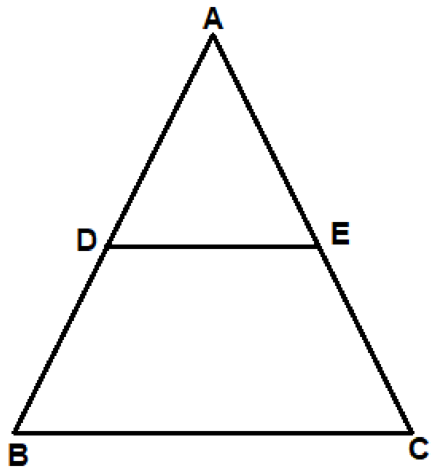
1. In ΔABC, DE is parallel to BC and AD/DB = 2/7. If AC = 5.6, find AE.
Solution:-



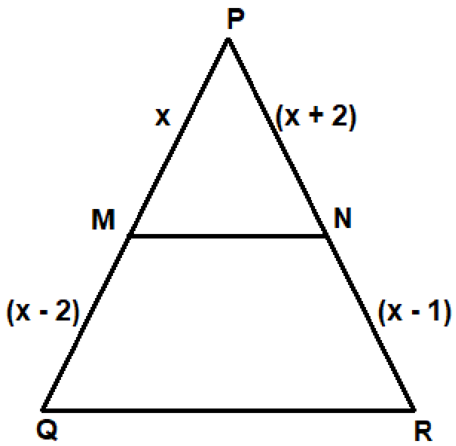
From the question it is given that, AD/DB = 2/7 and AC = 5.6
Consider ΔABC,
DE is parallel to BC,
Let us assume AE = y
From Basic Proportionality Theorem (BPT) AD/DB = AE/EC

2/7 = y/(5.6 - y)
2(5.6 - y) = 7y
11.2 – 2y = 7y
11.2 = 7y + 2y
11.2 = 9y
y = 11.2/9
y = 1.24

Therefore, AE is 1.24

2. In ΔPQR, MN is drawn parallel to QR. If PM = x, MQ = (x - 2) and NR = (x - 1), find the value of x.

Solution:-



From the question it is given that, PM = x, MQ = (x - 2) and NR = (x - 1)
Consider ΔPQR,
MN is drawn parallel to QR,
From Basic Proportionality Theorem (BPT) PM/MQ = PN/NR

