Exercise 5C

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1.

Solution:

(i) $A=\begin{bmatrix}2 & -1\\ 3 & 0\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}$

It can be written as

$$AB=\begin{bmatrix}2 & -1\\ 3 & 0\\ -1 & 4\end{bmatrix}\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}2(-2)-1(0) & 2(3)-1(4)\\ 3(-2)+0(0) & 3(3)+0(4)\\ -1(-2)+4(0) & -1(3)+4(4)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}-4-0 & 6-4\\ -6+0 & 9+0\\ 2+0 & -3+16\end{bmatrix}=\begin{bmatrix}-4 & 2\\ -6 & 9\\ 2 & 13\end{bmatrix}$$

The same way

$$BA=\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 0\\ -1 & 4\end{bmatrix}.$$

Here, B is 2 × 2 and A is 3 × 2 where the column of B and row of A are not equal.

Hence, BA does not exist.

(ii) $A=\begin{bmatrix}-1 & 1\\ -2 & 2\\ -3 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2 & 1\\ 0 & 1 & 2\\ -3 & 4 & -5\end{bmatrix}$

We know that

$$AB=\begin{bmatrix}-1 & 1\\ -2 & 2\\ -3 & 3\end{bmatrix}\begin{bmatrix}3 & -2 & 1\\ 0 & 1 & 2\\ -3 & 4 & -5\end{bmatrix}.$$

AB does not exist

Similarly

$$BA=\begin{bmatrix}3 & -2 & 1\\ 0 & 1 & 2\\ -3 & 4 & -5\end{bmatrix}\begin{bmatrix}-1 & 1\\ -2 & 2\\ -3 & 3\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}3(-1)+(-2)(-2)+1(-3) & 3(1)-2(2)+1(3)\\ 0(-1)+1(-2)+2(-3) & 0(1)+1(2)+2(3)\\ -3(-1)+4(-2)-5(-3) & -3(1)+4(2)-5(3)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}-3+4-3 & 3-4+3\\ 0-2-6 & 0+2+6\\ 3-8+15 & -3+8-15\end{bmatrix}=\begin{bmatrix}-2 & 2\\ -8 & 8\\ 10 & -10\end{bmatrix}$$

(iii) $A=\begin{bmatrix}0 & 1 & -5\\ 2 & 4 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -1 & 0\\ 0 & 5\end{bmatrix}$

It can be written as

$$AB=\begin{bmatrix}0 & 1 & -5\\ 2 & 4 & 0\end{bmatrix}\begin{bmatrix}1 & 3\\ -1 & 0\\ 0 & 5\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}0(1)+1(-1)-5(0) & 0(3)+1(0)-5(5)\\ 2(1)+4(-1)+0(0) & 2(3)+4(0)+0(5)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}0-1-0 & 0+0-25\\ 2-4+0 & 6+0+0\end{bmatrix}=\begin{bmatrix}-1 & -25\\ -2 & 6\end{bmatrix}$$

Similarly

$$BA=\begin{bmatrix}1 & 3\\ -1 & 0\\ 0 & 5\end{bmatrix}\begin{bmatrix}0 & 1 & -5\\ 2 & 4 & 0\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}1(0)+3(2) & 1(1)+3(4) & 1(-5)+3(0)\\ -1(0)+0(2) & -1(1)+0(4) & -1(-5)+0(0)\\ 0(0)+5(2) & 0(1)+5(4) & 0(-5)+5(0)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}0+6 & 1+12 & -5+0\\ 0+0 & -1+0 & 5+0\\ 0+10 & 0+20 & 0+0\end{bmatrix}=\begin{bmatrix}6 & 13 & -5\\ 0 & -1 & 5\\ 10 & 20 & 0\end{bmatrix}$$

