos^5 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{sin^5 x}{sin^5 x + cos^5 x} dx + \int_0^{\pi/2} \frac{cos^5 x}{sin^5 x + cos^5 x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{sin^5 x + cos^5 x}{sin^5 x + cos^5 x} dx$$

We get

$$2y = \int_0^{\pi/2} 1 \, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

5.

Solution:

It is given that

$$y = \int_0^{\pi/2} \frac{cos^4 x}{sin^4 x + cos^4 x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{cos^4\left(\frac{\pi}{2} - x\right)}{sin^4\left(\frac{\pi}{2} - x\right) + cos^4\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{sin^4 x}{sin^4 x + cos^4 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{cos^4 x}{sin^4 x + cos^4 x} dx + \int_0^{\pi/2} \frac{sin^4 x}{sin^4 x + cos^4 x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{cos^4 x + sin^4 x}{sin^4 x + cos^4 x} dx$$

We get

$$2y = \int_0^{\pi/2} 1 \, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \pi/4$

6.

Solution:

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx + \int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x + \sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

We get

$$2y = \int_0^{\pi/2} 1 \, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \pi/4$

7.

Solution:

It is given that

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx + \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

We get

$$2y = \int_0^{\pi/2} 1\,dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

8.

Solution:

It is given that

$$y = \int_0^{\pi/2} \frac{sin^n x}{sin^n x + cos^n x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{sin^n \left(\frac{\pi}{2} - x\right)}{sin^n \left(\frac{\pi}{2} - x\right) + cos^n \left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{cos^n x}{sin^n x + cos^n x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{sin^n x}{sin^n x + cos^n x} dx + \int_0^{\pi/2} \frac{cos^n x}{sin^n x + cos^n x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{sin^n x + cos^n x}{sin^n x + cos^n x} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \pi/4$

9.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\sin x}{\cos x}}}{\sqrt{\frac{\sin x}{\cos x}}+\sqrt{\frac{\cos x}{\sin x}}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx + \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x} dx$$

We get

$$2y = \int_0^{\pi/2} 1\,dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \pi/4$

10.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\sin x}{\cos x}}+\sqrt{\frac{\cos x}{\sin x}}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

11.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sin x}{\cos x}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{sinx + cosx}{sinx + cosx} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

12.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\cos x}{\sin x}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{sin\left(\frac{\pi}{2} - x\right)}{sin\left(\frac{\pi}{2} - x\right) + cos\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{cosx}{sinx + cosx} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{sinx}{sinx + cosx}\,dx + \int_0^{\pi/2} \frac{cosx}{sinx + cosx}\,dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{sinx + cosx}{sinx + cosx}\,dx$$

We get

$$2y = \int_0^{\pi/2} 1\,dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

13.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{sin^3x}{cos^3x}}\,dx$$

$$y = \int_0^{\pi/2} \frac{cos^3x}{sin^3x+cos^3x}\,dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{cos^3\left(\frac{\pi}{2}-x\right)}{sin^3\left(\frac{\pi}{2}-x\right)+cos^3\left(\frac{\pi}{2}-x\right)}\,dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx + \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{cos^3 x + sin^3 x}{sin^3 x + cos^3 x} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \pi/4$

14.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \frac{cos^3 x}{sin^3 x}} dx$$

$$y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

$y = \pi/4$

15.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\frac{\sin x}{\cos x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)}dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})}dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})}dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})}dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})}dx$$

We get

$$2y = \int_0^{\pi/2} 1\,dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

16.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\cos x}{\sin x}}}{1+\sqrt{\frac{\cos x}{\sin x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx$$

We get

$$2y = \int_0^{\pi/2} 1\,dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

17.

Solution:

We can write it as

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\sin x}{\cos x}}}{1+\sqrt{\frac{\sin x}{\cos x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

On further simplification

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$$

It can be written as

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\left(\sqrt{\sin x} + \sqrt{\cos x}\right)} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

18.

Solution:

It is given that

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$$

It can be written as

$$2y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x + \cos x - \sin x}{1 + \cos x \sin x} dx$$

Integrating w.r.t x

$$2y = \int_0^{\frac{\pi}{2}} 0 \, dx$$

y = 0

19.

Solution:

It is given that

$$y = \int_0^1 x(1-x)^5 dx$$

By using the king theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

We know that

$$y = \int_0^1 (1-x)x^5 dx$$

By further calculation

$$y = \int_0^1 x^5 - x^6 \, dx$$

Integration w.r.t x

$$y = \left(\frac{x^6}{6} - \frac{x^7}{7}\right)_0^1$$

Substituting the values

y = 1/6 – 1/7 = 1/42

20.

Solution:

It is given that

$$y = \int_0^2 x\sqrt{2-x}\,dx$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

We know that

$$y = \int_0^2 (2-x)\sqrt{x}\,dx$$

On further calculation

$$y = \int_0^2 2x^{\frac{1}{2}} - x^{\frac{3}{2}}\,dx$$

By integrating w.r.t x

$$y = \left(2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)_0^2$$

Substituting the values

$$y = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}$$

21.