$x/(x - 2) = (x + 2)/(x - 2)$

$x(x - 2) = (x + 2)(x - 2)$

$x^2 - 2x = x^2 - 4$

$x^2 - x^2 - 2x = -4$

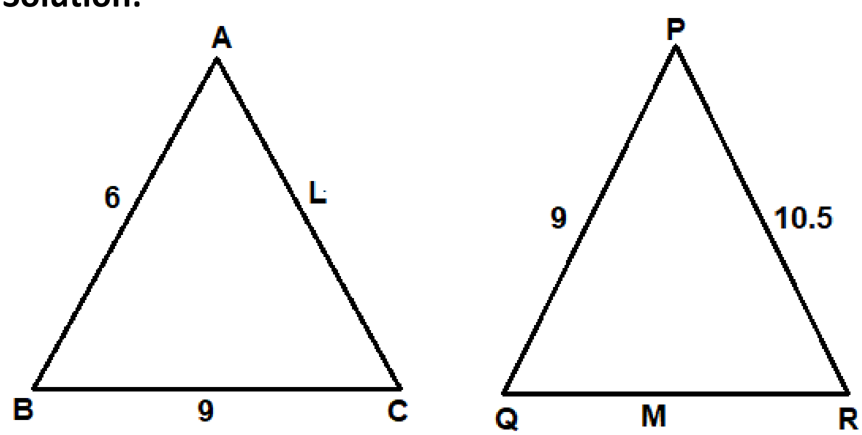
$-2x = -4$

$x = -4/-2$

$x = 2$

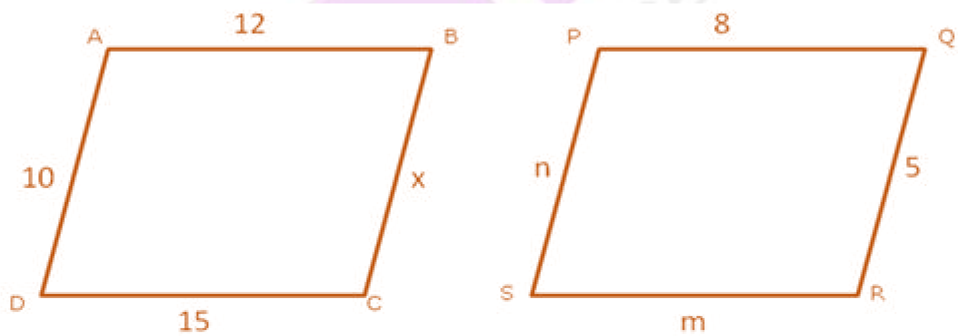
Therefore, PM = x = 2
MQ = x – 2 = 2 – 2 =0
NR = x – 1 = 2 – 1 = 1

3. ΔABC is similar to ΔPQR. If AB = 6 cm, BC = 9 cm, PQ = 9 cm and PR = 10.5 cm, find the lengths of AC and QR.
Solution:-



From the question it is given that, ΔABC ~ ΔPQR
AB = 6 cm, BC = 9 cm, PQ = 9 cm and PR = 10.5 cm
Now, consider ΔABC and ΔPQR
AB/PQ = BC/QR = AC/PR
6/9 = 9/M = L/10.5
Take first two fraction, 6/9 = 9/q
6M = 81
M = 81/6
M = 27/2
Then, QR = 13.5 cm
Now consider second and third fraction, 9/M = L/10.5
9/13.5 = L/10.5
9(10.5) = 13.5L
94.5 = 13.5L
L = 94.5/13.5
L = 7 cm
So, AC = 7 cm

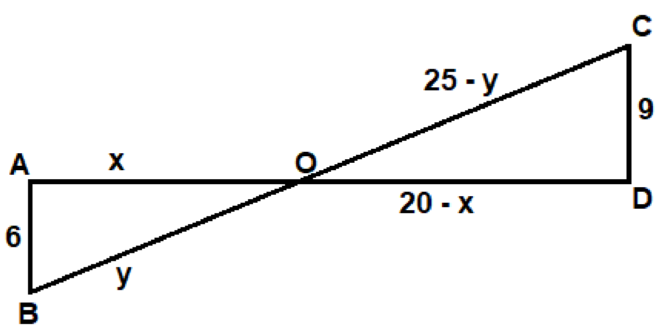
4. ABCD and PQRS are similar figures. AB = 12 cm, BC = x cm, CD = 15 cm, AD = 10 cm, PQ = 8 cm, QR = 5 cm, RS = m cm and PS = n cm. Find the values of x, m and n.
Solution:-



From the question it is given, quadrilateral ABCD ~ quadrilateral PQRS. AB = 12 cm, BC = x cm, CD = 15 cm, AD = 10 cm, PQ = 8 cm, QR = 5 cm, RS = m cm and PS = n cm.
Then, AB/PQ = BC/QR = DC/SR = AD/SR
12/8 = x/5 = 15/m = 10/n
Consider, 12/8 = x/5
By cross multiplication we get,
12 × 5 = 8 × x
60 = 8x

x = 60/8
x = 7.5 cm
Then, consider x/5 = 15/m
7.5/5 = 15/m
By cross multiplication we get,
7.5 × m = 15 × 5
7.5m = 75
m = 75/7.5
m = 750/75
m = 10 cm
Now, consider 15/m = 10/n
15/10 = 10/n
By cross multiplication we get,
15 × n = 10 × 10
15n = 100
n = 100/15
n = 6.67 cm
Therefore, the value of x is 7.5 cm, value of m is 10 cm and value of n is 6.67 cm.

