1. Find without division, the remainder in each of the following:

(i) $5x^2 - 9x + 4$ is divided by (x - 2)

Solution:-

From the question it is given that, $5x^2 - 9x + 4$ is divided by (x - 2)

Let us assume $x - 2 = 0$, $x = 2$

Now, substitute the value of x in given expression,

$= 5 \times (2)^2 - (9 \times 2) + 4$

$= (5 \times 4) - 18 + 4$

$= 20 - 18 + 4$

$= 24 - 18$

$= 6$

Therefore, the remainder of the given expression is 6.

(ii) $5x^3 - 7x^2 + 3$ is divided by (x - 1)

Solution:-

From the question it is given that, $5x^3 - 7x^2 + 3$ is divided by (x - 1)

Let us assume $x - 1 = 0$, $x = 1$

Now, substitute the value of x in given expression,

$= 5 \times (1)^2 - 7 \times (1)^2 + 3$

$= (5 \times 1) - (7 \times 1) + 3$

$= 5 - 7 + 3$

$= 8 - 7$

$= 1$

Therefore, the remainder of the given expression 1.

(iii) $8x^2 - 2x + 1$ is divided by (2x + 1)

Solution:-

From the question it is given that, $8x^2 - 2x + 1$ is divided by (2x + 1)

Let us assume $2x + 1 = 0$, $x = -½$

Now, substitute the value of x in given expression,

$= 8(-½)^2 - 2(-½) + 1$

$= (8 \times 1/4) + 1 + 1$

$= 2 + 1 + 1$

$= 4$

Therefore, the remainder of the given expression 4.

(iv) $x^3 + 8x^2 + 7x - 11$ is divided by (x + 4)

Solution:-

From the question it is given that, $x^3 + 8x^2 + 7x - 11$ is divided by (x + 4)

Let us assume x + 4 = 0, x = -4

Now, substitute the value of x in given expression,

$= (-4)^3 + 8(-4)^2 + 7(-4) - 11$

$= - 64 + 8(-16) – 28 - 11$

$= -64 + 128 – 28 - 11$

$= 25$

Therefore, the remainder of the given expression 25.

(v) $2x^3 – 3x^2 + 6x - 4$ is divided by (2x - 3)

Solution:-

From the question it is given that, $2x^3 – 3x^2 + 6x - 4$ is divided by (x + 4)

Let us assume 2x - 3 = 0, x = 3/2

Now, substitute the value of x in given expression,

$= 2(3/2)^3 – 3(3/2)^2 + 6(3/2) - 4$

$= (2 \times 27/8) – 3(9/4) + (3 \times 3) - 4$

$= 27/4 – 27/4 + 9 – 4$

$= 9 - 4$

$= 5$

Therefore, the remainder of the given expression 5.

2. Prove by factor theorem that,

(i) (x - 2) is a factor of $2x^3 – 7x – 2$

Solution:-

From the question it is given that, $f(x) = 2x^3 – 7x – 2$

Let us assume, x – 2 = 0, x = 2

Then, substitute the value of x,

$f(2) = 2(2)^3 – 7(2) – 2$

$= 2(8) – 14 – 2$

$= 16 – 14 – 2$

$= 16 – 16$

$= 0$

Now it is clear that (x - 2) is a factor of $2x^3 – 7x – 2$.

(ii) (2x + 1) is a factor of $4x^3 + 12x^2 + 7x + 1$

Solution:-

From the question it is given that, $f(x) = 4x^3 + 12x^2 + 7x + 1$
Let us assume, $2x + 1 = 0$, $x = -½$
Then, substitute the value of x,
$f(-½) = 4(-½)^3 + 12(-½)^2 + 7(-½) + 1$
$= 4(-1/8) + 12(1/4) – 7/2 + 1$
$= -½ + 3 – 7/2 + 1$
$= -½ - 7/2 + 4$
$= -8/2 + 4$
$= -4 + 4$
$= 0$
Now it is clear that $(2x + 1)$ is a factor of $4x^3 + 12x^2 + 7x + 1$.

