1. Find the length of the chord of a circle in each of the following when:

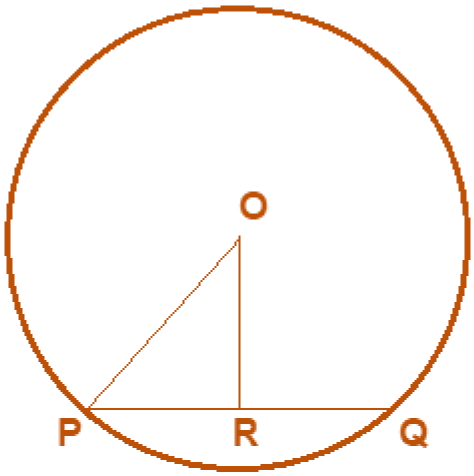
(i) Radius is 13 cm and the distance from the center is 12 cm

Solution:-

From the question it is given that,

Radius = 13 cm

Distance from the center is 12 cm



From the figure we can say that, PR = RQ

Because, perpendicular from center to a chord bisects the chord.

Consider the ΔPRO,

By using Pythagoras theorem, $OP^2 = OR^2 + PR^2$

$13^2 = 12^2 + PR^2$

$PR^2 = 13^2 - 12^2$

$PR^2 = 169 - 144$

$PR^2 = 25$

$PR = \sqrt{25}$

PR = 5 cm

Therefore, length of chord PQ = 2PR

= 2(5)

= 10 cm

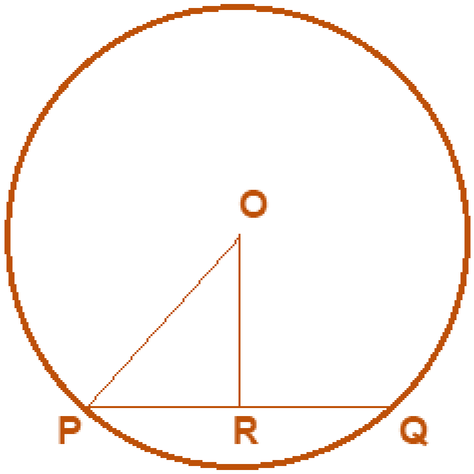
(ii) Radius is 1.7 cm and the distance from the center is 1.5 cm.

Solution:-

From the question it is given that,

Radius = 1.7 cm

Distance from the center is 1.5 cm



From the figure we can say that, PR = RQ

Because, perpendicular from center to a chord bisects the chord.

Consider the ΔPRO,

By using Pythagoras theorem, $OP^2 = OR^2 + PR^2$

$(1.7)^2 = (1.5)^2 + PR^2$

$PR^2 = (1.7)^2 – (1.5)^2$

$PR^2 = 2.89 – 2.25$

$PR^2 = 0.64$

$PR = \sqrt{0.64}$

PR = 0.8 cm

Therefore, length of chord PQ = 2PR

= 2(0.8)

= 1.6 cm

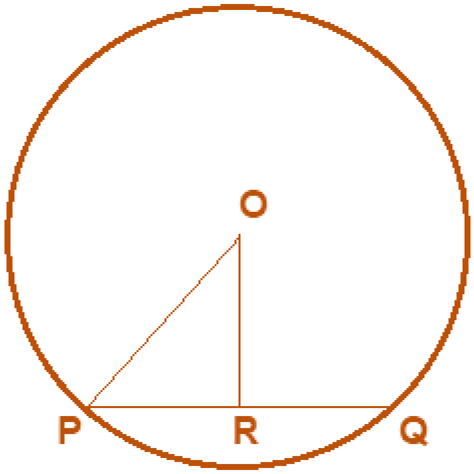
(iii) Radius is 6.5 cm and the distance from the center is 2.5 cm.

Solution:-

From the question it is given that,

Radius = 6.5 cm

Distance from the center is 2.5 cm



From the figure we can say that, PR = RQ

Because, perpendicular from center to a chord bisects the chord.