5. AD and BC are two straight lines intersecting at O. CD and BA are perpendiculars from B and C on AD. If AB = 6 cm, CD = 9 cm, AD = 20 cm and BC = 25 cm, find the lengths of AO, BO, CO and DO.
Solution:-



From the question it is given that, AB = 6 cm, CD = 9 cm, AD = 20 cm and BC = 25 cm
We have to find the lengths of AO, BO, CO and DO.
Consider ΔAOB and ΔCOD,
∠OAB = ∠ODC … [because both angles are equal to 90^o]
∠AOB = ∠DOC … [because vertically opposite angles are equal]
Therefore, ΔAOB ~ ΔDOC [from AA corollary]

Then, AO/DO = OB/OC = AB/DC

x/(20 - x) = y/(25 - y) = 6/9

Consider, x/(20 – x) = 6/9

x/(20 - x) = 2/3

By cross multiplication we get,

3x = 2(20 - x)

3x = 40 – 2x

3x + 2x = 40

5x = 40

x = 40/5

x = 8

Now, consider y/(25 – y) = 6/9

y/(25 - y) = 2/3

By cross multiplication we get,

3y = 2(25 - y)

3y = 50 – 2y

3y + 2y = 50

5y = 50

y = 50/5

y = 10

Therefore, AO = x = 8 cm

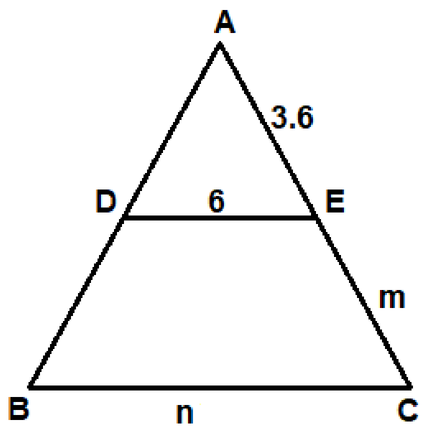
OD = 20 – x = 20 – 8 = 12 cm

BO = y = 10 cm

OC = 25 – y = 25 – 10 = 15 cm

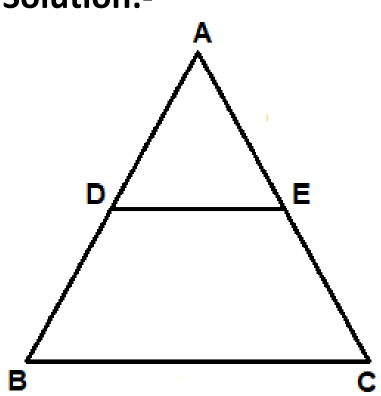
6. In ΔABC, DE is drawn parallel to BC. If AD: DB = 2: 3, DE = 6 cm and AE = 3.6 cm find BC and AC.

Solution:-



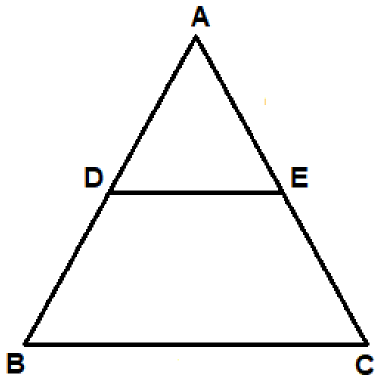
From the question it is given that, AD: DB = 2: 3, DE = 6 cm and AE = 3.6 cm
Now consider the ΔABC,
DE || BC … [given]
From Basic Proportionality Theorem (BPT) AD/DB = AE/EC
2/3 = 3.6/m
2m = 3 × 3.6
2m = 10.8
m = 10.8/2
m = 5.4 cm
So, EC = 5.4 cm
Therefore, AC = 3.6 + x
= 3.6 + 5.4
= 9 cm
Now, consider the ΔADE and ΔABC,
∠ABC = ∠ADE … [because corresponding angles are equal]
∠ACB = ∠AED … [because corresponding angles are equal]
Therefore, ΔABC ~ ΔADE … [from AA corollary]
AE/AC = DE/BC
3.6/9 = 6/n
By cross multiplication we get,
3.6n = 9 × 6
3.6n = 54
n = 54/3.6
n = 15
So, BC = 15 cm