(iv) $A=\begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$

It can be written as

$$AB=\left[\,1(1)+2(2)+3(3)+4(4)\,\right]$$

On further calculation

$$=[1+4+9+16]=[30]$$

Similarly

$$BA=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}1(1) & 1(2) & 1(3) & 1(4)\\2(1) & 2(2) & 2(3) & 2(4)\\3(1) & 3(2) & 3(3) & 3(4)\\4(1) & 4(2) & 4(3) & 4(4)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}1 & 2 & 3 & 4\\2 & 4 & 6 & 8\\3 & 6 & 9 & 12\\4 & 8 & 12 & 16\end{bmatrix}$$

(v) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

It can be written as

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

Similarly

$$BA = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 1(2)+0(3)+1(-1) & 1(1)+0(2)+1(1) \\ -1(2)+2(3)+1(-1) & -1(1)+2(2)+1(1) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 2+0-1 & 1+0+1 \\ -2+6-1 & -1+4+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

2.

Solution:

(i) $A=\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$

It can be written as

$$AB=\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}5(2)+(-1)3 & 5(1)-1(4)\\ 6(2)+7(3) & 6(1)+7(4)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}10-3 & 5-4\\ 12+21 & 6+28\end{bmatrix}=\begin{bmatrix}7 & 1\\ 33 & 34\end{bmatrix}$$

Similarly

$$BA=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}2(5)+1(6) & 2(-1)+1(7)\\ 3(5)+4(6) & 3(-1)+4(7)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}10+6 & -2+7\\ 15+24 & -3+28\end{bmatrix}=\begin{bmatrix}16 & 5\\ 39 & 25\end{bmatrix}$$

Hence, AB ≠ BA.

(ii) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

It can be written as

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Similarly

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)-1(0)+1(1) & 0(2)-1(1)+1(1) & 0(3)-1(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0+0+1 & 0-1+1 & 0-0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

Therefore, $AB \neq BA$.

3.

Solution:

(i) $A=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$ and $B=\begin{bmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{bmatrix}$

It can be written as

$$AB=\begin{bmatrix}\cos\theta\cos\phi-\sin\theta\sin\phi & -\cos\theta\sin\phi-\sin\theta\cos\phi \\ \sin\theta\cos\phi+\cos\theta\sin\phi & -\sin\theta\sin\phi+\cos\theta\cos\phi\end{bmatrix}$$

So we get

$$AB=\begin{bmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi)\end{bmatrix}$$

Similarly

$$BA=\begin{bmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$$

On further calculation

$$BA=\begin{bmatrix}\cos\phi\cos\theta-\sin\phi\sin\theta & \cos\phi(-\sin\theta)+(-\sin\phi)\cos\theta \\ \sin\phi\cos\theta+\cos\phi\sin\theta & \sin\phi(-\sin\theta)+\cos\phi\cos\theta\end{bmatrix}$$

So we get

$$BA=\begin{bmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi)\end{bmatrix}$$

Hence, AB = BA.

(ii) $A=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$ and $B=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}$

It can be written as

$$AB=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}1(10)+2(-11)+1(9)&1(-4)+2(5)+1(-5)&1(-1)+2(0)+1(1)\\3(10)+4(-11)+2(9)&3(-4)+4(5)+2(-5)&3(-1)+4(0)+2(1)\\1(10)+3(-11)+2(9)&1(-4)+3(5)+2(-5)&1(-1)+3(0)+2(1)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}10-22+9&-4+10-5&-1+0+1\\30-44+18&-12+20-10&-3+0+2\\10-33+18&-4+15-10&-1+0+2\end{bmatrix}=\begin{bmatrix}-3&1&0\\4&-2&-1\\-5&1&1\end{bmatrix}$$

Similarly

$$BA=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}10(1)+(-4)(3)-1(1)&10(2)-4(4)-1(3)&10(1)-4(2)-1(2)\\-11(1)+5(3)+0(1)&-11(2)+5(4)+0(3)&-11(1)+5(2)+0(2)\\9(1)-5(3)+1(1)&9(2)-5(4)+1(3)&9(1)-5(2)+1(2)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}10-12-1&20-16-3&10-8-2\\-11+15+0&-22+20+0&-11+10+0\\9-15+1&18-20+3&9-10+2\end{bmatrix}=\begin{bmatrix}-3&1&0\\4&-2&-1\\-5&1&1\end{bmatrix}$$

Hence, AB = BA.