Solution:

It is given that

$$y = \int_0^{\pi} x\cos^2 x\,dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi} (\pi - x)\cos^2(\pi - x)\,dx$$

On further simplification

$$y = \int_0^{\pi} \pi\cos^2 x - x\cos^2 x\,dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi} x\cos^2 x\,dx + \int_0^{\pi} \pi\cos^2 x - x\cos^2 x\,dx$$

It can be written as

$$2y = \int_0^{\pi} \pi\cos^2 x\,dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} \frac{1+\cos 2x}{2}dx$$

Integrating w.r.t x

$$y = \frac{\pi}{2}\left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\pi}$$

Substituting the values

$$y = \frac{\pi}{2}\left(\frac{\pi}{2} + \frac{\sin 2\pi}{4}\right) = \frac{\pi^2}{4}$$

22.

Solution:

It is given that

$$y = \int_0^{\pi} \frac{x \tan x}{\sec x \; cosec \, x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\pi} \frac{(\pi - x) \tan(\pi - x)}{\sec(\pi - x) \; cosec \; (\pi - x)} dx$$

$$y = \int_0^{\pi} \frac{-(\pi - x) \tan x}{-\sec x \; cosec \; x} dx$$

On further simplification

$$y = \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x \; cosec \, x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi} \frac{x \tan x}{\sec x \; cosec \; x} dx + \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x \; cosec \; x} dx$$

It can be written as

$$2y = \int_0^{\pi} \frac{\pi \tan x}{\sec x \ cosec\ x} dx$$

We get

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} \times \frac{1}{\sin x}} dx$$

On further simplification

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{1 - \cos 2x}{2} dx$$

Integrating w.r.t x

$$y = \frac{\pi}{2} \left(\frac{x}{2} - \frac{\sin 2x}{4} \right)_0^{\pi}$$

Substituting the values

$$y = \frac{\pi}{2} \left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right) = \frac{\pi^2}{4}$$

23.

Solution:

It is given that

$$y = \int_0^{\frac{\pi}{2}} \frac{cos^2 x}{\sin x + \cos x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

So we get

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

On further simplification

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$$

It can be written as

$$2y = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$$

We get

$$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x} dx$$

On further simplification

$$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\sin\left(x + \frac{\pi}{4}\right)} dx$$

Here

$$y = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} cosec\left(x + \frac{\pi}{4}\right) dx$$

Integrating w.r.t x

$$y = \frac{1}{2\sqrt{2}}\left(\log\left(\operatorname{cosec}\left(x+\frac{\pi}{4}\right) - \cot\left(x+\frac{\pi}{4}\right)\right)\right)_0^{\frac{\pi}{2}}$$

Substituting the values

$$y = \frac{1}{2\sqrt{2}}\left(\log\left(\operatorname{cosec}\frac{3\pi}{4} - \cot\frac{3\pi}{4}\right) - \log\left(\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right)\right)$$

We get

$$y = \frac{1}{2\sqrt{2}}\log\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$y = \frac{1}{2\sqrt{2}}\log(\sqrt{2}+1)^2 = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$$

24.

Solution:

We can write it as

$$y = \int_0^{\pi} \frac{x\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\cos x}\,dx$$

$$y = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\,dx$$

On further simplification

$$y = \int_0^{\pi} \frac{\pi\sin x - x\sin x}{1+\cos^2 x}\,dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi} \frac{x \sin x}{1 + cos^2 x} dx + \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1 + cos^2 x} dx$$

It can be written as

$$2y = \int_0^{\pi} \frac{\pi \sin x}{1 + cos^2 x} dx$$

Consider cos x = t

We get

- sin x dx = dt

Here if x = 0 we get t = 1

And if x = π we get t = - 1

Substituting the values

$$y = -\frac{\pi}{2} \int_1^{-1} \frac{1}{1 + t^2} dt$$

By integrating w.r.t t

$$y = -\frac{\pi}{2} (\tan^{-1} t)_1^{-1}$$

By substituting the value of t

$$y = -\frac{\pi}{2} (\tan^{-1}(-1) - \tan^{-1} 1)$$

We get

$$y = \frac{\pi^2}{4}$$

25.

Solution:

It is given that

$$y = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin(\pi - x)} dx$$

On further simplification

$$y = \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} - \frac{x \sin x}{1+\sin x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx + \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} - \frac{x \sin x}{1+\sin x} dx$$

It can be written as

$$2y = \int_0^{\pi} \frac{\pi(\sin x + 1 - 1)}{1+\sin x} dx$$

On further simplification

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \frac{1}{1+\sin x} dx$$

So we get

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \frac{1-\sin x}{cos^2 x} dx$$

Here

$$y = \frac{\pi}{2}\int_0^{\pi} 1 - sec^2 x + \frac{\sin x}{cos^2 x}\,dx$$

Consider cos x = t

We get

- sin x dx = dt

Here if x = 0 we get t = 1

And if x = π we get t = - 1

Substituting the values

$$y = \frac{\pi}{2}\left((x - \tan x)_0^{\pi} - \int_1^{-1} \frac{1}{t^2}\,dt\right)$$

By integrating w.r.t t

$$y = \frac{\pi}{2}\left(\pi - \tan\pi - \left(\frac{-1}{t}\right)_1^{-1}\right)$$

By substituting the value of t

$$y = \frac{\pi}{2}(\pi - 2) = \pi\left(\frac{\pi}{2} - 1\right)$$

26.

