WB-JEE-2016 (Mathematics)

1. Let A and B be two events such that $P(A \cap B) = \frac{1}{6}$, $P(A \cup B) = \frac{31}{45}$ and $P(\overline{B}) = \frac{7}{10}$ then
   a. A and B are independent
   b. A and B are mutually exclusive
   c. $P\left(\frac{A}{B}\right) < \frac{1}{6}$
   d. $P\left(\frac{B}{A}\right) < \frac{1}{6}$

2. The value of $\cos 15° \cos 7\frac{1°}{2} \sin 7\frac{1°}{2}$ is
   a. $\frac{1}{2}$
   b. $\frac{1}{8}$
   c. $\frac{1}{4}$
   d. $\frac{1}{16}$

3. The smallest positive root of the equation tan x – x = 0 lies in
   a. $(0, \pi/2)$
   b. $(\pi/2, \pi)$
   c. $\left(\pi, \frac{3\pi}{2}\right)$
   d. $\left(\frac{3\pi}{2}, 2\pi\right)$

4. If in a triangle ABC, AD, BE and CF are the altitudes and R is the circumradius, then the radius of the circumcircle of ΔEF is
   a. $\frac{R}{2}$
   b. $\frac{2R}{3}$
   c. $\frac{1}{3}R$
   d. None of these

5. The points $(-a, -b)$, (a, b), $(0, 0)$ and (a^2, ab), $a \neq 0$, $b \neq 0$ always lie on this line. Hence, collinear
   a. collinear
   b. vertices of a parallelogram
   c. vertices of a rectangle
   d. lie on a circle

6. The line AB cuts off equal intercepts 2a from the axes. From any point P on the line AB perpendiculars PR and PS are drawn on the axes. Locus of mid-point of RS is
   a. $x - y = \frac{a}{2}$
   b. $x + y = a$
   c. $x^2 + y^2 = 4a^2$
   d. $x^2 - y^2 = 2a^2$

7. $x + 8y - 22 = 0$, $5x + 2y - 34 = 0$, $2x - 3y + 13 = 0$ are the three sides of a triangle. The area of the triangle is
   a. 36 square unit
   b. 19 square unit
   c. 42 square unit
   d. 72 square unit

8. The line through the points (a, b) and $(-a, -b)$ passes through the point
   a. $(1, 1)$
   b. $(3a, -2b)$
   c. (a^2, ab)
   d. (a, b)

9. The locus of the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = K$ and $\frac{x}{a} - \frac{y}{b} = \frac{1}{k}$, where k is a non-zero real variable, is given by
   a. a straight line
   b. an ellipse
   c. a parabola
   d. a hyperbola

10. The equation of a line parallel to the line $3x + 4y = 0$ and touching the circle $x^2 + y^2 = 9$ in the first quadrant is
   a. $3x + 4y = 15$
   b. $3x + 4y = 45$
   c. $3x + 4y = 9$
   d. $3x + 4y = 27$

11. A line passing through the point of intersection of $x+y=4$ and $x-y=2$ makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the x-axis. It intersects the parabola $y^2=4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1-x_2|$ is equal to
   a. $\frac{16}{9}$
   b. $\frac{32}{9}$
   c. $\frac{40}{9}$
   d. $\frac{80}{9}$

12. Then equation of auxiliary circle of the $16x^2+25y^2+32x-100y=284$ is
   a. $x^2 + y^2 + 2x - 4y - 20 = 0$
   b. $x^2 + y^2 + 2x - 4y = 0$
   c. $(x+1)^2+(y-2)^2 = 400$
   d. $(x+1)^2+(y-2)^2=225$

13. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOPQ is equilateral, O being the centre. Then the eccentricity e satisfies
   a. $1 < e < \frac{2}{\sqrt{3}}$
   b. $e = \frac{2}{\sqrt{2}}$
   c. $e = \frac{\sqrt{3}}{2}$
   d. $e > \frac{2}{\sqrt{3}}$

14. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines; $x+y=3$ and $2x+2y-1=0$ then
    a. $2<a<4$
    b. $-\frac{1}{2}<a<2$
    c. $0<a<2$
    d. $-\frac{1}{2}<a<\frac{3}{2}$

15. A straight line joining the points (1,1,1) and (0,0,0) intersects the plane $2x+2y+z=10$ at
    a. (1, 2, 5)
    b. (2, 2, 2)
    c. (2, 1, 5)
    d. (1, 1, 6)

16. Angle between the planes $x+y+2z=6$ and $2x-y+z=9$ is
    a. $\frac{\pi}{4}$
    b. $\frac{\pi}{6}$
    c. $\frac{\pi}{3}$
    d. $\frac{\pi}{2}$

17. If $y=(1+x)(1+x^2)(1+x^4)........(1+x^{2n})$ then the value of $\left(\frac{dy}{dx}\right)$ at $x = 0$ is
    a. 0
    b. −1
    c. 1
    d. 2

18. If f(x) is an odd differentiable function defined on $(-\infty,\infty)$ such that $f'(3) = 2$, then $f'(-3)$ equal to
    a. 0
    b. 1
    c. 2
    d. 4

19. $\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{(1-\sqrt{x})}{(1-x)}}$

a. is 1
b. does not exist
c. is $\sqrt{\frac{2}{3}}$
d. is/n 2

20. If $f(x)=\tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right]+\tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$ then the value of f'' (x) is

a. x^2
b. x
c. 1
d. 0

21. $\int\frac{\log\sqrt{x}}{3x}\,dx$ is equal to

a. $\frac{1}{3}\left(\log\sqrt{x}\right)^2+c$
b. $\frac{2}{3}\left(\log\sqrt{x}\right)^2+c$
c. $\frac{2}{3}(\log x)^2+c$
d. $\frac{1}{3}(\log x)^2+c$

22. $\int 2^x(f'(x)+f(x)\log 2)$ is equal to

a. $2^x f'(x)+c$
b. $2^x\log 2+c$
c. $2^x f(x)+c$
d. 2^x+c

23. $\int_0^1 \log\left(\frac{1}{x}-1\right)dx =$

a. 1

b. 0

c. 2

d. None of these

24. The value of $\lim_{n\to\infty}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+....+\sqrt{2n-1}}{n^{\frac{3}{2}}}\right\}$ is

a. $\frac{2}{3}\left(2\sqrt{2}-1\right)$

b. $\frac{2}{3}\left(\sqrt{2}-1\right)$

c. $\frac{2}{3}\left(\sqrt{2}+1\right)$

d. $\frac{2}{3}\left(2\sqrt{2}+1\right)$

25. If the solution of the differential equation $x\frac{dy}{dx}+y=xe^x$ be, $xy = e^x\varphi(x) + c$ then $\varphi(x)$ is equal to

a. x+1

b. x–1

c. 1–x

d. x

26. The order of the differential equation of all parabolas whose axis of symmetry along x-axis is

a. 2

b. 3

c. 1

d. None of these

27. The line $y = x + \lambda$ is tangent to the ellipse $2x^2 + 3y^2 = 1$. Then λ is
    a. -2
    b. 1
    c. $\sqrt{\frac{5}{6}}$
    d. $\sqrt{\frac{2}{3}}$

28. The area enclosed by $y = \sqrt{5-x^2}$ and $y = |x-1|$ is
    a. $\left(\frac{5\pi}{4}-2\right)$sq.units
    b. $\frac{5\pi-2}{2}$sq.units
    c. $\left(\frac{5\pi}{4}-\frac{1}{2}\right)$sq.units
    d. $\left(\frac{\pi}{2}-5\right)$sq.units

29. Let S be the set of points whose abscissas and ordinates are natural numbers. Let $P \in S$ such that the sum of the distance of P from (8,0) and (0,12) is minimum among all elements in S. Then the number of such points P in S is
    a. 1
    b. 3
    c. 5
    d. 11

30. Time period T of a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. If the length is increased by 2%, then an approximate change in the time period is
    a. 2%
    b. 1%
    c. $\frac{1}{2}$%
    d. None of these

31. The cosine of the angle between any two diagonals of a cube is

a. $\frac{1}{3}$

b. $\frac{1}{2}$

c. $\frac{2}{3}$

d. $\frac{1}{\sqrt{3}}$

32. If x is a positive real number different from 1 such that $\log_a x$, $\log_b x$, $\log_c x$ are in A.P., then

a. $b = \frac{a+c}{2}$

b. $b = \sqrt{ac}$

c. $c^2 = (ac)^{\log_a b}$

d. None of (A), (B), (C) are correct

33. If a, x are real numbers and $|a| < 1$, $|x| < 1$, then $1 + (1+a)x + (1+a+a^2)x^2 + \ldots\infty$ is equal to

a. $\frac{1}{(1-a)(1-ax)}$

b. $\frac{1}{(1-a)(1-x)}$

c. $\frac{1}{(1-x)(1-ax)}$

d. $\frac{1}{(1+ax)(1-a)}$

34. if $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

a. $(2, \infty)$

b. $(1, 2)$

c. $(-2, -1)$

d. None of these

35. The value of $\sum_{n=1}^{13}(i^n + i^{n+1}), i = \sqrt{-1}$, is
   a. i
   b. i–1
   c. 1
   d. 0

36. If z_1, z_2, z_3 are imaginary numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ then $|z_1+z_2+z_3|$ is
   a. Equal to 1
   b. Less than 1
   c. Greater than 1
   d. Equal to 3

37. If p, q are the roots of the equation $x^2 + px + q = 0$, then
   a. p = 1, q = –2
   b. p = 0, q = 1
   c. p = –2, q = 0
   d. p = –2, q = 1

38. The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) are
   a. Three
   b. Two
   c. Infinitely many
   d. No values of k satisfies the requirement

39. The number of ways in which the letters of the word ARRANGE can be permuted such that the R's occur together is
   a. $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$
   b. $\frac{\lfloor 7}{\lfloor 2}$
   c. $\frac{\lfloor 6}{\lfloor 2}$
   d. $\lfloor 5 \times \lfloor 2$