(iii) $A=\begin{bmatrix}1 & 3 & -1\\ 2 & 2 & -1\\ 3 & 0 & -1\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 3 & -1\\ -1 & 2 & -1\\ -6 & 9 & -4\end{bmatrix}$

It can be written as

$$AB=\begin{bmatrix}1 & 3 & -1\\ 2 & 2 & -1\\ 3 & 0 & -1\end{bmatrix}\begin{bmatrix}-2 & 3 & -1\\ -1 & 2 & -1\\ -6 & 9 & -4\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}1(-2)+3(-1)-1(-6) & 1(3)+3(2)-1(9) & 1(-1)+3(-1)-1(-4)\\ 2(-2)+2(-1)-1(-6) & 2(3)+2(2)-1(9) & 2(-1)+2(-1)-1(-4)\\ 3(-2)+0(-1)+(-1)(-6) & 3(3)+0(2)-1(9) & 3(-1)+0(-1)-1(-4)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}-2-3+6 & 3+6-9 & -1-3+4\\ -4-2+6 & 6+4-9 & -2-2+4\\ -6-0+6 & 9+0-9 & -3-0+4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Similarly

$$BA=\begin{bmatrix}-2 & 3 & -1\\ -1 & 2 & -1\\ -6 & 9 & -4\end{bmatrix}\begin{bmatrix}1 & 3 & -1\\ 2 & 2 & -1\\ 3 & 0 & -1\end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}-2(1)+3(2)-1(3) & -2(3)+3(2)-1(0) & -2(-1)+3(-1)-1(-1)\\ -1(1)+2(2)-1(3) & -1(3)+2(2)-1(0) & -1(-1)+2(-1)-1(-1)\\ -6(1)+9(2)-4(3) & -6(3)+9(2)+(-4)(0) & -6(-1)+9(-1)-4(-1)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}-2+6-3 & -6+6-0 & 2-3+1\\ -1+4-3 & -3+4-0 & 1-2+1\\ -6+18-12 & -18+18-0 & 6-9+4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Therefore, AB = BA.

4.

Solution:

It can be written as

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 2(2)-3(-1)-5(1) & 2(-2)-3(3)-5(-2) & 2(-4)-3(4)-5(-3) \\ -1(2)+4(-1)+5(1) & -1(-2)+4(3)+5(-2) & -1(-4)+4(4)+5(-3) \\ 1(2)-3(-1)-4(1) & 1(-2)-3(3)-4(-2) & 1(-4)-3(4)-4(-3) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A$$

Therefore, AB = A.

Similarly

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-3)-2(4)-4(-3) & 2(-5)-2(5)-4(-4) \\ -1(2)+3(-1)+4(1) & -1(-3)+3(4)+4(-3) & -1(-5)+3(5)+4(-4) \\ 1(2)-2(-1)-3(1) & 1(-3)-2(4)-3(-3) & 1(-5)-2(5)-3(-4) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

Therefore, BA = B.

5.

Solution:

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 0(a^2)+c(ab)-b(ac) & 0(ab)+c(b^2)-b(bc) & 0(ac)+c(bc)-b(c^2) \\ -c(a^2)+0(ab)+a(ac) & -c(ab)+0(b^2)+a(bc) & -c(ac)+0(bc)+a(c^2) \\ b(a^2)-a(ab)+0(ac) & b(ab)-a(b^2)+0(bc) & b(ac)-a(bc)+0(c^2) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 0+abc-abc & 0+cb^2-b^2c & 0+bc^2-bc^2 \\ -a^2c^2+0+a^2c & -abc+0+abc & -c^2a+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, AB = 0.

6.