7. D and E are points on the sides AB and AC respectively of ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE is parallel to BC.
Solution:-



From the question it is given that, AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm
We have to show that, DE ∥ BC
Consider the ΔABC,
AD/DB = 1.4/4.2
= 14/42
AD/DB = 1/3 … (i)
AE/EC = 1.8/5.4
= 18/54
AE/EC = 1/3 … (ii)
From (i) and (ii)
AD/DB = AE/EC
Hence, it is proved that DE ∥ BC by converse BPT.

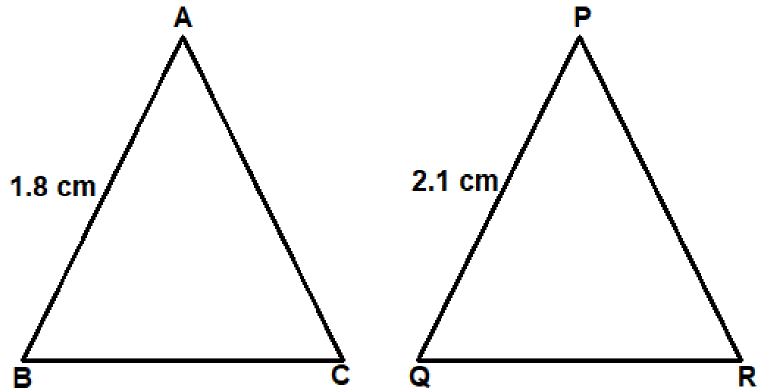
8. In ΔABC, D and E are points on the sides AB and AC respectively. If AD = 4 cm, DB = 4.5 cm, AE = 6.4 cm and EC = 7.2 cm, find if DE is parallel to BC or not.
Solution:-



From the question it is given that, AD = 4 cm, DB = 4.5 cm, AE = 6.4 cm and EC = 7.2 cm
We have to show that, DE ∥ BC
Consider the ΔABC,
AD/DB = 4/4.5
= 40/45
AD/DB = 8/9 … (i)
AE/EC = 6.4/7.2
= 64/72
AE/EC = 8/9 … (ii)
From (i) and (ii)
AD/DB = AE/EC
Hence, it is proved that DE ∥ BC by converse BPT.

9. ΔABC ~ ΔPQR such that AB = 1.8 cm and PQ = 2.1 cm. Find the ratio of areas of ΔABC and ΔPQR.

Solution:-



From the question it is given that, ΔABC ~ ΔPQR and AB = 1.8 cm, PQ = 2.1 cm

We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔABC/Area of ΔPQR = AB^2/PQ^2