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40. If $\frac{1}{{}^5C_r}+\frac{1}{{}^6C_r}=\frac{1}{{}^4C_r}$, then the value of r equals to

a. 4

b. 2

c. 5

d. 3

41. For +ve integer n, $n^3 + 2n$ is always divisible

a. 3

b. 7

c. 5

d. 6

42. In the expansion of (x – 1) (x – 2) .... (x – 18), the coefficient of x^{17} is

a. 684

b. –171

c. 171

d. –342

43. $1+{}^nC_1\cos\theta+{}^nC_2\cos 2\theta+\ldots\ldots+{}^nC_n\cos n\theta$ equals

a. $\left(2\cos\frac{\theta}{2}\right)^n\cos\frac{n\theta}{2}$

b. $2\cos^2\frac{n\theta}{2}$

c. $2\cos^{2n}\frac{\theta}{2}$

d. $\left(2\cos 2\frac{\theta}{2}\right)^n$

44. If x, y and z be greater than 1, then the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

a. log x. logy. log z

b. log x + logy + log z

c. 0

d. 1 – {(log x).(logy).(logz)}

45. Let A is a 3 × 3 matrix and B is its adjoint matrix. If |B| = 64, then |A| =
   a. ±2
   b. ±4
   c. ±8
   d. ±12

46. Let $Q = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ then Q^3x is equal to
   a. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
   b. $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
   c. $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
   d. $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

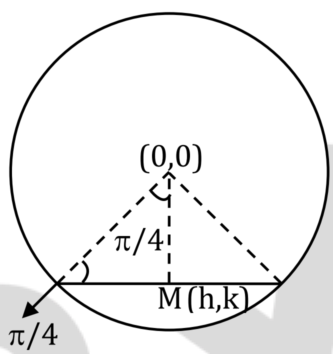
47. Let R be a relation defined on the set Z of all integers and xRy when x + 2y is divisible by 3. Then
   a. R is not transitive
   b. R is symmetric only
   c. R is an equivalence relation
   d. R is not an equivalence relation

48. If $A = \{5^n - 4n - 1 : n \in N\}$ and $B = \{16(n-1) : n \in N\}$, then
   a. A = B
   b. $A \cap B = \phi$
   c. $A \subseteq B$
   d. $B \subseteq A$

49. If the function $f : R \to R$ is defined by $f(x) = (x^2+1)^{35} \ \forall\ x \in R$, then f is
    a. one-one but not onto
    b. onto but not one-one
    c. neither one-one nor onto
    d. both one-one and onto

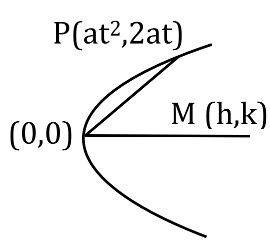
50. Standard Deviation of n observations a_1, a_2, a_3 .....an is σ. Then the standard deviation of the observations $\lambda a_1, \lambda a_2$ ...... λa_n is
    a. $\lambda\sigma$
    b. $-\lambda\sigma$
    c. $|\lambda|\sigma$
    d. $\lambda^n\sigma$

51. The locus of the midpoints of chords of the circle $x^2+y^2=1$ which subtends a right angle at the origin is



    a. $x^2+y^2=\frac{1}{4}$
    b. $x^2+y^2=\frac{1}{2}$
    c. $xy = 0$
    d. $x^2 - y^2 = 0$

52. The locus of the midpoints of all chords of the parabola $y^2 = 4ax$ through its vertex is another parabola with directrix



    a. $x = -a$
    b. $x = a$
    c. $x = 0$
    d. $x = -\frac{a}{2}$

53. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2[x]dx$ equals

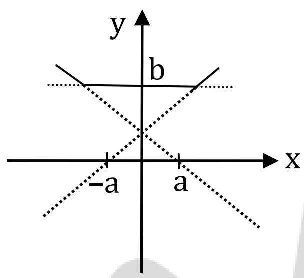
a. $\frac{5}{3}$

b. $\frac{7}{3}$

c. $\frac{8}{3}$

d. $\frac{4}{3}$

54. The number of points at which the function f(x) = max {a – x, a + x, b}, $-\infty < x < \infty$, 0 < a < b cannot be differentiable



a. 0
b. 1
c. 2
d. 3

55. For non-zero vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

a. Collinear

b. Perpendicular to each other

c. Inclined at an acute angle

d. Inclined at an obtuse angle

56. General solution of $y\frac{dy}{dx} + by^2 = a\cos x$, 0 < x < 1 is Here c is an arbitrary constant

a. $y^2 = 2a(2b \sin x + \cos x) + ce^{-2bx}$

b. $(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$

c. $(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{2bx}$

d. $y^2 = 2a(2b\sin x + \cos x) + ce^{-2bx}$

57. The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abscissa increases is/are given by

a. $\left(3, \frac{16}{3}\right) \& \left(-3, \frac{-16}{3}\right)$

b. $\left(3, \frac{-16}{3}\right) \& \left(-3, \frac{16}{3}\right)$

c. $\left(\frac{1}{16}, \frac{1}{9}\right) \& \left(-\frac{1}{16}, \frac{1}{9}\right)$

d. $\left(\frac{1}{16}, \frac{1}{9}\right) \& \left(-\frac{1}{16}, \frac{1}{9}\right)$

58. The letters of the word COCHIN are permuted and all permutation are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

a. 96
b. 48
c. 183
d. 267

59. If the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$, then $A^n = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{pmatrix}$, $n \in N$ where

a. $a = 2n, b = 2^n$
b. $a = 2^n, b = 2n$
c. $a = 2^n, b = n2^{n-1}$
d. $a = 2^n, b = n2^n$

60. The sum of n terms of the following series; $1^3 + 3^3 + 5^3 + 7^3 + \ldots$ is

a. $n^2(2n^2 - 1)$
b. $n^3(n - 1)$
c. $n^3 + 8n + 4$
d. $2n^4 + 3n^2$

61. If α and β are roots of $ax^2 + bx + c = 0$ then the equation whose roots are α^2 and β^2 is

a. $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$
b. $a^2x^2 + (b^2 - ac)x + c^2 = 0$
c. $a^2x^2 + (b^2 + ac)x + c^2 = 0$
d. $a^2x^2 + (b^2 + 2ac)x + c^2 = 0$

62. If ω is an imaginary cube root of unity, then the value of
$(2-\omega)(2-\omega^2)+2(3-\omega)(3-\omega^2)+\ldots\ldots+(n-1)(n-\omega)(n-\omega^2)$ is

a. $\frac{n^2}{4}(n+1)^2-n$

b. $\frac{n^2}{4}(n+1)^2+n$

c. $\frac{n^2}{4}(n+1)^2$

d. $\frac{n^2}{4}(n+1)-n$

63. If ${}^nC_{r-1}=36$, ${}^nC_r=84$ and ${}^nC_{r+1}=126$ then the value of ${}^nC_8$ is

a. 10
b. 7
c. 9
d. 8

64. In a group 14 males and 6 females, 8 and 3 of the males and females respectively are aged above 40 years. The probability that a person selected at random from the group is aged above 40 years, given that the selected person is female, is

a. $\frac{2}{7}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{5}{6}$

65. The equation $x^3-yx^2+x-y=0$ represents

a. a hyperbola and two straight lines
b. a straight line
c. a parabola and two straight lines
d. a straight line and a circle

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66. If the first and the $(2n+1)^{th}$ terms of an AP, GP and HP are equal and their n^{th} terms are respectively a, b, c then always
    a. $a = b = c$
    b. $a \geq b \geq c$
    c. $a + c = b$
    d. $ac - b^2 = 0$

67. The coordinates of a point on the line $x + y + 1 = 0$ which is at a distance $\frac{1}{5}$ unit from the line $3x + 4y + 2 = 0$ are
    a. $(2, -3)$
    b. $(-3, 2)$
    c. $(0, -1)$
    d. $(-1, 0)$

68. If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ unit on the line $y - 2x = 1$ then a is equal to
    a. 1
    b. -2
    c. -1
    d. 2

69. if f(x) is a function such that $f'(x) = (x-1)^2(4-x)$, then
    a. $f(0) = 0$
    b. f(x) is increasing in (0, 3)
    c. $x = 4$ is a critical point of f(x)
    d. f(x) is decreasing in (3, 5)

70. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are
    a. $\left(\frac{2}{5}, \frac{1}{5}\right)$
    b. $\left(-\frac{2}{5}, \frac{1}{5}\right)$
    c. $\left(-\frac{2}{5}, -\frac{1}{5}\right)$
    d. $\left(\frac{2}{5}, -\frac{1}{5}\right)$

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71. If $\int_{-3000}^{3000} \varphi(t) = \begin{cases} 1, \text{for } 0 \le t < 1 \\ 0 \text{ otherwise} \end{cases}$ then $\int_{-3000}^{3000} \left(\sum_{-3000}^{2016} \varphi(t - r')\varphi(t - 2016) \right) dt =$
    a. a real number
    b. 1
    c. 0
    d. does not exist

72. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots x = a and y = β then
    a. $3 \le x \le 7$
    b. $3 \le y \le 7$
    c. $-2 \le y \le 2$
    d. $-2 \le x \le 2$

73. If z = sin θ – i cos θ then for any integer n,
    a. $z^n + \frac{1}{z^n} = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$
    b. $z^n + \frac{1}{z^n} = 2\sin\left(\frac{n\pi}{2} - n\theta\right)$
    c. $z^n - \frac{1}{z^n} = 2i\cos\left(\frac{n\pi}{2} - n\theta\right)$
    d. $z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$

74. Let $f : X \to X$ be such that f(f(x)) = x for all $x \in X$ and $X \subseteq R$, then
    a. f is one-to-one
    b. f is onto
    c. f is one-to-one but not onto
    d. f is onto but not one-to-one

75. If A, B are two events such that $P(A \cup B)^3 \frac{3}{4}$ and $\frac{3}{8} \le P(A \cap B) \le \frac{3}{8}$ then
    a. $P(A) + P(B) \le \frac{11}{8}$
    b. $P(A).P(B) \le \frac{3}{8}$
    c. $P(A) + P(B) \ge \frac{7}{8}$
    d. None of these