Solution:

(i) $\text{LHS} = A(BC) = A\left(\begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}\right)$

So we get

$$= A\begin{bmatrix} 2(1)+3(4)+0(5) \\ 1(1)+0(4)+4(5) \\ 1(1)-1(4)+2(5) \end{bmatrix}$$

On further calculation

$$= A\begin{bmatrix} 2+12+0 \\ 1+0+20 \\ 1-4+10 \end{bmatrix}$$

By substituting the value of A

$$= \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix}$$

We get

$$= \begin{bmatrix} 1(14)+2(21)+5(7) \\ 0(14)+1(21)+3(7) \end{bmatrix}$$

So,

$$= \begin{bmatrix} 14+42+35 \\ 0+21+21 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

Similarly

$$\text{RHS} = (AB)C = \left(\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}\right)C$$

On further calculation

$$=\begin{bmatrix} 1(2)+2(1)+5(1) & 1(3)+2(0)+5(-1) & 1(0)+2(4)+5(2) \\ 0(2)+1(1)+3(1) & 0(3)+1(0)+3(-1) & 0(0)+1(4)+3(2) \end{bmatrix}$$

We get,

$$=\begin{bmatrix} 2+2+5 & 3+0-5 & 0+8+10 \\ 0+1+3 & 0+0-3 & 0+4+6 \end{bmatrix}C$$

By substituting the value of C

$$=\begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

So,

$$=\begin{bmatrix} 9(1)-2(4)+18(5) \\ 4(1)-3(4)+10(5) \end{bmatrix}$$

Here,

$$=\begin{bmatrix} 9-8+90 \\ 4-12+50 \end{bmatrix}=\begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

Therefore, A (BC) = (AB) C.

(ii) LHS $= A(BC) = A\left(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \end{bmatrix}\right)$

It can be written as

$$= A\begin{bmatrix} 1(1) & 1(-2) \\ 1(1) & 1(-2) \\ 2(1) & 2(-2) \end{bmatrix}$$

On further calculation

$$= A\begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$$

By substituting the value of A

$$= \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$$

By further simplification

$$= \begin{bmatrix} 2(1)+3(1)-1(2) & 2(-2)+3(-2)-1(-4) \\ 3(1)+0(1)+2(2) & 3(-2)+0(-2)+2(-4) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 2+3-2 & -4-6+4 \\ 3+0+4 & -6+0-8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

Similarly

$$\text{RHS} = (AB)C = \left(\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}\right)C$$

It can be written as

$$= \begin{bmatrix} 2(1)+3(1)-1(2) \\ 3(1)+0(1)+2(2) \end{bmatrix}C$$

On further calculation

$$= \begin{bmatrix} 2+3-2 \\ 3+0+4 \end{bmatrix}C$$

By substituting the value of C

$$= \begin{bmatrix} 3 \\ 7 \end{bmatrix}\begin{bmatrix} 1 & -2 \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 3(1) & 3(-2) \\ 7(1) & 7(-2) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

Therefore, A (BC) = (AB) C.

2

7.

Solution:

(i) $\text{LHS} = A(B+C) = A\left[\begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right]$

On further calculation

$$= A\begin{bmatrix} 2+1 & 0-1 \\ 1+0 & -3+1 \end{bmatrix}$$

By substituting the value of A

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 1(3)+2(1) & 1(-1)+2(-2) \\ 3(3)+4(1) & 3(-1)+4(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 3+2 & -1-4 \\ 9+4 & -3-8 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

Similarly

$$\text{RHS} = AB + AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 1(2)+2(1) & 1(0)+2(-3) \\ 3(2)+4(1) & 3(0)+4(-3) \end{bmatrix} + \begin{bmatrix} 1(1)+2(0) & 1(-1)+2(1) \\ 3(1)+4(0) & 3(-1)+4(1) \end{bmatrix}$$

It can be written as

$$= \begin{bmatrix} 2+2 & 0-6 \\ 6+4 & 0-12 \end{bmatrix} + \begin{bmatrix} 1+0 & -1+2 \\ 3+0 & -3+4 \end{bmatrix}$$

So

$$=\begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix};$$

By addition

$$=\begin{bmatrix} 4+1 & -6+1 \\ 10+3 & -12+1 \end{bmatrix}=\begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

Therefore, A (B + C) = (AB + AC).