$= (1.8)^2/(2.1)^2$

$= (18/21)^2$

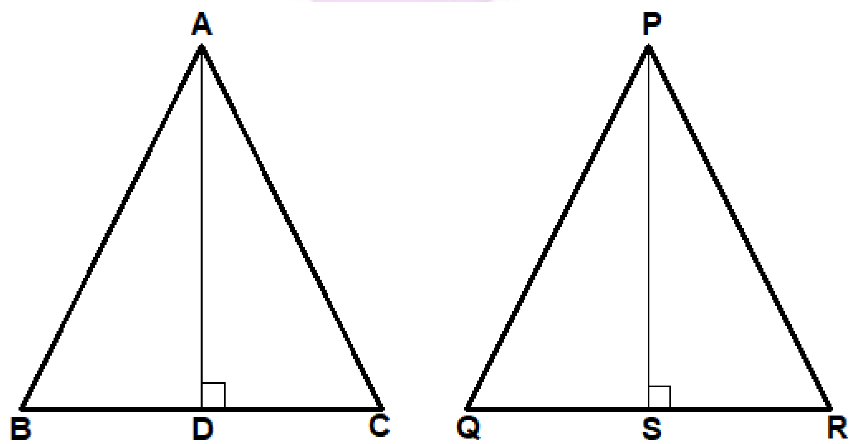
$= (6/7)^2$

$= 36/49$

Therefore, the ratio of areas of ΔABC and ΔPQR is 36: 49.

10. ΔABC ~ ΔPQR, AD and PS are altitudes from A and P on sides BC and QR respectively. If AD: PS = 4: 9, find the ratio of the areas of ΔABC and ΔPQR.

Solution:-



From the question it is given that, ΔABC ~ ΔPQR and AD: PS = 4: 9

We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔABC/Area of ΔPQR = AB^2/PQ^2 … [equation (i)]

Now, consider the ΔBAD and ΔQPS

∠ABD = ∠PQS … [because ΔABC ~ ΔPQR]

∠AOB = ∠PSQ … [because both angles are equal to 90^o]

Therefore, ΔBAD ~ ΔQPS … [from AA corollary]

So, AB/PQ = AD/PS … [equation (ii)]

From equation (i) and equation (ii),

Area of ΔABC/Area of ΔPQR = AD^2/PS^2

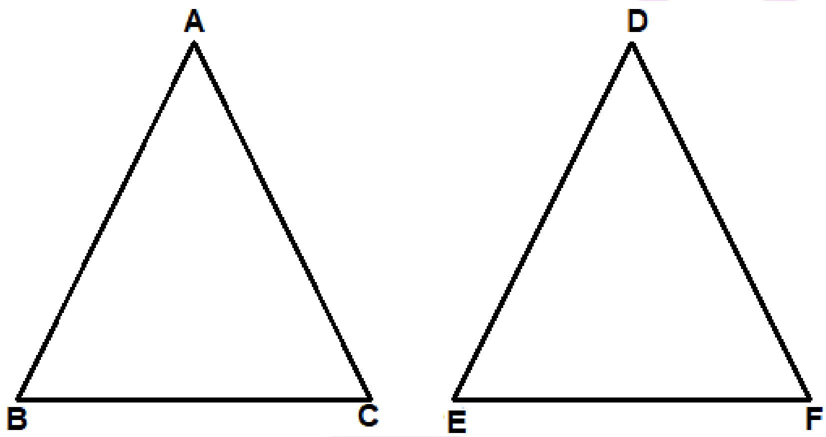
= $(4)^2/(9)^2$

= 16/81

Therefore, the ratio of areas of ΔABC and ΔPQR is 16: 81.

11. ΔABC ~ ΔDEF. If BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm^2, find the area of ΔDEF.

Solution:-



From the question it is given that, ΔABC ~ ΔDEF and BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm^2

We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔABC/Area of ΔDEF = BC^2/EF^2

54/Area of ΔDEF = $(3)^2/(4)^2$

54/Area of ΔDEF = 9/16

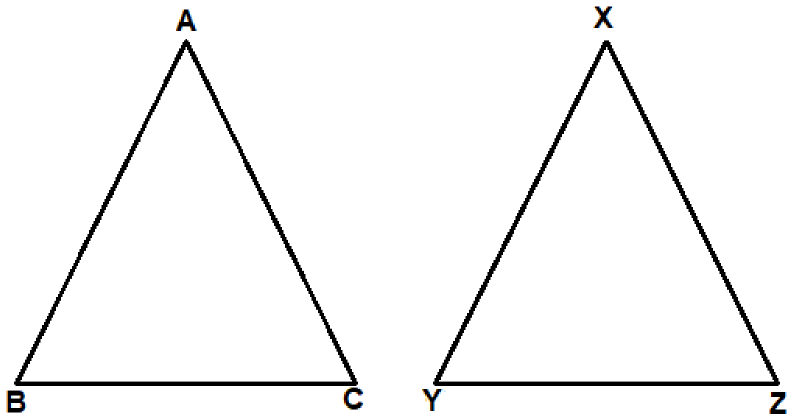
Area of ΔDEF = (54 × 16)/9

Area of ΔDEF = 96 cm^2

Therefore, area of ΔDEF is 96 cm^2.