Solution:

It is given that

$$y = \int_0^{\pi} \frac{x}{1+sin^2 x}\,dx \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

So we get

$$y = \int_0^{\pi} \frac{(\pi - x)}{1 + sin^2(\pi - x)} dx$$

On further simplification

$$y = \int_0^{\pi} \frac{\pi}{1+sin^2 x} - \frac{x}{1+sin^2 x} dx \quad \ldots\ldots (2)$$

By adding both the equations

$$2y = \int_0^{\pi} \frac{x}{1 + sin^2 x} dx + \int_0^{\pi} \frac{\pi}{1 + sin^2 x} - \frac{x}{1 + sin^2 x} dx$$

It can be written as

$$2\,y = \pi \int_0^{\pi} \frac{1}{1 + sin^2 x} dx$$

So we get

$$2I = 2\pi \int_0^{\pi/2} \frac{1}{1 + sin^2 x} dx$$

$On\ further\ calculation$

$$I = \pi \int_0^{\pi/2} \frac{1}{1 + sin^2 x} dx$$

$Here$

$$I = \pi \int_0^{\pi/2} \frac{sec^2 x}{sec^2 x + tan^2 x} dx$$

$By\ further\ simplification$

$$I = \pi \int_0^{\pi/2} \frac{sec^2 x}{1 + 2tan^2 x} dx$$

$Consider\ tan\ x = t\ we\ get\ sec^2 x dx = dt$

$$I = \pi \int_0^{\infty} \frac{dt}{1 + \sqrt{2}t^2}$$

$Integration\ w.r.t\ t$

$$I = \pi\frac{[tan^{-1}\sqrt{2}t]_0^{\varpi}}{\sqrt{2}}$$

$Substituting\ the\ values$

$$I = \pi\frac{[tan^{-1}\varpi - tan^{-1}0]}{\sqrt{2}}$$

$So\ we\ get$

$$I = \frac{\pi}{\sqrt{2}}(\frac{\pi}{2} - 0) = \frac{\pi^2}{2\sqrt{2}}$$

28.

Solution:

Let us take

$$y = \int_0^{\infty}\frac{x}{(1+x)(1+x^2)}dx$$

Consider x = tan t

We get dx = sec^2 t dt

Here if x = 0 we get t = 0

And if x = ∞ we get t = π/2

Substituting the values

$$y = \int_0^{\frac{\pi}{2}}\frac{\tan t}{(1+\tan t)(1+tan^2 t)}sec^2 t\ dt$$

We get

$$y = \int_0^{\frac{\pi}{2}} \frac{\tan t}{(1+\tan t)}\, dt$$

By further simplification

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin t}{(\cos t + \sin t)}\, dt \quad \ldots\ldots (1)$$

By using the king theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

So we get

$$y = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-t\right)}{\sin\left(\frac{\pi}{2}-t\right)+\cos\left(\frac{\pi}{2}-t\right)} dt$$

$$y = \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dt \quad \ldots. (2)$$

By adding both the equations

$$2y = \int_0^{\pi/2} \frac{\sin t}{\sin t + \cos t} dx + \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dx$$

On further calculation

$$2y = \int_0^{\pi/2} \frac{\sin t + \cos t}{\sin t + \cos t} dx$$

We get

$$2y = \int_0^{\pi/2} 1\, dx$$

Integrating w.r.t x

$$2y = (x)_0^{\frac{\pi}{2}}$$

y = π/4

29.

Solution:

Consider x = a sin t

We get dx = a cos t dt

Here if x = 0 we get t = 0

And if x = a we get t = π/2

So we get

$$y = \int_0^{\frac{\pi}{2}} \frac{a\cos t}{a\sin t + \sqrt{a^2 - a^2 sin^2 t}} dt$$

By taking a as common

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos t}{\sin t + \cos t} dt$$

It can be written as

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos t + \cos t - \sin t + \sin t}{\sin t + \cos t} dt$$

By further simplification

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \frac{\cos t - \sin t}{\sin t + \cos t} dt$$

Integrating the first term

$$y = \frac{1}{2}\left((t)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos t - \sin t}{\sin t + \cos t} dt \right)$$

Consider sin t + cos t = z

We get (cos t – sin t) dt = dz

Here if t = 0 we get z = 1

And if $t = \pi/2$ we get $z = 1$

We get

$$y = \frac{1}{2}\left(\frac{\pi}{2} + \int_1^1 \frac{1}{z} dz\right)$$

Integrating w.r.t z

$$y = \frac{1}{2}\left(\frac{\pi}{2} + (\log z)_1^1\right)$$

$y = \pi/4$