(ii) $\text{LHS} = A(B+C) = A\left[\begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}\right]$

On further calculation

$$= A\begin{bmatrix} 5+(-1) & -3+2 \\ 2+3 & 1+4 \end{bmatrix} = A\begin{bmatrix} 5-1 & -1 \\ 5 & 5 \end{bmatrix}$$

By substituting the value of A

$$= \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 2(4)+3(5) & 2(-1)+3(5) \\ -1(4)+4(5) & -1(-1)+4(5) \\ 0(4)+1(5) & 0(-1)+1(5) \end{bmatrix}$$

$$= \begin{bmatrix} 8+15 & -2+15 \\ -4+20 & 1+20 \\ 0+5 & 0+5 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

Similarly

$$\text{RHS} = AB + BC = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

On further calculation

$$=\begin{bmatrix}2(5)+3(2) & 2(-3)+3(1)\\ -1(5)+4(2) & -1(-3)+4(1)\\ 0(5)+1(2) & 0(-3)+1(1)\end{bmatrix}+\begin{bmatrix}2(-1)+3(3) & 2(2)+3(4)\\ -1(-1)+4(3) & -1(2)+4(4)\\ 0(-1)+1(3) & 0(2)+1(4)\end{bmatrix}$$

So we get

$$=\begin{bmatrix}10+6 & -6+3\\ -5+8 & 3+4\\ 0+2 & 0+1\end{bmatrix}+\begin{bmatrix}-2+9 & 4+12\\ 1+12 & -2+16\\ 0+3 & 0+4\end{bmatrix}$$

It can be written as

$$=\begin{bmatrix}16 & -3\\ 3 & 7\\ 2 & 1\end{bmatrix}+\begin{bmatrix}7 & 16\\ 13 & 14\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}16+7 & -3+16\\ 3+13 & 7+14\\ 2+3 & 1+4\end{bmatrix}=\begin{bmatrix}23 & 13\\ 16 & 21\\ 5 & 5\end{bmatrix}$$

Therefore, A (B + C) = AB + AC.

8.

Solution:

We know that

$$\text{LHS} = A(B-C) = A\left(\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right)$$

On further calculation

$$= A\begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ -1-0 & 0+1 & 2-1 \end{bmatrix}$$

By substituting the value of A

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

It can be written as

$$= \begin{bmatrix} 1(-1)+0(-1)-2(-1) & 1(0)+0(0)-2(1) & 1(-6)+0(3)-2(1) \\ 3(-1)-1(-1)+0(-1) & 3(0)-1(0)+0(1) & 3(-6)-1(3)+0(1) \\ -2(-1)+1(-1)+1(-1) & -2(0)+1(0)+1(1) & -2(-6)+1(3)+1(1) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} -1-0+2 & 0+0-2 & -6+0-2 \\ -3+1-0 & 0-0+0 & -18-3+0 \\ 2-1-1 & 0+0+1 & 12+3+1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

Similarly

$$\text{RHS} = AB - AC = \left(\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\right) - \left(\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right)$$

On further calculation

$$= \begin{bmatrix} 1(0)+0(-2)-2(-1) & 1(5)+0(1)-2(0) & 1(-4)+0(3)-2(3) \\ 3(0)-1(-2)+0(-1) & 3(5)-1(1)+0(0) & 3(-4)-1(3)+0(2) \\ -2(0)+1(-2)+1(-1) & -2(5)+1(1)+1(0) & -2(-4)+1(3)+1(2) \end{bmatrix}$$

$$- \begin{bmatrix} 1(1)+0(-1)-2(0) & 1(5)+0(1)-2(-1) & 1(2)+0(0)+(-2)1 \\ 3(1)-1(-1)+0(0) & 3(5)-1(1)+0(-1) & 3(2)-1(0)+0(1) \\ -2(1)+1(-1)+1(0) & -2(5)+1(1)+1(-1) & -2(2)+1(0)+1(1) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 0-0+2 & 5+0-0 & -4+0-4 \\ 0+2-0 & 15-1+0 & -12-3+0 \\ 0-2-1 & -10+1+0 & 8+3+2 \end{bmatrix} - \begin{bmatrix} 1-0-0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15-1+0 & 6-0+0 \\ -2-1+0 & -10+1-1 & -4+0+1 \end{bmatrix}$$

Here

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-7 & -8-0 \\ 2-4 & 14-14 & -15-6 \\ -3+3 & -9+10 & 13+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

Here, LHS = RHS

Therefore, A (B – C) = AB – AC.