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ANSWER KEYS

1. (a)	2. (b)	3. (c)	4. (a)	5. (a)	6. (b)	7. (b)	8. (c,d)	9. (d)	10. (a)
11. (b)	12. (a)	13. (d)	14. (b)	15. (b)	16. (c)	17. (c)	18. (c)	19. (c)	20. (d)
21. (a)	22. (c)	23. (b)	24. (a)	25. (b)	26. (a)	27. (c)	28. (c)	29. (b)	30. (b)
31. (a)	32. (c)	33. (c)	34. (a)	35. (b)	36. (a)	37. (a)	38. (c)	39. (c)	40. (b)
41. (a)	42. (b)	43. (a)	44. (c)	45. (c)	46. (c)	47. (c)	48. (c)	49. (c)	50. (c)
51. (b)	52. (d)	53. (b)	54. (c)	55. (d)	56. (b)	57. (a)	58. (a)	59. (d)	60. (a)
61. (a)	62. (a)	63. (c)	64. (b)	65. (b)	66. (b,d)	67. (b,d)	68. (a,b)	69. (b,c)	70. (b,d)
71. (a,b)	72. (a,c)	73. (a,c)	74. (a,b)	75. (a,c)					

Solution

1. (a)

$P(A\cap B)=\frac{1}{6}$, $P(A\cup B)=\frac{31}{45}$ $P(\overline{B})=\frac{7}{10}$ (given)

$P(B) = 1-\frac{7}{10}=\frac{3}{10}$

$\Rightarrow P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{31}{45}=P(A)+\frac{3}{10}-\frac{1}{6}\Rightarrow P(A)=\frac{5}{9}$

$\therefore P(A)\times P(B)=\frac{5}{9}\times\frac{3}{10}=\frac{1}{6}=P(A\cap B)$

$\Rightarrow$ A, B are independent events

2. (b)

$\text{Cos } 15° \cos 7\frac{1°}{2}\sin 7\frac{1°}{2}$

$\Rightarrow \frac{2\cos 7\frac{1°}{2}\sin 7\frac{1°}{2}\cos 15°}{2}$

$\Rightarrow \frac{\sin 15°\cos 15°}{2}\times\frac{2}{2}$

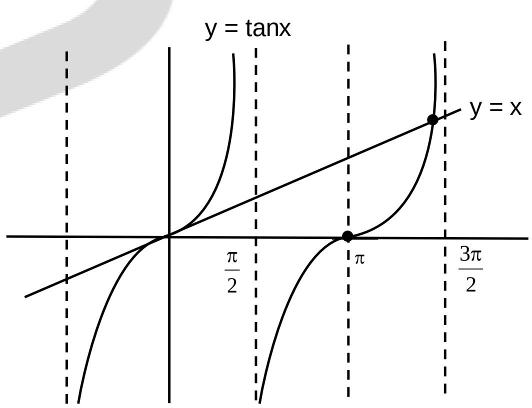
$\Rightarrow \frac{\sin 30°}{4}=\frac{1}{8}$

3. (c)

$\Rightarrow \tan x = x$

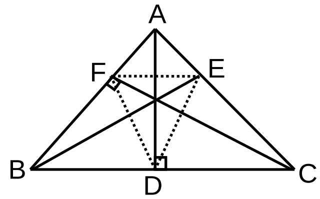
$y = \tan x$ ....(1)

$y = x$ .... (2)



It is clearly visible that solution lies in $\left(\pi,\frac{3\pi}{2}\right)$

4. (a)
Here, DEF is a pedal triangle of ΔABC
We know



EF = a cosA = R sinA (side of pedal Δ)
$\angle FDE = 180^\circ - 2A$
Let, circumradius of ΔDEF be R′.
Now by sine rule in ΔDEF

$$2R' = \frac{EF}{\sin\angle FDE} = \frac{R\sin 2A}{\sin(180^\circ - 2A)} \Rightarrow R' = \frac{R}{2}$$

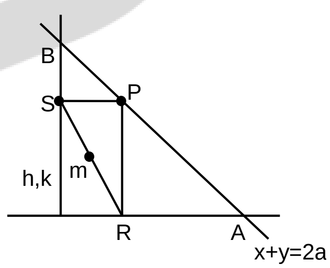
5. (a)

$$\Delta = \frac{1}{2}\begin{vmatrix} -a & -b \\ a & b \\ 0 & 0 \\ a^2 & ab \\ -a & -b \end{vmatrix}$$

$$\Delta = \frac{1}{2}\left(-ab + ab + 0 - 0 + 0 - 0 - a^2b + a^2b\right)$$

$\Delta = 0$
$\Rightarrow$ Hence, points are collinear

6. (b)
Equation of line which cuts equal intercepts from axes is x + y = 2a



Let, co-ordinates of the midpoint be m (h, k).
So, R and S are (2h, 0) and (0, 2k)
Therefore, p must be (2h, 2k)
$\because$ P lies on AB
$\therefore$ 2h + 2k = 2a
$\Rightarrow$ x + y = a

7. (b)

L_1: x + 8y = 22

L_2: 5x + 2y = 34

L_3: 2x – 3y = –13

On solving L_1, L_2 & L_3 we get

A = (–2, 3),   B = (6, 2) and C = (4, 7)

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} -2 & 3 \\ 6 & 2 \\ 4 & 7 \\ -2 & 3 \end{vmatrix} \right|$$

$$= \left| \frac{1}{2}(-4-18+42-8+12+14) \right| = 19 \text{ square units}$$

8. (c,d)

Equation of line passes through points (a, b) & (–a, –b) is

$$\Rightarrow (y - b) = \frac{-2b}{-2a}(x - a)$$

$\Rightarrow$ ay – ab = bx – ab

$\Rightarrow$ ay = bx

Now check options $\Rightarrow$ (C) & (D) are correct

9. (d)

$$L_1 = \frac{x}{a} + \frac{y}{b} = k$$

$$L_2 = \frac{x}{a} - \frac{y}{b} = \frac{1}{k}$$

Let point of intersection be (α, β)

So, $\frac{\alpha}{a} + \frac{\beta}{b} = k$ and $\frac{\alpha}{a} - \frac{\beta}{b} = \frac{1}{k}$

$$\Rightarrow \left(\frac{\alpha}{a} + \frac{\beta}{b}\right)\left(\frac{\alpha}{a} - \frac{\beta}{b}\right) = 1$$

$$\Rightarrow \frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1$$

Locus: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which is hyperbola

10. (a)

Let, equation of line which is parallel to given line is

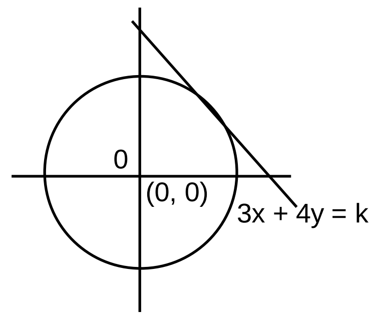
$3x + 4y = k$

This line is tangent to circle

$\Rightarrow d = r$

$\Rightarrow \left|\frac{0+0-k}{5}\right| = 3$

$\Rightarrow k = \pm 15$



So, equation of tangent in first quadrant is $3x + 4y = 15$

11. (b)

$L_1: x + y = 4$

$L_2: x - y = 2$

$\Rightarrow$ line passing through the point of intersection is

$\Rightarrow L = L_1 + \lambda L_2 = 0$

$\Rightarrow L = (x + y - 4) + \lambda (x - y - 2) = 0$

$\Rightarrow L = x(1 + \lambda) + y (1 - \lambda) - 4 - 2\lambda = 0$

$\Rightarrow M_L = \frac{3}{4}$ (given)

$\Rightarrow \frac{1+\lambda}{-1+\lambda} = \frac{3}{4}$

$\Rightarrow \lambda = -7$

Equation of line is $L = -6x + 8y + 10 = 0$

$y = \frac{3x-5}{4}$ (put in equation of parabola)

$\Rightarrow \left(\frac{3x-5}{4}\right)^2 = 4(x-3) \Rightarrow 9x^2 - 94x + 217 = 0$

$\Rightarrow x_1 + x_2 = \frac{94}{9}, x_1x_2 = \frac{217}{9}$

$\Rightarrow |x_1 - x_2| = \sqrt{(x_1+x_2)^2 - 4x_1x_2} = \frac{32}{9}$

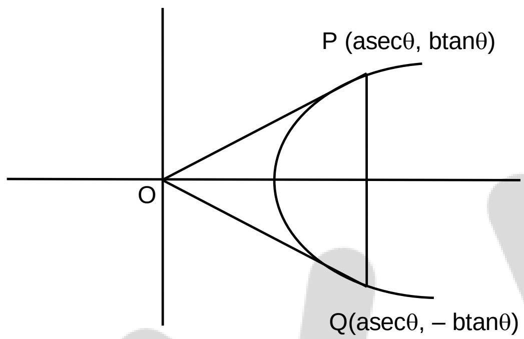
12. (a)

$\Rightarrow 16x^2 + 25y^2 + 32x - 100y = 284$
$\Rightarrow 16(x^2 + 2x) + 25(y^2 + 4y) = 284$
$\Rightarrow 16(x + 1)^2 + 25(y - 2)^2 = 284 + 16 + 100$
$\Rightarrow 16(x + 1)^2 + 25(y - 2)^2 = 400$
$\Rightarrow \frac{(x+1)^2}{25} + \frac{(y-2)^2}{16} = 1$
So, the auxiliary circle is $(x + 1)^2 + (y - 2)^2 = 25$
$\Rightarrow x^2 + y^2 + 2x - 4y - 20 = 0$

13. (d)

$\because \Delta OPQ$ is equilateral

$(OP)^2 = (PQ)^2$



$\Rightarrow a^2\sec^2\theta + b^2\tan^2\theta = (2b\tan\theta)^2$
$\Rightarrow a^2\sec^2\theta = 3b^2\tan^2\theta$
$\Rightarrow \sin^2\theta = \frac{a^2}{3b^2}$
$\Rightarrow$ Now, $\sin^2\theta < 1$
$\Rightarrow \frac{a^2}{3b^2} < 1$
$\Rightarrow \frac{b^2}{a^2} > \frac{1}{3}$
On adding 1 both sides
$\Rightarrow 1 + \frac{b^2}{a^2} > 1 + \frac{1}{3}$
$\Rightarrow e^2 > \frac{4}{3}$
On taking root both sides
$\Rightarrow e > \frac{2}{\sqrt{3}}$

14. (b)