(iii) (3x - 2) is a factor of $18x^3 - 3x^2 + 6x - 8$
Solution:-
From the question it is given that, $f(x) = 18x^3 - 3x^2 + 6x - 8$
Let us assume, $3x - 2 = 0$, $x = 2/3$
Then, substitute the value of x,
$f(2/3) = 18(2/3)^3 - 3(2/3)^2 + 6(2/3) - 8$
$= 18(8/27) - 3(4/9) + (2 \times 2) - 8$
$= 2(8/3) – (4/3) + 4 - 8$
$= 16/3 - 4/3 + 4 - 8$
$= 12/3 + 4 - 8$
$= 4 + 4 - 8$
$= 8 – 8$
$= 0$
Now it is clear that $(3x - 2)$ is a factor of $18x^3 - 3x^2 + 6x - 8$.

(iv) (2x - 1) is a factor of $6x^3 - x^2 - 5x + 2$
Solution:-
From the question it is given that, $f(x) = 6x^3 - x^2 - 5x + 2$
Let us assume, $2x - 2 = 0$, $x = ½$
Then, substitute the value of x,
$f(½) = 6(½)^3 - (½)^2 - 5(½) + 2$
$= 6(1/8) - (1/4) – (5/2) + 2$
$= 3(1/4) – (1/4) – 5/2 + 2$
$= 3/4 - 1/4 – 5/2 + 2$
$= 2/4 – 5/2 + 2$

$= 1/2 – 5/2 + 2$
$= -4/2 + 2$
$= -2 + 2$
$= 0$

Now it is clear that $(2x - 1)$ is a factor of $6x^3 - x^2 - 5x + 2$.

(v) $(x - 3)$ is a factor of $5x^2 – 21x + 18$

Solution:-

From the question it is given that, $f(x) = 5x^2 – 21x + 18$
Let us assume, $x – 3 = 0$, $x = 3$
Then, substitute the value of x,
$f(3) = 5(3)^2 – 21(3) + 18$
$= 5(9) – 63 + 18$
$= 45 – 63 + 18$
$= 63 – 63$
$= 0$

Now it is clear that $(x - 3)$ is a factor of $5x^2 – 21x + 18$.

3. Find the values of a and b in the polynomial $f(x) = 2x^3 + ax^2 + bx + 10$, if it is exactly divisible by $(x + 2)$ and $(2x - 1)$

Solution:-

From the question it is given that,
$f(x) = 2x^3 + ax^2 + bx + 10$
let us assume $x + 2 = 0$
$x = -2$
$2x – 1 = 0$
$x = ½$
Now, substitute the value of x,
$x = -2$
$f(-2) = 2(-2)^3 + a(-2)^2 + b(-2) + 10 = 0$
$2(-8) + a(4) – 2b + 10 = 0$
$- 16 + 4a – 2b + 10 = 0$
$- 6 + 4a – 2b = 0$
Divide both side by 2 we get,
$-6/2 + 4a/2 – 2b/2 = 0$
$- 3 + 2a – b = 0$
$2a = b + 3$

$a = b/2 + 3/2$ … [equation (i)]

Then,

$f(½) = 2(½)^3 + a(½)^2 + b(½) + 10 = 0$

$2(1/8) + a(1/4) + b/2 + 10 = 0$

$¼ + a/4 + b/2 + 10 = 0$

Multiply by 4 for each terms we get,

$1 + a + 2b + 40 = 0$

$41 + a + 2b = 0$

$a = - 2b – 41$ … [equation (ii)]

By combining both equation (i) and equation (ii) we get,

$b/2 + 3/2 = -2b – 41$

$(b + 3)/2 = -2b – 41$

$b + 3 = - 4b – 82$

By transposing we get,

$4b + b = -82 – 3$

$5b = - 85$

$b = -85/5$

$b = - 17$

So, $a = - 2b – 41$

$= -2(-17) – 41$

$= 34 – 41$

$= - 7$

Therefore, the value of a is – 7 and b is – 17.

4. Using remainder theorem, find the value of m if the polynomial $f(x) = x^3 + 5x^2 – mx + 6$ leaves a remainder 2m when divided by (x - 1).