12. ΔABC ~ ΔXYZ. If area of ΔABC is 9 cm^2 and area of ΔXYZ is 16 cm^2 and if BC = 2.1 cm, find the length of YZ.

Solution:-



From the question it is given that, ΔABC ~ ΔXYZ, area of ΔABC is 9 cm^2 and area of ΔXYZ is 16 cm^2 and BC = 2.1 cm

We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔABC/Area of ΔXYZ = BC^2/YZ^2

$9/16 = (2.1)^2/(YZ)^2$

Taking square root on both sides we get,

$3/4 = 2.1/YZ$

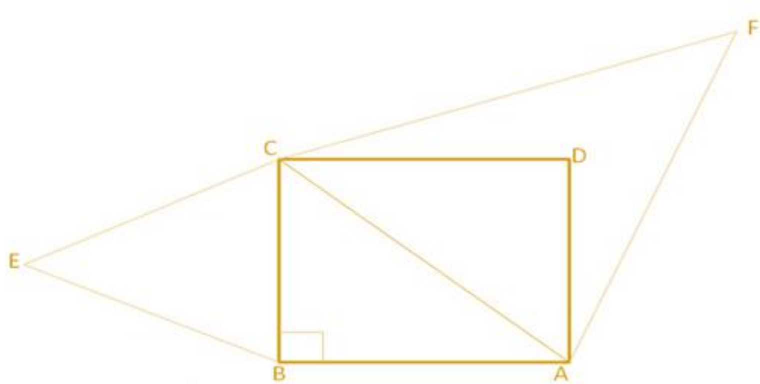
$YZ = (2.1 \times 4)/3$

$YZ = 2.8$

Therefore, YZ is 2.8 cm.

13. Prove that the area of ΔBCE described on one side BC of a square ABCD is one half the area of the similar ΔACF described on the diagonal AC.

Solution:-



Consider the right angle triangle ABC,

We know that, Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$AC^2 = 2BC^2$ … [because AB = BC]

From the question it is given that, ΔBCE ~ ΔACF

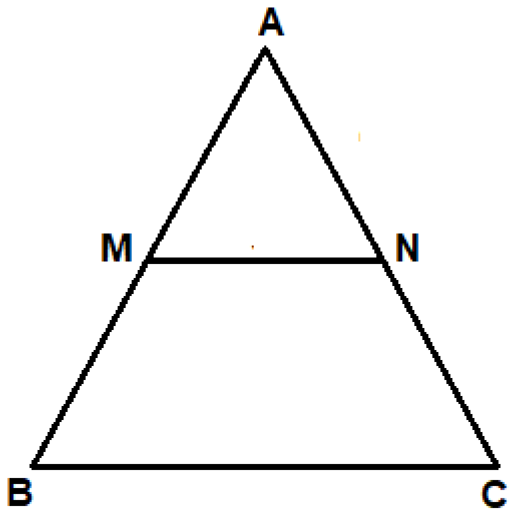
We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔBCE/Area of ΔACF = BC^2/AC^2

= ½

Therefore, the ratio of areas of ΔBCE and ΔACF is 1: 2.