$\Rightarrow y^2 - 4y = 4x - 4a$

$\Rightarrow y^2 - 4y + 4 = 4x - 4a + 4$

$\Rightarrow (y - 2)^2 = 4(x-(a-1))$

$\therefore$ Vertex: (a–1, 2)

Vertex lies in between lines

$\therefore L_1 \times L_2 < 0$

$\Rightarrow (a - 1 + 2 - 3) \times (2(a-1) + 4 - 1) < 0$

$\Rightarrow (a - 2)(2a + 1) < 0$

$\Rightarrow a \in \left(-\frac{1}{2}, 2\right)$

15. (b)

Equation of line joining points (1, 1, 1) & (0, 0, 0) is

$\Rightarrow L: \frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$

i.e. L : x = y = z = k (let) ……..(1)

Any point on L is p(k, k, k)

Since line & plane intersect at a point so, P lies on 2x + 2y + z = 10

$\Rightarrow 2k + 2k + k = 10$

$\Rightarrow k = 5$

So, point is (2, 2, 2)

16. (c)

Angle between planes = $\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$

$\Rightarrow \cos\theta = \frac{1 \times 2 + 1 \times (-1) + 2 \times 1}{\sqrt{1^2 + 1^2 + 2^2} \cdot \sqrt{2^2 + (-1)^2 + 1^2}}$

$\Rightarrow \cos\theta = \frac{3}{6}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

17. (c)

$y = (1+x)(1+x^2)(1+x^4) \ldots\ldots\ldots(1+x^{2n})$

Taking log both sides

$\log y = \log(1+x) + \log(1+x)^2 + \ldots\ldots + \log(1+x^{2n})$

On differentiating both sides

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \ldots\ldots\ldots + \frac{2n\,x^{2n-1}}{1+x^{2n}}$$

$$\frac{dy}{dx} = y\left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \ldots\ldots\ldots + \frac{2n\,x^{2n-1}}{1+x^{2n}}\right) \because \text{ at } x = 0 \Rightarrow y = 1$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 1 \cdot 1 = 1$$

18. (c)

$\because$ f(x) is odd differentiable

$\therefore$ $f(-x) = -f(x)$

On differentiating both sides

$\Rightarrow -f'(-x) = -f'(x)$

$\Rightarrow f'(x) = f'(-x)$ ....(1)

$\Rightarrow$ put $x = 3$ in eq. (1)

$\Rightarrow f'(3) = f'(-3)$

$\Rightarrow f'(-3) = 2 \quad \because f'(3) = 2$ (given)

19. (c)

$$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\left(\frac{1-\sqrt{x}}{1-x}\right)}$$

$$\Rightarrow \lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\left(\frac{1-\sqrt{x}}{(1-\sqrt{x})(1-\sqrt{x})}\right)}$$

$$\Rightarrow \lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\left(\frac{1}{1+\sqrt{x}}\right)}$$

$$\Rightarrow \left(\frac{1+1}{2+1}\right)^{\frac{1}{1+1}}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{1}{2}}.$$

20. (d)

$$f(x) = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$f(x) = \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-3.2\log x}\right)$$

$$f(x) = \tan^{-1} 1 - \tan^{-1}(2\log x) + \tan^{-1} 3 + \tan^{-1}(2\log x)$$

$f(x) = \tan^{-1}1 + \tan^{-1}3 =$ constant

On differentiating with respect to x

$f'(x) = 0$

Again differentiating with respect to x

$f''(x) = 0$

21. (a)

$$I = \int \frac{\log\sqrt{x}}{3x}dx$$

Let $\log\sqrt{x} = t$

$$\Rightarrow \frac{1}{\sqrt{x}}.\frac{1}{2\sqrt{x}}dx = dt \Rightarrow \frac{dx}{x} = 2dt$$

$$\Rightarrow I = \int \frac{t}{3}2dt$$

$$\Rightarrow I = \frac{2}{3}\left(\frac{t^2}{2}\right) + c \Rightarrow I = \frac{1}{3}\left(\log\sqrt{x}\right)^2 + c$$

22. (c)

$$I = \int 2^x(f'(x) + f(x)\log 2)\,dx$$

Let $g(x) = 2^x f(x) \Rightarrow g'(x) = 2^x(f(x) + f'(x)\ell n2)$

$$I = \int g'(x)\,dx$$

$$I = g(x) + C$$

$I = 2^x f(x) + C$

23. (b)

$$I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$$

$$\Rightarrow I = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

On applying king property

$$\Rightarrow I = \int_0^1 \log\left(\frac{x}{1-x}\right)dx$$

$$= -\int_0^1 \log\left(\frac{1-x}{x}\right)dx = -I$$

$\Rightarrow I = -I$

$\Rightarrow 2I = 0$

$\Rightarrow I = 0$

24. (a)

$$\lim_{n\to\infty}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+.......+\sqrt{2n-1}}{n^{3/2}}\right\}$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\left\{\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+.....+\sqrt{1+\frac{n-1}{n}}\right\}$$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n}\sqrt{1+\frac{r}{n}}$$

$$\Rightarrow \int_0^1 \sqrt{1+x}\,dx \qquad \because \text{ upper limit } = \lim_{n\to\infty}\frac{n-1}{n} = 1$$

$$\Rightarrow \frac{2}{3}(1+x)^{3/2}\Big|_0^1 \qquad \text{lower limit } = \lim_{n\to\infty}\frac{1}{n} = 0$$

$$\Rightarrow \frac{2}{3}\left(2\sqrt{2}-1\right)$$

25. (b)

$$x\frac{dy}{dx}+y=xe^x$$

$\frac{dy}{dx}+\frac{y}{x}=e^x$ …… linear differential equation

I.F = $e^{\int \frac{1}{x}dx} = e^{\ell nx} = x$

Solution of L.D.E

$\Rightarrow y.\,x = \int e^x.x\,dx$

$\Rightarrow y\,.\,x = xe^x - \int e^x dx$

$\Rightarrow y.x = xe^x - e^x + c$

$\Rightarrow xy = e^x(x-1) + c$

On comparing with given relation $xy = e^x\,\phi(x) + c$

$\phi(x) = (x-1)$

26. (a)

Let, equation of parabola $y^2 = 4a(x-b)$ ……..(1)

On differentiating with respect to x

$\Rightarrow 2yy' = 4a$

Again differentiating with respect to x

$\Rightarrow 2yy'' + 2(y')^2 = 0$

Order of differential equation = 2

27. (c)

E: $\frac{x^2}{3}+\frac{y^2}{2}=\frac{1}{6}$ ……….(1)

L: $y = x + \lambda$ …………(2)

Line is tangent to ellipse

$\Rightarrow 2x^2 + 3(x+\lambda)^2 = 1$

$\Rightarrow 2x^2 + 3(x^2 + 2x\lambda + \lambda^2) = 1$

$\Rightarrow 5x^2 + 6x\lambda + 3\lambda^2 - 1 = 0$

$\Rightarrow D = 0$

$\Rightarrow 36\lambda^2 - 20(3\lambda^2 - 1) = 0$

$\Rightarrow -24\lambda^2 + 20 = 0 \Rightarrow 6\lambda^2 - 5 = 0 \Rightarrow \lambda = \pm\sqrt{\frac{5}{6}}$

28. (c)

$y = \sqrt{5-x^2}$ ...... (1)

y = |x – 1| ...... (2)

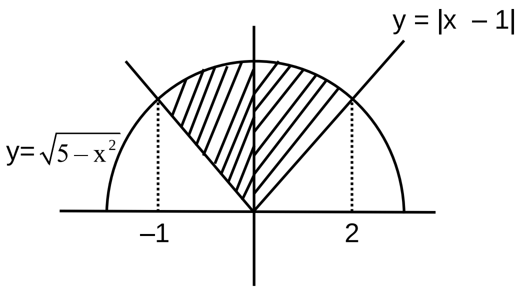
From equation (1) & (2)

$\Rightarrow 5 - x^2 = (x-1)^2$

$\Rightarrow 5 - x^2 = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow x = -1, 2$



Required area = A = $\int_{-1}^{2}\left(\sqrt{5-x^2} - |x-1|\right)dx$

$= \int_{-1}^{2}\left(\sqrt{5-x^2}\right)dx - \int_{-1}^{1}(1-x)\,dx - \int_{1}^{2}(x-1)\,dx$

$= 2 + \frac{5\pi}{4} - \frac{5}{2}$

$= \frac{5\pi}{4} - \frac{1}{2}$

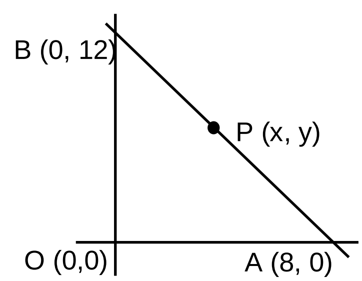
29. (b)

Equation of line AB =

$\Rightarrow \frac{x}{8} + \frac{y}{12} = 1$

$\Rightarrow 3x + 2y = 2y$

$\because$ Sum of distance of P is minimum,
so P will be on line AB



AB: 3x + 2y = 24

	x	y
Possible points	2	9
	4	6
	6	3

$\therefore$ (x, y) ≡ (2, 9), (4, 6), (6, 3)

30. (b)

Wwe have

$T = 2\pi\sqrt{\frac{\ell}{g}}$

Taking log both sides

$\log T = \log 2\pi + \frac{1}{2}(\log \ell - \log g)$

Differentiating both sides we get

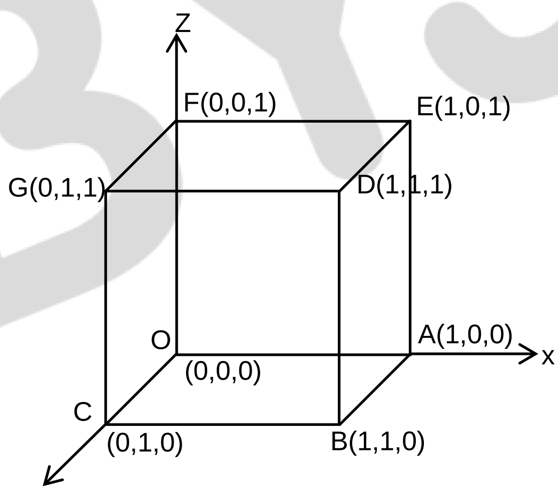
$\Rightarrow \frac{1}{T}dT = \frac{1}{2}\cdot\frac{1}{\ell}d\ell$