Solution:-

From the question it is given that,

$f(x) = x^3 + 5x^2 – mx + 6$

Remainder = 2m

Let us assume that $x – 1 = 0$, $x = 1$

Now, substitute the value of x in f(x) we get,

$f(1) = 1^3 + 5(1)^2 – m(1) + 6 = 2m$

$1 + 5 – m + 6 = 2m$

$6 – m + 6 = 2m$

By transposing we get,

$12 = 2m + m$

$12 = 3m$
$m = 12/3$
$m = 4$
Therefore, the value of m is 4.

5. Find the value of m when $x^3 + 3x^2 – mx + 4$ is exactly divisible by (x - 2)
Solution:-
From the question it is given that,
$f(x) = x^3 + 3x^2 – mx + 4$
Let us assume that $x – 2 = 0$, $x = 2$
Now, substitute the value of x in f(x) we get,
$f(2) = 2^3 + 3(2)^2 – m(2) + 4 = 0$
$8 + 3(4) – 2m + 4 = 0$
$8 + 12 – 2m + 4 = 0$
$24 – 2m = 0$
By transposing we get,
$24 = 2m$
$m = 24/2$
$m = 12$
Therefore, the value of m is 12.

6. Find the values of p and q in the polynomial $f(x) = x^3 – px^2 + 14x – q$, if it is exactly divisible by (x - 1) and (x - 2).
Solution:-
From the question it is given that, $f(x) = x^3 – px^2 + 14x – q$
Let us assume that, $x – 1= 0$, $x = 1$
$x – 2 = 0$, $x = 2$
Now, substitute the value of x in f(x) we get,
$f(1) = 1^3 – p(1)^2 + 14(1) – q = 0$
$1 – p + 14 – q = 0$
$15 – p – q = 0$
$p = 15 – q$ … [equation (i)]
Then, $f(2) = 2^3 – p(2)^2 + 14(2) – q = 0$
$8 – 4p + 28 – q = 0$
$36 – 4p – q = 0$
$q = 36 – 4p$ … [equation (ii)]
So, substitute the value of q in equation (i) we get,

p = 15 – (36 – 4p)
p = 15 – 36 + 4p
By transposing we get,
36 – 15 = 4p – p
21 = 3p
p = 21/3
p = 7
Consider the equation (ii) to find the value of q,
q = 36 – 4(7)
q = 36 – 28
q = 8
Therefore, the value of p is 7 and q is 8.

7. Find the values of a and b when the polynomial $f(x) = ax^3 + 3x^2 + bx – 3$ is exactly divisible by (2x + 3) and leaves a remainder – 3 when divided by (x + 2).

Solution:-

From the question it is given that, $f(x) = ax^3 + 3x^2 + bx – 3$
Remainder = -3
Let us assume that, 2x + 3 = 0, x = -3/2
x + 2 = 0, x = -2
Now, substitute the value of x in f(x) we get,
$f(-3/2) = a(-3/2)^3 + 3(-3/2)^2 + b(-3/2) – 3= 0$
a(-27/8) + 3(9/4) – b(3/2) – 3 = 0
a(-27/8) + (27/4) - 3 – b(3/2) = 0
a(-27/8) + (27- 12)/4 – b(3/2) = 0
a(-27/8) + 15/4 – b(3/2) = 0
-27a + 30 – 12b = 0
27a = - 12b + 30
Then,
$f(- 2) = a(-2)^3 + 3(-2)^2 + b(-2) – 3= -3$
a(-8) + 3(4) - 2b – 3 = -3
-8a + 12 – 2b - 3 + 3= 0
-4a + 6 – b = 0
b = 6 – 4a … [equation (ii)]
By combining both equation (i) and equation (ii) we get,
27a = - 12(6 – 4a) + 30
27a = - 72 + 48a + 30

$27a - 48a = -42$
$-21a = -42$
$a = -42/-21$
$a = 2$
Then,
$b = 6 - 4a$
$b = 6 - 4(2)$
$b = 6 - 8$
$b = -2$
Therefore, the value of a is 2 and b is – 2.