14. In ΔABC, MN ∥ BC



(a) If AN: AC = 5: 8, find the ratio of area of ΔAMN: area of ΔABC

Solution:-

From the it is given that, AN: AC = 5: 8

We have to find the ratio of area of ΔAMN: area of ΔAB

So, consider the ΔAMN and ΔABC

∠AMN = ∠ABC … [because corresponding angles are equal]

∠ANM = ∠ACB … [because corresponding angles are equal]

Therefore, ΔAMN ~ ΔABC … [from AA corollary]

We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔAMN/Area of ΔABC = AN^2/AC^2

= $(5/8)^2$

= 25/64

Therefore, the ratio of areas of ΔAMN and ΔABC is 25: 64.

(b) If AB/AM = 9/4, find area of trapezium MBCN/area of ΔABC.

Solution:-

From the it is given that, AB/AM = 9/4

In the above solution we prove that, ΔAMN ~ ΔABC

So, Area of ΔAMN/Area of ΔABC = AM^2/AB^2

$= 4^2/9^2$

$= 16/81$

(Area of ΔABC – Area of trapezium MBCN)/area of ΔABC = 16/81

By cross multiplication we get,

81(Area of ΔABC – Area of trapezium MBCN) = area of ΔABC × 16

(81 × Area of ΔABC) – (81 × area of trapezium MBCN) = 16 × area of ΔABC

64 area of ΔABC = 81 area of trapezium MBCN

Then, area of trapezium MBCN/area of ΔABC = 65/81

Therefore, the ratio of areas of trapezium MBCN and ΔABC is 65: 81.

(c) If BC = 14 cm and MN = 6 cm, find area of ΔAMN/area of trapezium MBCN

Solution:-

From the it is given that, BC = 14 cm and MN = 6 cm

In the above solution (a) we prove that, ΔAMN ~ ΔABC

So, Area of ΔAMN/Area of ΔABC = MN^2/BC^2

$= 6^2/14^2$

$= 36/196$

$= 9/49$

(Area of ΔAMN/Area of ΔAMN + Area of trapezium MBCN) = 9/49

By cross multiplication we get,

49(Area of ΔAMN) = 9 (Area of ΔAMN + Area of trapezium MBCN)

49 Area of ΔAMN = 9 Area of ΔAMN + 9 Area of trapezium MBCN

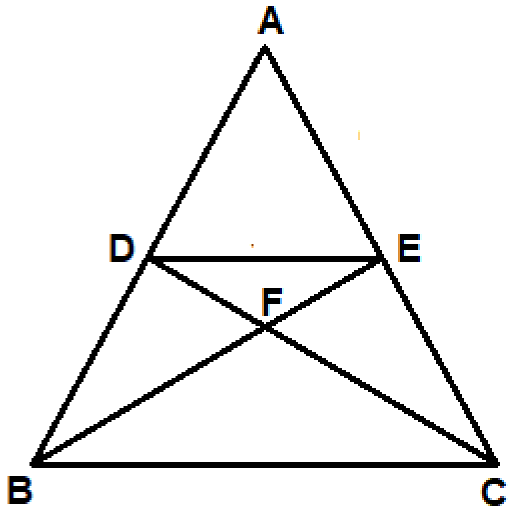
49 area of ΔAMN = 9 area of trapezium MBCN

Then, area of ΔABC/area of trapezium MBCN = 9/40

Therefore, the ratio of area of ΔABC and area of trapezium is 9: 40.

15. In ΔABC, DE ∥ BC; DC and EB intersect at F, if DE/BC = 2/7, find area of ΔFDE/area of ΔFBC

Solution:-



From the question it is given that, DE/BC = 2/7, DE ∥ BC

Consider the ΔFDE and ΔFCB

∠FDE = ∠FCB … [because interior alternate angles are equal]

∠ANM = ∠ACB … [because interior alternate angles are equal]

Therefore, ΔFDE ~ ΔFCB … [from AA corollary]

We know that, the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, Area of ΔFDE/Area of ΔFBC = DE^2/BC^2

$= (2/7)^2$

$= 4/49$

Therefore, the ratio of areas of ΔFDE and ΔFBC is 4: 49.