$\Rightarrow \frac{dT}{T}\times 100 = \frac{1}{2}\left(\frac{d\ell}{\ell}\times 100\right) = \frac{1}{2}\times 2 = 1\%$

31. (a)

Direction cosine of diagonal OD = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\right)$

Direction cosine of diagonal FB = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\frac{-1}{\sqrt{3}}\right)$



$\cos\theta = \frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\left(\frac{-1}{\sqrt{3}}\right) = \frac{1}{3}$

32. (c)

$\log_a x, \ \log_b x, \ \log_c x \rightarrow$ A.P

Then,

$\Rightarrow 2\log_b x = \log_a x + \log_c x$

$\Rightarrow \dfrac{2}{\log_x b} = \dfrac{1}{\log_x a} + \dfrac{1}{\log_x c}$

$\Rightarrow 2\log_x a \log_x c = \log_x ac \cdot \log_x b$

$\Rightarrow \log_x c^2 = \log_x ac \cdot \log_x b$

$\Rightarrow c^2 = (ac)^{\log_a b}$

33. (c)

$1 + (1 + a)\,x + (1 + a + a^2)\,x^2 + \ldots\ldots\ldots$

Multiply & divide by $(1 - a)$ we get

$= \dfrac{1}{1-a}\ [(1-a) + (1 - a^2)\,x + (1 - a^3)x^2 + \ldots\ldots\ \infty]$

$= \dfrac{1}{1-a}\ [(1 + x + x^2 + \ldots\ldots\ldots \infty) - (a + a^2 x + a^3 x^2 + \ldots\ldots\ldots \infty)$

$= \dfrac{1}{1-a}\left[\dfrac{1}{1-x} - \dfrac{a}{1-ax}\right]$

$= \dfrac{1}{(1-a)}\left[\dfrac{1-ax-a+ax}{(1-x)(1-ax)}\right]$

$= \dfrac{1}{(1-x)(1-a\,x)}$

34. (a)

$\log_{.3}(x-1) < \log_{.09}(x-1) \qquad \because x - 1 > 0 \Rightarrow x > 1 \qquad \ldots\ldots\ldots(1)$

$\Rightarrow \log_{.3}(x-1) < \log_{(.3)^2}(x-1)$

$\Rightarrow 2\log_{.3}(x-1) < \log_{.3}(x-1)$

$\Rightarrow \log_{.3}(x-1)^2 < \log_{.3}(x-1)$

$\Rightarrow (x - 1)^2 > (x - 1)$

$\Rightarrow x^2 - 3x + 2 > 0$

$x \in (-\infty, 1) \cup (2, \infty) \qquad \ldots\ldots\ldots(2)$

From (1) & (2)

$x \in (2, \infty)$

35. (b)

$$\sum_{n=1}^{13}(i^n + i^{n+1})$$

$\because$ we know,

$i + i^2 + i^3 + i^4 = 0$

$$\Rightarrow \sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$$

$\Rightarrow i^{13} + i^{14}$

$\Rightarrow i + i^2$

$\Rightarrow i - 1$

36. (a)

$$\left|\frac{1}{z} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1 \qquad \because z\bar{z} = |z|^2 = 1 \Rightarrow \bar{z} = \frac{1}{z}$$

$$\Rightarrow \left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 1$$

$$\Rightarrow |\overline{z_1 + z_2 + z_3}| = 1 \qquad \because |\bar{z}| = |z|$$

$$\Rightarrow |z_1 + z_2 + z_3| = 1$$

37. (a)

$x^2 + px + q = 0 <^{\alpha}_{\beta}$

$\Rightarrow p + q = -p \quad \Rightarrow \quad 2p + q = 0 \qquad .....(1)$

$\Rightarrow pq = q \quad \Rightarrow \quad q(p-1) = 0 \qquad ... \quad (2)$

From equation (1) & (2)

$\Rightarrow (-2p)(p-1) = 0$

$\Rightarrow \quad p = 0 \quad$ or $\quad p = 1$

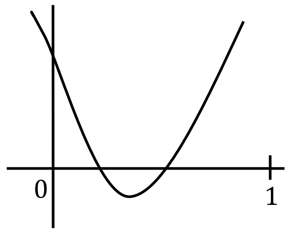
$q = 0 \quad$ or $\quad q = -2$

38. (c)

(1) D > 0

9 – 4 k > 0

$k < \frac{9}{4}$ .....(1)



(2) f(0) > 0

$\Rightarrow k > 0$ ...... (2)

(3) f(1) > 0

$\Rightarrow 1 - 3 + k > 0$

$\Rightarrow k > 2$ .....(3)

From (1), (2) & (3)

$k \in \left(2, \frac{9}{4}\right)$

39. (c)

A $\boxed{\text{RR}}$ ANGE

Number of ways when R's occurs together = $\frac{\lfloor 6}{\lfloor 2}$

$= \frac{120.6}{2}$

$= 360$

40. (b)

$\Rightarrow \frac{1}{{}^5C_r} + \frac{1}{{}^6C_r} = \frac{1}{{}^4C_r}$

$\Rightarrow \frac{r!(5-r)!}{5!} + \frac{r!(6-r)!}{6!} = \frac{r!(4-r)!}{4!}$

$\Rightarrow 6(5 - r) + (6 - r)(5 - r) = 6 \times 5$

$\Rightarrow 30 - 6r + r^2 - 11r + 30 = 30$

$\Rightarrow r^2 - 17r + 30 = 0$

$\Rightarrow r = 2, 15$ (not possible)

$\therefore r = 2$

41. (a)

$n^3 + 2n$, where n is positive integer i.e. n = 1, 2, 3

When

$n = 1,\ 1^3 + 2.1 = 3 = 3 \times 1$

$n = 2,\ 2^3 + 2.2 = 12 = 3 \times 4$

$n = 3,\ 3^3 + 2\times3 = 33 = 3 \times 11$

$n = 4,\ 4^3 + 2\times4 = 72 = 3 \times 24$

$n = 5,\ 5^3 + 2\times5 = 135 = 3 \times 45$

Hence, $n^3 + 3n$ is divisible by 3

Option (a) is correct

42. (b)

$(x-1)(x-2)(x-3) \ldots\ldots (x-18)$

$(x-1)(x-2) = x^2-3x + 2$, coefficient of $x = -3 = -(1+2)$

$(x-1)(x-2)(x-3) = (x^2-3x + 2)(x - 3)$

$= x^3 - 6x^2 + 11x - 6$, coefficient of $x^2 = -6 = -(1 + 2 + 3)$

As, similarly,

Coefficient of $x^{17} = -(1+2+3+\ldots\ldots+18)$

$$= -\left(\frac{18(18+1)}{2}\right)$$

$= -9 \times 19 = -171$

Hence option (b) is correct

43. (a)

$= 1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots\ldots + {}^nC_n\cos n\theta$

$\because\ (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots.. + {}^nC_n x^n$

Put $x = e^{i\theta}$

$(1+e^{i\theta}) = 1 + {}^nC_1 e^{i\theta} + {}^nC_2 (e^{i\theta})^2 + \ldots..\ {}^nC_n (e^{i\theta})^n$

$= 1 + {}^nC_1 e^{i\theta} + {}^nC_2 e^{2i\theta} + \ldots..\ {}^nC_n (e^{ni\theta})$

$(\because e^{i\theta} = \cos\theta + i\sin\theta)$

$\Rightarrow (1+\cos\theta + i\sin\theta)^n = 1 + {}^nC_1 e^{i\theta} + {}^nC_2 e^{2i\theta} + {}^nC_3 e^{3i\theta} + \ldots.. + {}^nC_n e^{ni\theta}$

$\Rightarrow \left(1+2\cos^2\frac{\theta}{2}-1+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n = 1 + {}^nC_1(\cos\theta + i\sin\theta) + {}^nC_2(\cos 2\theta + i\sin 2\theta)+\ldots.+{}^nC_n$ $(\cos n\theta + i\sin n\theta)$

$\Rightarrow \left(2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n = (1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_2\cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2$ $\sin 2\theta + \ldots.+ {}^nC_n\sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^n = (1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_n\cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2$ $\sin 2\theta + \ldots.+ {}^nC_n\sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(e^{i\frac{\theta}{2}}\right)^n = (1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_n\cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2\sin 2\theta$ $+\ldots.+ {}^nC_n\sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right) = (1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_n\cos n\theta) + i({}^nC_1\sin\theta +$ ${}^nC_2\sin 2\theta + \ldots.+ {}^nC_n\sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right) = (1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_n\cos n\theta) + i({}^nC_1\sin\theta +$ ${}^nC_2\sin 2\theta + \ldots.+ {}^nC_n\sin n\theta)$

$\Rightarrow \left(2\cos\frac{\theta}{2}\right)^n.\cos\frac{n\theta}{2} + i\left(2\cos\frac{\theta}{2}\right)^n.\sin\frac{n\theta}{2} = (1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_n\cos n\theta) +$ $i({}^nC_1\sin\theta + {}^nC_2\sin 2\theta + {}^nC_3\sin 3\theta + \ldots.. + {}^nC_n\sin n\theta)$

Comparing both side with real part we get

$\left(2\cos\frac{\theta}{2}\right)^n\cos\frac{n\theta}{2} = 1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \ldots.. + {}^nC_n\cos n\theta)$

Hence option (a) is correct.

44. (c)

$$= \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log_x x & \log_x y & \log_x z \\ \log_y x & \log_y y & \log_y z \\ \log_z x & \log_z y & \log_z z \end{vmatrix} \qquad (\because \log_a^a = 1)$$

$$= \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix} \begin{matrix} \leftarrow R_1 \\ \leftarrow R_2 \\ \leftarrow R_3 \end{matrix}$$

Taking $\frac{1}{\log x}, \frac{1}{\log y}, \frac{1}{\log z}$ Common R_1, R_2, R_3 we get

$$= \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

= 0

Since all rows are identical.