8. Find the values of m and n when the polynomial $f(x) = x^3 - 2x^2 + mx + n$ has a factor (x + 2) and leaves a remainder 9 when divided by (x + 1).
Solution:-
From the question it is given that,
$f(x) = x^3 - 2x^2 + mx + n$
Remainder = 9
Let us assume that, $x + 2 = 0$, $x = -2$
$x + 1 = 0$, $x = -1$
Now, substitute the value of x in f(x) we get,
$f(-2) = (-2)^3 - 2(-2)^2 + m(-2) + n = 0$
$-8 - 8 - 2m + n = 0$
$-16 - 2m + n = 0$
$n = 2m + 16$ … [equation (i)]
Then, $f(-1) = (-1)^3 - 2(-1)^2 + m(-1) + n = 9$
$-1 - 2 - m + n = 9$
$-3 - m + n = 9$
$m = -3 - 9 + n$
$m = n - 12$ … [equation (ii)]
Now, combining both equation (i) and equation (ii) we get,
$n = 2(n - 12) + 16$
$n = 2n - 24 + 16$
$n = 2n - 8$
$2n - n = 8$
$n = 8$
Consider the equation (ii) to find out the value of m,
$m = n - 12$

$m = 8 – 12$
$m = -4$
Therefore, the value of n is 8 and m is – 4.

9. Find the values of a and b when the polynomials $f(x) = 2x^2 – 5x + a$ and $g(x) = 2x^2 + 5x + b$ both have a factor $(2x + 1)$

Solution:-

From the question it is given that,
$f(x) = 2x^2 – 5x + a$
$g(x) = 2x^2 + 5x + b$
Let us assume $2x + 1 = 0$, $x = -½$
Now, substitute the value of x in f(x) we get,
$f(-½) = 2(-½)^2 - 5(-½) + a = 0$
$2(1/4) + 5/2 + a = 0$
$½ + 5/2 + a = 0$
$6/2 + a = 0$
$3 + a = 0$
$a = - 3$
Then,
$g(-½) = 2(-½)^2 + 5(-½) + b = 0$
$2(1/4) - 5/2 + b = 0$
$½ - 5/2 + b = 0$
$- 4/2 + b = 0$
$- 2 + b = 0$
$b = 2$
Therefore, the value of a is – 3 and b is 2.

10. Find the values of a and b when the factors of the polynomial $f(x) = ax^3 + bx^2 + x – a$ are $(x + 3)$ and $(2x - 1)$. Factorize the polynomial completely.

Solution:-

From the question it is given that, $f(x) = ax^3 + bx^2 + x – a$
Let us assume, $x + 3 = 0$, $x = -3$
$2x – 1 = 0$, $x = ½$
Now, substitute the value of x in f(x) we get,
$f(-3) = a(-3)^3 + b(-3)^2 + (-3) – a = 0$
$-27a + 9b – 3 – a = 0$
$-28a + 9b – 3 = 0$

By transposing we get,

$-28a = -9b + 3$

$a = -9b/-28 + 3/-28$

$a = 9b/28 – 3/28$ … [equation (i)]

Then, $f(½) = a(½)^3 + b(½)^2 + (½) – a = 0$

$a(1/8) + b(1/4) + ½ - a= 0$

$(a – 8a)/8 + b(1/4) + ½ = 0$

$-7a/8 + b(¼) + ½ = 0$

$b(¼) = -½ + 7a/8$

$b = -4/2 + 28a/8$

$b = -2 + 7a/2$ … [equation (ii)]

Now, combining equation (i) and equation (ii) we get,

$a = 9/28 \times (7a/2 - 2) – 3/28$

By simplification we get,

$56a = 63a – 42$

$a = 6$

Consider the equation (ii) to find out the value of b,

$b = 7a/2 – 2$

$b = (7 \times 6)/2 – 2$

$b = 42/2 – 2$

$b = 21 – 2$

$b = 19$

Substitute the value of a and b in f(x) we get,

$f(x) = 6x^3 + 19x^2 + x – 6$

Therefore, equation becomes $(x + 3) (2x - 1) (3x + 2) = 0$

11. What number should be subtracted from $x^2 + x + 1$ so that the resulting polynomial is exactly divisible by (x - 2)?