9.

Solution:

We know that

$$A^2 = A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} ab(ab)+b^2(-a^2) & ab(b^2)+b^2(-ab) \\ -a^2(ab)-ab(-a^2) & -a^2(b^2)-ab(-ab) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} a^2b^2-a^2b^2 & ab^3-ab^3 \\ -a^3b+a^3b & -a^2b^2+a^2b^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Therefore, $A^2 = 0$.

10.

Solution:

It can be written as

$$A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-2)-2(3)-4(-2) & 2(-4)-2(4)-4(-3) \\ -1(2)+3(-1)+4(1) & -1(-2)+3(3)+4(-2) & -1(-4)+3(4)+4(-3) \\ 1(2)-2(-1)-3(1) & 1(-2)-2(3)-3(-2) & 1(-4)-2(4)-3(-3) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Therefore, $A^2 = A$.

11.

Solution:

It can be written as

$$A^2 = A.A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 4(4)-1(3)-4(3) & 4(-1)-1(0)-4(-1) & 4(-4)-1(-4)-4(-3) \\ 3(4)+0(3)-4(3) & 3(-1)+0(0)-4(-1) & 3(-4)+0(-4)-4(-3) \\ 3(4)-1(3)-3(3) & 3(-1)-1(0)-3(-1) & 3(-4)-1(-4)-3(-3) \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 16-3-12 & -4-0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12-0+12 \\ 12-3-9 & -3-0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Therefore, $A^2 = I$.

12.

Solution:

It is given that

$$3A^2-2B+I=3\begin{bmatrix}2&-1\\3&2\end{bmatrix}\begin{bmatrix}2&-1\\3&2\end{bmatrix}-2\begin{bmatrix}0&4\\-1&7\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

On further calculation

$$=3\begin{bmatrix}2(2)-1(3)&2(-1)-1(2)\\3(2)+2(3)&3(-1)+2(2)\end{bmatrix}-\begin{bmatrix}0&8\\-2&14\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

It can be written as

$$=3\begin{bmatrix}4-3&-2-2\\6+6&-3+4\end{bmatrix}-\begin{bmatrix}0&8\\-2&14\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

So we get

$$=3\begin{bmatrix}1&-4\\12&1\end{bmatrix}-\begin{bmatrix}0&8\\-2&14\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}3&-12\\36&3\end{bmatrix}-\begin{bmatrix}0&8\\-2&14\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

By further simplification

$$=\begin{bmatrix}3&-20\\38&-11\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}4&-20\\38&-10\end{bmatrix}$$

13.

Solution:

We know that

$$-A^2+6A=-\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}+6\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

On further calculation

$$=-\begin{bmatrix} 2(2)-2(-3) & 2(-2)-2(4) \\ -3(2)+4(-3) & -3(-2)+4(4) \end{bmatrix}+\begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$$

So we get

$$=\begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix}+\begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$$

$$=\begin{bmatrix} -10+12 & 12+(-12) \\ 18+(-18) & -22+24 \end{bmatrix}=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

14.

Solution:

We know that

$$\text{LHS} = A^2 - 5A + 7I$$

By substituting the values

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

It can be written as

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore, $A^2 - 5A + 7I = 0$.

15.