45. (c)

$\because$ B is adjoint matrix of A

$\Rightarrow$ B = adj(A)

$\Rightarrow$ |B| = |adj(A)|

$\Rightarrow$ 64 = |adj(A)| ($\because$ |B| = 64)

$\Rightarrow |A|^{(n-1)} = 64$ ($\because$ n = 3)

$\Rightarrow |A|^{(3-1)} = 64$

$\Rightarrow |A|^2 = 8^2$

$\Rightarrow |A| = \pm 8$

Hence option (c) is correct.

46. (c)

Given Q = $\begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$

Let Q(θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Q(θ). Q(θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

= $\begin{bmatrix} \cos^2\theta - \sin^2\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$

$Q^2(\theta)$ = $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

Now $Q^2(\theta)$. Q(θ) = $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} . \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

= $\begin{bmatrix} \cos 2\theta.\cos\theta - \sin 2\theta.\sin\theta & -\cos 2\theta\sin\theta - \sin 2\theta.\cos\theta \\ \cos\theta.\sin 2\theta + \cos 2\theta.\sin\theta & -\sin\theta.\sin 2\theta + \cos 2\theta.\cos\theta \end{bmatrix}$

$\Rightarrow Q^3(\theta)$ = $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

$\Rightarrow Q^3(\pi/4)$ = $\begin{bmatrix} \cos\left(3\frac{\pi}{4}\right) & -\sin\left(3\frac{\pi}{4}\right) \\ \sin\left(3\frac{\pi}{4}\right) & \cos\left(3\frac{\pi}{4}\right) \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

Now $Q^3(\pi/4)$. x ≡ $\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} \end{bmatrix}$

= $\begin{bmatrix} -\frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

Hence option (c) is correct

47. (c)

R = {x, y : x, y $\in$ z, x + 2y is divisible by 3}

Reflexive: Let x, y $\in$ z

x = y

x + 2x = 3x

it is divisible by 3

(x, x) $\in$ R

So it is reflexive

Symmetric: If x R y $\Rightarrow$ x + 2y is divisible by 3.

Now, y + 2x = 3x + 3y – (x + 2y) is divisible by 3.

$\Rightarrow$ y R x

i.e. it is symmetric.

Transitive: x R y $\Rightarrow$ x + 2y is divisible by 3.

y R z $\Rightarrow$ y + 2z is divisible by 3.

$\Rightarrow$ x + 2y + y + 2z is divisible by 3.

$\Rightarrow$ x + 3y + 2z is divisible by 3.

$\Rightarrow$ x + 2z is divisible by 3.

$\Rightarrow$ x R z

Hence transitive

Therefore, R is equivalence relation.

48. (c)

A = $\{5^n - 4n-1 : n \in N\}$

When n = 1, $5^1 - 4 \times 1-1 = 0$

n = 2, $5^2 - 4 \times 2-1 = 16$

n = 3, $5^3 - 4 \times 3-1 = 112$

n = 4, $5^4 - 4 \times 4-1 = 608$

. . . .

$\Rightarrow$ A = {0, 16, 112, 608}

While, B = {16(n–1), n $\in$ N}

B = {0, 16, 32, 48……}

Hence it is clear that A $\subseteq$ B

Hence option (c) is correct.

49. (c)

$f(x) = (x^2 + 1)^{35}. \forall x \in R$

$f(-x) = ((-x)^2 + 1)^{35}$

$= (x^2 + 1)^{35} = f(x)$

Hence f(x) is an even function

So its range $\neq R \quad (f(x) > 0 \ \forall x \in R)$

And also it is not one-one and not onto.

Hence option (c) is correct.

50. (c)

Observations are $a_1, a_2, a_3 \ldots\ldots a_n$

$$\text{Mean } (\bar{x}) = \frac{a_1 + a_2 + \ldots. + a_n}{n}$$

$$\because \sigma = \sqrt{\frac{\Sigma x_i^{\ 2}}{n} - (\bar{x})^2}$$

$$\sigma = \sqrt{\frac{a_1^2 + a_2^2 + a_3^2 + \ldots. a_n^2}{n} - \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^2}$$

$\lambda a_1, \lambda a_2, \lambda a_3, \ldots\ldots., \lambda a_n$

$$\sigma_1 = \sqrt{\frac{(\lambda a_1)^2 + (\lambda a_2)^2 + \ldots.. + (\lambda a_n)^2}{n} - \left(\frac{\lambda a_1 + \lambda a_2 + \ldots.. + \lambda a_n}{n}\right)^2}$$

$$= \sqrt{\lambda^2 \left(\frac{a_1^2 + a_2^2 + \ldots.. + a_n^2}{n}\right) - \lambda^2 \left(\frac{a_1 + a_2 + \ldots.. + a_n}{n}\right)^2}$$

$$= |\lambda| \sqrt{\frac{a_1^2 + a_2^2 + \ldots.. + a_n^2}{n} - \left(\frac{a_1 + a_2 + \ldots.. + a_n}{n}\right)^2}$$

$\sigma_1 = |\lambda| \, \sigma$

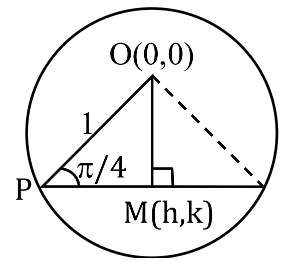
Hence option (c) is correct.

51. (b)

$OM = \sqrt{(0-h)^2+(0-k)^2}$

$OM = \sqrt{h^2+k^2}$

$\therefore$ In ΔOPM



$\sin\left(\frac{\pi}{4}\right) = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{h^2+k^2}}{1}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{h^2+k^2}}{1}$

Squaring both side, we get

$\Rightarrow \frac{1}{2} = \frac{h^2+k^2}{1}$

$\Rightarrow 2(h^2 + k^2) = 1$

$(h, k) \to (x, y)$

$2(x^2+y^2) = 1$

$\Rightarrow x^2+y^2 = \frac{1}{2}$

Hence option (b) is correct.

52. (d)

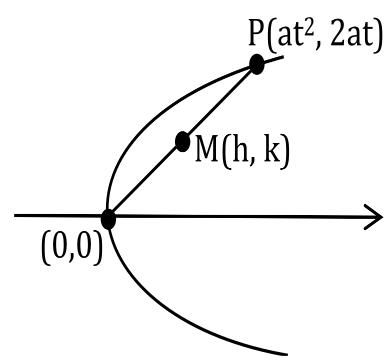
Let midpoint of chord = (h, k)

$\therefore h = \frac{at^2+0}{2} \Rightarrow at^2 = 2h$ …..(1)

$K = \frac{2at-0}{2} \Rightarrow at = k$

$\Rightarrow t = \frac{k}{a}$

Putting value of t in (1) we get



$\Rightarrow a\left(\frac{k}{a}\right)^2 = 2h$

$\Rightarrow k^2 = 2ah \Rightarrow y^2 = 2ax$

Directrix of parabola $y^2 = 2ax$ is

$x = \frac{-a}{2}$

Hence option (d) is correct

53. (b)

Let I = $\int_0^2 x^2[x]dx$

I = $\int_0^1 x^2[x]dx + \int_1^2 x^2[x]dx$

$\because \quad 0 \le x < 1 \Rightarrow [x] = 0$

$1 \le x < 2 \Rightarrow [x] = 1$

$\therefore$ I = $\int_0^1 x^2.0dx + \int_1^2 x^2.1dx$

$= 0 + \int_1^2 x^2 dx$

$= 0 + \left(\frac{x^3}{3}\right)^2$

$= \frac{2^3}{3} - \frac{1^3}{3}$

$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

Hence option (b) is correct.

54. (c)

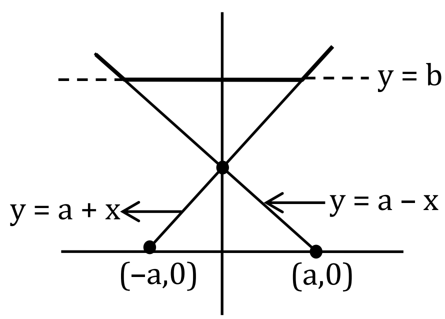
f(x) = $\{a - x, a + x.b\}$

$-a < x < \infty$, a < a < b

y = a − x

y = a + x

y = b



Possible graph of f(x) is shown.

There are two sharp turn. Hence f(x) is not differentiable at two points.

55. (d)

$|\vec{a} + \vec{b}| < |\vec{a} - \vec{b}|$

Squaring both sides we get

$|\vec{a}+\vec{b}|^2 < |\vec{a}-\vec{b}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|.|\vec{b}|\cos\alpha < |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}|.|\vec{b}|\cos\alpha$

(α is the angle between $\vec{a}$ & $\vec{b}$)

$\Rightarrow 2\,|\vec{a}||\vec{b}|\cos\alpha < -2\,|\vec{a}|\,|\vec{b}|\cos\alpha$

$\Rightarrow 4\,|\vec{a}||\vec{b}|\cos\alpha < 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\alpha < 0$

$\Rightarrow \cos\alpha < 0$

$\Rightarrow \alpha$ is an obtuse angle.

Hence (d) option is correct.

56. (b)

$y\frac{dy}{dx} + by^2 = a\cos x,\ 0 < x < 1$

Put $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{1}{2}\frac{dt}{dx} + bt = a\cos x$

$\Rightarrow \frac{dt}{dx} + 2bt = 2a\cos x$

It is linear in 't' we get

$\therefore$ IF $= e^{\int 2b\,dx} = e^{2bx}$

Solution is

t. IF $= \int 2a\cos x\,.\,\text{IF}\,dx$

$\Rightarrow t.\,e^{2bx} = \int 2a\cos x\,.\,e^{2bx}\,dx$

$\Rightarrow t.e^{2bx} = \frac{2a}{4b^2+1}(\sin x + 2b\cos x).\,e^{2bx} + c$

$\Rightarrow y^2.\,e^{2bx} = \frac{2a(\sin x + 2b\cos x)}{4b^2+1}\,e^{2bx} + c$

$\Rightarrow (4b^2+1)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$

Hence option (b) is correct.