Solution:-

From the question it is given that, $f(x) = x^2 + x + 1$

Then, $x – 2 = 0$, $x = 2$

Let us assume the number should be subtracted from $x^2 + x + 1$ be b,

$f(2) = 2^2 + 2 + 1 – b = 0$

$4 + 2 + 1 – b = 0$

$7 – b = 0$

$b = 7$

Therefore, the number is 7.

12. What number should be added to $2x^3 - 3x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by (x - 1)?

Solution:-

From the question it is given that, $f(x) = 2x^3 - 3x^2 + 7x - 8$

Then, $x – 1 = 0$, $x = 1$

Let us assume the number should be added to $2x^3 - 3x^2 + 7x - 8$ be b,

$f(1) = 2(1)^3 – 3(1)^2 + 7(1) – 8 + b = 0$

$2 – 3 + 7 – 8 + b = 0$

$9 – 11 + b = 0$

$- 2 + b = 0$

$b = 2$

Therefore, the number is 2.

13. What number should be subtracted from polynomial f(x) $2x^3 - 5x^2 + 8x - 17$ so that the resulting polynomial is exactly divisible by (2x - 5)?

Solution:-

From the question it is given that, $f(x) = 2x^3 - 5x^2 + 8x - 17$

Then, $2x – 5 = 0$, $x = 5/2$

Let us assume the number should be subtracted from $2x^3 - 5x^2 + 8x - 17$ be b,

$f(5/2) = 2(5/2)^3 – 5(5/2)^2 + 8(5/2) – 17 – b = 0$

$2(125/8) – 5(25/4) + 40/2 – 17 – b = 0$

$125/4 – 125/4 + 20 – 17 – b = 0$

$3 – b = 0$

$b = 3$

Therefore, the number is 3.

14. What number should be added to polynomial f(x) $12x^3 + 16x^2 - 5x - 8$ so that the resulting polynomial is exactly divisible by (2x - 1)?

Solution:-

From the question it is given that, $f(x) = 12x^3 + 16x^2 - 5x - 8$

Then, $2x – 1 = 0$, $x = ½$

Let us assume the number should be added to $2x^3 - 3x^2 + 7x - 8$ be b,

$f(½) = 12(½)^3 + 16(½)^2 - 5(½) – 8 + b = 0$

$12(1/8) + 16(1/4) – 5/2 – 8 + b = 0$

$3/2 + 4 – 5/2 – 8 + b = 0$

$- 4 – 2/2 + b = 0$

$-4 - 1 + b = 0$
$-5 + b = 0$
$b = 5$
Therefore, the number is 5.

15. Use the remainder theorem to find the factors of $(a - b)^3 + (b - c)^3 + (c - a)^3$
Solution:-
From the question it is given that, $f(x) = (a - b)^3 + (b - c)^3 + (c - a)^3$
We know the formula, $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ … [equation (i)]
Let us assume that $a - b = 0$, $a = b$
Now substitute the above value in f(x), we get,
$f(x) = 0 + (a - c)^3 + (c - a)^3 = 0$
$(a - c)^3 - (a - c)^3 = 0$
$0 = 0$
Therefore, (a - b) is a factor. … [equation (ii)]
Again, $f(x) = 0 + (b^3 - 3b^2c + 3bc^2 - c^3) + (c^3 - 3c^2a + 3ca^2 - a^2)$
$= -3b^2c + 3bc^2 - 3ca^2 + 3ca^2$
$= 3(-b^2c + bc^2 - ca^2 + ca^2)$
So, now we put $b - c = 0$, $b = c$
Substitute the above value in f(x), we get,
Then, $f(b = c)$, $3((-c^2 \times c) + (c \times c^2) - (c \times c^2) + (c \times c^2)) = 0$
Factors are 3(a - b) (b - c) … [equation (iii)]
similarly if we put c = a,
(c - a) is a factor … [equation (iv)]
By combining equation (ii), equation (iii) and (iv), we get,
3(a - b)(b - c)(c - a).