Solution:

We know that

$$\text{LHS} = A^3 - 4A^2 + A$$

By substituting the values

$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

It can be written as

$$= \begin{bmatrix} 2(2)+3(1) & 2(3)+3(2) \\ 1(2)+2(1) & 1(3)+2(2) \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2(2)+3(1) & 2(3)+3(2) \\ 1(2)+2(1) & 1(3)+2(2) \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

On further calculation

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

By addition

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

By multiplication

$$= \begin{bmatrix} 7(2)+12(1) & 7(3)+12(2) \\ 4(2)+7(1) & 4(3)+7(2) \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

So we get

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

By further simplification

$$=\begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}-\begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}+\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=0$$

Therefore, $A^3 - 4A^2 + A = 0$.

16.

Solution:

We know that

$$A^2 = kA - 2I$$

It can be written as

$$kA = A^2 + 2I$$

By substituting the values

$$kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On further calculation

$$kA = \begin{bmatrix} 3(3)-2(4) & 3(-2)-2(-2) \\ 4(3)-2(4) & 4(-2)-2(-2) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

So we get

$$kA = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

It can be written as

$$kA = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

By further simplification

$$kA = \begin{bmatrix} 1+2 & -2+0 \\ 4+0 & -4+2 \end{bmatrix}$$

We get

$$kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

By substituting the value of A

$$k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

We can write it as

$$\begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

Here, 3k = 3, - 2k = -2 and 4k = 4.

Therefore, k = 1.

17.

Solution:

We know that

$$f(x)=x^2-2x+3$$

It can be written as

$$f(A)=A^2-2A+3I$$

By substituting the value of A

$$f(A)=\begin{bmatrix}-1 & 2\\ 3 & 1\end{bmatrix}\begin{bmatrix}-1 & 2\\ 3 & 1\end{bmatrix}-2\begin{bmatrix}-1 & 2\\ 3 & 1\end{bmatrix}+3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

On further calculation

$$f(A)=\begin{bmatrix}-1(-1)+2(3) & -1(2)+2(1)\\ 3(-1)+1(3) & 3(2)+1(1)\end{bmatrix}-\begin{bmatrix}-2 & 4\\ 6 & 2\end{bmatrix}+3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

So we get

$$f(A)=\begin{bmatrix}1+6 & -2+2\\ -3+3 & 6+1\end{bmatrix}-\begin{bmatrix}-2 & 4\\ 6 & 2\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

By addition

$$f(A)=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}-\begin{bmatrix}-2 & 4\\ 6 & 2\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

We get

$$f(A)=\begin{bmatrix}7+2+3 & 0-4+0\\ 0-6+0 & 7-2+3\end{bmatrix}=\begin{bmatrix}12 & -4\\ -6 & 8\end{bmatrix}$$

18.

Solution:

We know that

$$f(x)=2x^3+4x+5$$

It can be written as

$$f(A)=2A^3+4A+5I$$

By substituting the values

$$f(A)=2\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}+4\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}+5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

On further calculation

$$f(A)=2\begin{bmatrix}1(1)+2(4) & 1(2)+2(-3)\\ 4(1)-3(4) & 4(2)+(-3)(-3)\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}+\begin{bmatrix}4 & 8\\ 16 & -12\end{bmatrix}+\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

So we get

$$f(A)=2\begin{bmatrix}1+8 & 2-6\\ 4-12 & 8+9\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}+\begin{bmatrix}4+5 & 8+0\\ 16+0 & -12+5\end{bmatrix}$$

By addition

$$f(A)=2\begin{bmatrix}9 & -4\\ -8 & 17\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}+\begin{bmatrix}9 & 8\\ 16 & -7\end{bmatrix}$$

We get

$$f(A)=2\begin{bmatrix}9(1)-4(4) & 9(2)-4(-3)\\ -8(1)+17(4) & -8(2)+17(-3)\end{bmatrix}+\begin{bmatrix}9 & 8\\ 16 & -7\end{bmatrix}$$

By further simplification

$$f(A)=2\begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

Here

$$f(A)=2\begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

Multiplying by 2

$$f(A)=\begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

Where

$$f(A)=\begin{bmatrix} -14+9 & 60+8 \\ 120+16 & -134-7 \end{bmatrix}=\begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$$