57. (a)

Given, $16x^2 + 9y^2 = 400$

$$\Rightarrow \frac{16x^2}{400} + \frac{9y^2}{400} = 1$$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{\frac{400}{9}} = 1$$

Here $a^2 = 25 \Rightarrow a = 5$

$b^2 = \frac{400}{9} \Rightarrow b = \frac{20}{3}$

Any point on ellipse is $(a \cos\theta, b \sin \theta)$

$\equiv \left(5\cos\theta, \frac{20}{3}\sin\theta\right)$

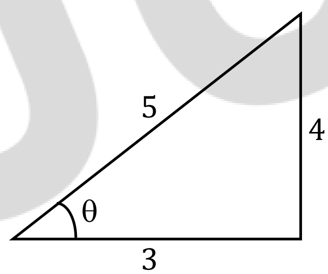
$x = 5\cos\theta,\ y = \frac{20}{3}\sin\theta$

$\Rightarrow \frac{dx}{d\theta} = -5\sin\theta,\ \frac{dy}{d\theta} = \frac{20}{3}\cos\theta$

$\Rightarrow \frac{dx}{d\theta} = -\frac{dy}{d\theta}$ (Since ordinate is decreasing)

$\Rightarrow -5\sin\theta = -\frac{20}{3}\cos\theta$

$\Rightarrow \tan\theta = \frac{4}{3}$



$\cos\theta = \frac{3}{5}$ or $-\frac{3}{5}$

$\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

$\therefore$ Points are $\left(3, \frac{16}{3}\right)$ and $\left(-3, \frac{-16}{3}\right)$

Hence option (a) is correct.

58. (a)

Arranging in alphabetical order $\rightarrow$ C, C, H, I, N, O

Number of words that appear before the word COCHIN is

CC …….. $\rightarrow$ 4!

CH …….. $\rightarrow$ 4!

CI …….. $\rightarrow$ 4!

CN …….. $\rightarrow$ 4!

COCHIN …….. $\rightarrow$ 1

$\therefore$ No. of words before …….. COCHIN = 4! + 4! + 4! + 4! = 96

Hence option (a) is correct.

59. (d)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 8 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 2^2 & 0 \\ 2.2^2 & 0 & 0 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 8 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 24 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 2^3 & 0 & 0 \\ 0 & 2^3 & 0 \\ 3.2^3 & 0 & 2^3 \end{bmatrix}$$

. . . .

$$A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ n.2^n & 0 & 2^n \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{bmatrix}$$

$\therefore$ On comparing both matrix we get

$a = 2^n$, $b = n.2^n$

Hence option (d) is correct.

60. (a)

Given series is

$Sn = 1^3 + 3^3 + 5^3 + 7^3 + \ldots\ldots\ldots$

$t_r = (2r-1)^3 = 8r^3 - 1 - 3.2r.1(2r-1)$

$= 8r^3 - 1 - 12r^2 + 6r$

$$\therefore Sn = \sum_{r=1}^{n} tr = \sum_{r=1}^{n}(8r^3 - 1 - 12r^2 + 6r)$$

$$= 8\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} 1 - 12\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r$$

$$= 8\left(\frac{n(n+1)}{2}\right)^2 - n - 12\ \frac{n(n+1)(2n+1)}{6} + 6\ \frac{n(n+1)}{2}$$

$= 2n^2(n+1)^2 - n - 2n(n+1)(2n+1) + 3n(n+1)$

$= 2n^2(n^2+1+2n) - n - (2n^2+2n)(2n+1) + 3n^2 + 3n$

$= 2n^4 + 2n^2 + 4n^3 - n - 4n^3 - 4n^2 - 2n^2 - 2n + 3n^2 + 3n$

$= 2n^4 - n^2$

$S_n = n^2(2n^2 - 1)$

Hence option (a) is correct

61. (a)

$\because$ α & β are roots of $ax^2 + bx + c = 0$

Let $y = x^2 \Rightarrow x = \sqrt{y}$

Putting $\sqrt{y}$ in the given equation, we get

$\Rightarrow a(\sqrt{y})^2 + b(\sqrt{y}) + c = 0$

$\Rightarrow ay + b\sqrt{y} + c = 0$

$\Rightarrow b(\sqrt{y}) = -ay - c$

Squaring both sides we get

$\Rightarrow b^2y = a^2y^2 + c^2 + 2acy$

$\Rightarrow a^2y^2 - (b^2-2ac)\, y + c^2 = 0$

So the required equation is

$a^2x^2-(b^2-2ac)\, x + c^2 = 0$

Hence option (a) is correct

62. (a)

$\because (2-\omega)(2-\omega^2) + 2(3-\omega)(3-\omega^2) + \ldots + (n-1)(n-\omega)(n-\omega^2)$

$\Rightarrow T_r = \sum_{r=2}^{n}(r-1)(r-\omega)(r-\omega^2)$

$= \sum_{r=2}^{n}(r^2 - r\omega - r + \omega)(r-\omega^2)$

$= \sum_{r=2}^{n}\left(r^3 - r^2\omega - r^2 + r\omega - \omega^2 r^2 + r\omega^3 + r\omega^2 - \omega^3\right)$

$= \sum_{r=2}^{n}\left((r^3 - r^2(\omega + 1 + \omega^2) + r(\omega + \omega^3 + \omega^2) - \omega^3\right)$

$(\because 1 + \omega + \omega^2 = 0,\ \omega^3 = 1)$

$= \sum_{r=2}^{n}\left(r^3 - r^2 \times 0 + r\left(\omega + 1 + \omega^2\right) - 1\right)$

$= \sum_{r=2}^{n}(r^3 - 1)$

$= \sum_{r=2}^{n} r^3 - \sum_{r=2}^{n} 1$

$= \left[\left(\frac{n(n+1)^2}{2}\right) - 1\right] - (n-1)$

$= \frac{n^2(n+1)^2}{4} - n$

Hence option (a) is correct.

63. (c)

Given $^nC_{r-1} = 36\ ^nC_r = 84,\ ^nC_{r+1} = 126$

$$\frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1} \text{......(i)}, \quad \frac{\lfloor n}{\lfloor r \lfloor n-r} = 84\text{.....(ii)}, \quad \frac{\lfloor n}{\lfloor r+1 \lfloor n-r-1} = 126 \text{ ....(iii)}$$

Dividing (i) ÷(ii) we get

$$\frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1} \div \frac{\lfloor n}{\lfloor r \lfloor n-r} = \frac{36}{84}$$

$$\frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1} \times \frac{\lfloor r \lfloor n-r}{\lfloor n} = \frac{3}{7}$$

$$\frac{r\lfloor r-1 \lfloor n-r}{\lfloor r-1 (n-r+1)\lfloor n-r} = \frac{3}{7}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{7}$$

$7r = 3n - 3r + 3$

$10r - 3n = 3$ .....(iv)

Now dividing eq. (ii) ÷ (iii) we get

$$\frac{\lfloor n}{\lfloor r \lfloor n-r} \div \frac{\lfloor n}{\lfloor r+1 \lfloor n-r-1} = \frac{84}{126}$$

$$\frac{\lfloor n}{\lfloor r \lfloor n-r} \times \frac{\lfloor r+1 \lfloor n-r-1}{\lfloor n} = \frac{84}{126}$$

$$\frac{r+1}{n-r} = \frac{2}{3} \Rightarrow 5r - 2n = -3 \text{ ....(v)}$$

Solving eq. (iv) & (v) we get r = 3 & n = 9

$\therefore\ ^nC_8 = {}^9C_8 = 9$

Hence option (c) is correct.

64. (b)

Since there are total 14 males and 6 females in a group. In which 8 males and 3 females are aged above 40 years.

Here out of 6 females 3 are above 40 and 3 are aged below 40. So probability of person aged above 40 given female person = $\frac{1}{2}$

65. (b)

Given equation, $x^3 - yx^2 + x - y = 0$

$\Rightarrow x^2(x-y) + (x-y) = 0$

$\Rightarrow (x-y)(x^2+1) = 0$

So only possibility is $x - y = 0$ or $x^2 + 1 = 0$

$\Rightarrow x = y$ or $x^2 + 1 = 0$ (not possible)

Hence, given equation represents straight line. Hence option (B) is correct.

66. (b, d)
There is mistake in question.
If there are $(2n-1)^{th}$ terms instead of $(2n+1)$ terms then n^{th} terms of the AP, GP and HP are the AM, GM & HM of the 1^{st} and the last terms.
So, $a \geq b \geq c \;\&\; ac - b^2$ (B, D)
Otherwise if there are $(2n+1)$ terms then the n^{th} terms should be in decreasing order of AP. GP & HP.
i.e. $a \geq b \geq c$ (B)

67. (b,d)

Let any parametric point on the line $x + y + 1 = 0$ is $(t, -1-t)$.

Distance of $(t, -1-t)$ from $3x + 4y + 2 = 0$ is

$$\Rightarrow \left|\frac{3\times t + 4(-1-t) + 2}{\sqrt{3^2+4^2}}\right| = \frac{1}{5}$$

$$\Rightarrow \left|\frac{3t - 4 - 4t + 2}{5}\right| = \frac{1}{5}$$

$$\Rightarrow \left|\frac{-t-2}{5}\right| = \frac{1}{5}$$

$\Rightarrow |t+2| = 1$

$\Rightarrow t + 2 = \pm 1$

$\Rightarrow t + 2 = \pm 1$

$\Rightarrow t + 2 = 1$ or $t + 2 = -1$

$t = -1$ or $t = -3$

$\therefore$ Possible points on the line $x + y + 1 = 0$

$(t, -1-t) \equiv (-1, -1+1) = (-1, 0)$

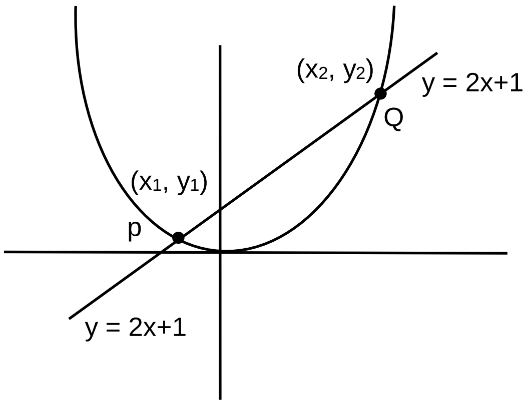
$(t, -1-t) \equiv (-3, -1+3) = (-3, 2)$

Hence option (B, D) are correct

68. (a, b)

Parabola: $x^2 = ay$

Line: $y - 2x = 1$



Solving parabola and line

$x^2 = a(1 + 2x)$

$\Rightarrow x^2 = a + 2ax$

$\Rightarrow x^2 - 2ax - a = 0$

Let x_1 and x_2 are roots.

$\therefore x_1 + x_2 = 2a$

$x_1 x_2 = -a$

$\therefore (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

$= (2a)^2 - 4(-a)$

$= 4a^2 + 4a$

$(x_1 - x_2)^2 = 4a(a + 1)$

Point (x_1, y_1) lie on line $y = 2x + 1$

$\therefore y_1 = 2x_1 + 1$

Also point (x_2, y_2) line on line $y = 2x + 1$

$\therefore y_2 = 2x_2 + 1$

$\therefore y_1 - y_2 = 2(x_1 - x_2)$

$\Rightarrow (y_1 - y_2)^2 = 4(x_1 - x_2)^2 = 4.\ 4a(a + 1)$

$(y_1 - y_2)^2 = 16a(a + 1)$

Length PQ $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{4a(a+1) + 16a(a+1)}$

PQ $= \sqrt{20a(a+1)}$

$\Rightarrow \sqrt{40} = \sqrt{20a(a+1)}$

Squaring both sides we get

$\Rightarrow 40 = 20a(a + 1)$

$\Rightarrow 2 = a(a + 1)$

$\Rightarrow a^2 + a - 2 = 0$

$\Rightarrow (a + 2)(a - 1) = 0$

$\Rightarrow a = -2, 1$

Hence option (A, B) are correct.

69. (b, c)

$\because f'(x) = (x-1)^2 (4-x)$

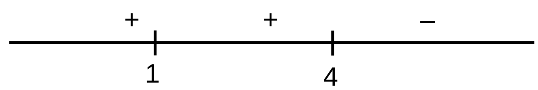
For critical point put $f'(x) = 0$

$\Rightarrow (x-1)^2 (4-x) = 0$

$\Rightarrow x = 1, 4$

Therefore x = 1 & 4 are critical point of f(x)

Now sign scheme for $f'(x)$



$\therefore$ f(x) is increasing in the interval $(-\infty, 4)$

Hence also increasing in the interval (0, 3)

And f(x) is decreasing in the interval $(4, \infty)$

Hence (B, C) option is correct.

From $f'(x)$, we can't determine f(x) uniquely so f(o) can't be predicted.

70. (b,d)

Given ellipse is $4x^2 + 9y^2 = 1$

$$\frac{x^2}{\frac{1}{1/4}} + \frac{y^2}{1/9} = 1$$

Here, $a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$

$b^2 = \frac{1}{9} \Rightarrow b = \frac{1}{3}$

$\therefore$ Any point on ellipse is $(a\cos\theta, b\sin\theta)$

$\therefore$ Point on ellipse is $\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{3}\right)$

$\therefore$ Equation of tangent at point $\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{3}\right)$ is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\Rightarrow \frac{x\frac{\cos\theta}{2}}{\frac{1}{4}} + \frac{y\frac{\sin\theta}{3}}{\frac{1}{9}} = 1$$

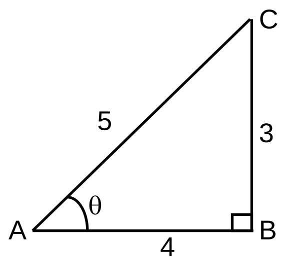
$\Rightarrow \frac{2x\cos\theta}{1} + \frac{3y\sin\theta}{1} = 1$

$\Rightarrow 2x\cos\theta + 3y\sin\theta = 1$ ...... (1)

Slope of line $9y = 8x \Rightarrow y = \frac{8}{9}x$ .... (2)

Equation slope of (1) and (2) we get

$\Rightarrow \frac{-2\cos\theta}{3\sin\theta} = \frac{8}{9} \Rightarrow \tan\theta = \frac{-3}{4}$



Either, $\cos\theta = \frac{-4}{5}, \sin\theta = \frac{3}{5}$

Or

$\text{Cos}\theta = \frac{4}{5}, \sin\theta = -\frac{3}{5}$

So, a point on ellipse is $(a\cos\theta, b\sin\theta) = \left(\frac{1}{2}\cos\theta, \frac{1}{3}\sin\theta\right)$

$\equiv \left(\frac{1}{2}\left(\frac{-4}{5}\right), \frac{1}{3}\left(\frac{3}{5}\right)\right) \text{or} \left(\frac{1}{2}\left(\frac{4}{5}\right), \frac{1}{3}\left(\frac{-3}{5}\right)\right)$

$\equiv \left(\frac{-4}{10}, \frac{1}{5}\right) \text{or} \left(\frac{4}{10}, \frac{-1}{5}\right)$

$= \left(-\frac{2}{5}, \frac{1}{5}\right) \text{ or } \left(\frac{2}{5}, -\frac{1}{5}\right)$

Hence option (B, D) are correct

71. (a,b)

Given $\phi(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$

So $\int_{-3000}^{3000}\left(\sum_{r'=2014}^{2016}\phi(t-r')+\phi(t-2016)\right)dt$

$= \int_{-3000}^{3000}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)\,dt$

$= \int_{-3000}^{2016}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)\,dt$

$+ \int_{2016}^{2017}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)\,dt$

$+ \int_{2017}^{3000}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)]\phi(t-2016)\,dt$

$= \int_{3000}^{2016} o dt + \int_{2016}^{2017}(0+0+1).1dt + \int_{2017}^{3000} o dt$

$= 0 + \int_{2016}^{2017} dt + 0$

$= [t]_{2016}^{2017}$

= 2017 – 296

= 1.

Hence option (A, B) is correct

72. (a,c)

Given equation is $x^2 + y^2 - 10x + 21 = 0$

$\Rightarrow x^2 - 10x + (y^2 + 21) = 0$ have roots

x = a and y = b

$\therefore$ for real roots $D \geq 0$

$\Rightarrow (-10)^2 - 4\cdot 1\cdot (y^2 + 21) \geq 0$

$\Rightarrow 100 - 4y^2 - 84 \geq 0$

$\Rightarrow -4y^2 + 16 \geq 0$

$\Rightarrow y^2 \leq 4$

$\Rightarrow -2 \leq y \leq 2$

Hence option (c) is correct

Now, $y^2 = -x^2 + 10x - 2$

For real roots of y

$\Rightarrow -x^2 + 10x - 21 \geq 0$

$\Rightarrow x^2 + -10x + 21 \leq 0$

$\Rightarrow x^2 - 7x - 3x + 21 \leq 0$

$\Rightarrow x(x-7) - 3(x-7) \leq 0$

$\Rightarrow (x-7)(x-3) \leq 0$

$\Rightarrow 3 \leq x \leq 7$

Option (A) is correct

Hence option (A, C) are correct

73. (a,c)

$\therefore z = \sin\theta - i\cos\theta$

$= \cos\left(\theta - \frac{\pi}{2}\right) + i\sin\left(\theta - \frac{\pi}{2}\right)$

$= z\ e^{i(\theta - \frac{\pi}{2})}$

So, $z^n = \left(e^{i(\theta-\frac{\pi}{2})}\right)^n = e^{\left(in\left(\theta - \frac{\pi}{2}\right)\right)}$

$z^n = \cos\left(n\left(\theta - \frac{\pi}{2}\right)\right) + i\sin n\left(\theta - \frac{\pi}{2}\right)$ .....(1)

Now $\frac{1}{z^n} = \frac{1}{(z)^n} = \frac{1}{\left(e^{i\left(\theta - \frac{\pi}{2}\right)}\right)^n} = \frac{1}{e^{ni\left(\theta - \frac{\pi}{2}\right)}}$

$\frac{1}{z^n} = e^{-ni(\theta - \frac{\pi}{2})} = \cos(-n(\theta - \frac{\pi}{2})) + i\sin(-n(\theta - \frac{\pi}{2}))$

$= \cos\left(n\left(\theta - \frac{\pi}{2}\right)\right) - i\sin\left(n\left(\theta - \frac{\pi}{2}\right)\right)$ .....(2)

Subtracting

$Z^n - \frac{1}{z^n} = \cos n(\theta - \frac{\pi}{2}) + i\sin(n(\theta - \frac{\pi}{2})) - \cos(n(\theta - \frac{\pi}{2})) + i\sin(n(\theta - \frac{\pi}{2})$

$= 2i\sin n\left(\theta - \frac{\pi}{2}\right)$

$z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$ (option (c))

Adding (1) & (2) we get

$Z^n + \frac{1}{z^n} = \cos\left(n\left(\theta - \frac{\pi}{2}\right)\right) + i\sin\left(n\left(\theta - \frac{\pi}{2}\right)\right) + \cos\left(n\left(\theta - \frac{\pi}{2}\right)\right) - i\sin\left(n\left(\theta - \frac{\pi}{2}\right)\right)$

$= 2\cos\left(n\left(\theta - \frac{\pi}{2}\right)\right)$

$= 2\cos\left(n\theta - \frac{n\pi}{2}\right)$ option (A)

Hence option (A, C) are correct

74. (a,b)

$f(f(x)) = x\ \ \forall x \in X \ \&\ x \subseteq R$

So, $f(x) = f^{-1}(x)$

$\Rightarrow$ f(x) is self inverse

Hence f(x) is one-one and onto

Therefore, option (A, B) is correct.

75. (a,c)

$P\left(A \cup B\right) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$ ……..(1)

$\therefore \frac{3}{4} \leq P(A \cup B) \leq 1$

$\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$

$\Rightarrow \frac{3}{4} + \frac{1}{8} \leq P(A \cup B) + P(A \cap B) \leq 1 + \frac{3}{8}$

$\Rightarrow \frac{7}{8} = \frac{6+1}{8} \leq P\left(A \cup B\right) + P(A \cap B) \leq \frac{11}{8}$

From (1)

$\frac{7}{8} \leq P\left(A\right) + P(B) \leq \frac{11}{8}$

Hence $P(A) + P(B) \geq \frac{7}{8}$ option (c)

$P(A) + P(B) \leq \frac{11}{8}$ option (A)

Hence option (A, C) are correct