Consider the ΔPRO,

By using Pythagoras theorem, $OP^2 = OR^2 + PR^2$

$(6.5)^2 = (2.5)^2 + PR^2$

$PR^2 = (6.5)^2 – (2.5)^2$

$PR^2 = 42.25 – 6.25$

$PR^2 = 36$

$PR = \sqrt{36}$

PR = 6 cm

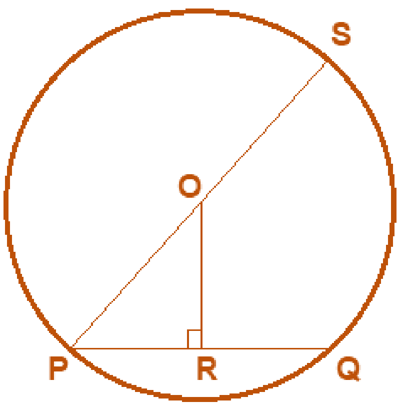
Therefore, length of chord PQ = 2PR

= 2(6)

= 12 cm

2. Find the diameter of the circle if the length of a chord is 3.2cm and its distance from the center is 1.2cm.

Solution:-



From the question it is given that,
length of a chord is 3.2cm
Distance from the center is 1.2cm
Then,
From the figure we can say that, PR = RQ = 1.6 cm
Because, perpendicular from center to a chord bisects the chord.
Consider the ΔPRO,
By using Pythagoras theorem, $OP^2 = OR^2 + PR^2$

$OP^2 = (1.6)^2 + (1.2)^2$

$OP^2 = 2.56+1.44$

$OP^2 = 4$

$OP = \sqrt{4}$

OP = 2 cm

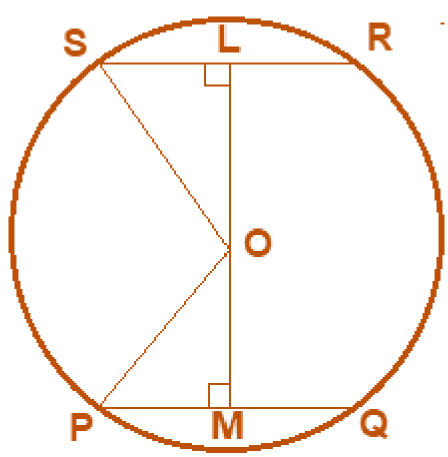
Therefore, Diameter of the circle PS = 2OP

= 2(2)

= 4 cm

3. A chord of length 16.8cm is at a distance of 11.2cm from the center of a circle. Find the length of the chord of the same circle which is at a distance of 8.4cm from the center.

Solution:-



From the figure we can say that, PM = MQ = 8.4 cm
Because, perpendicular from center to a chord bisects the chord.
Consider the ΔPMO,
By using Pythagoras theorem, $OP^2 = PM^2 + OM^2$

$OP^2 = (8.4)^2 + (11.2)^2$

$OP^2 = 70.56 + 125.44$

$OP^2 = 196$

$OP = \sqrt{196}$

$OP = 14$ cm

Then, OP = OS = 14 cm because radii of same circle.

Now consider the ΔSLO

By using Pythagoras theorem, $OS^2 = SL^2 + LO^2$

$14^2 = (SL)^2 + (8.4)^2$

$SL^2 = 14^2 - 8.4^2$

$SL^2 = 196 - 70.56$

$SL^2 = 125.44$

$SL = \sqrt{125.44}$

$SL = 11.2$ cm

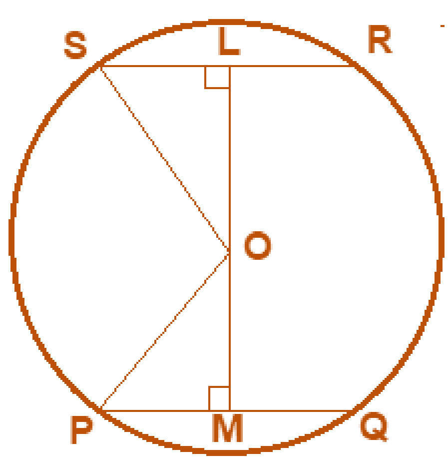
Therefore, the length of chord SR = 2SL

= 2(11.2)

= 22.4 cm

4. A chord of length 6 cm is at a distance of 7.2 cm from the center of a circle. Another chord of the same circle is of length 14.4 cm. Find its distance from the center.

Solution:-



From the figure we can say that, PM = MQ = 3 cm

Because, perpendicular from center to a chord bisects the chord.

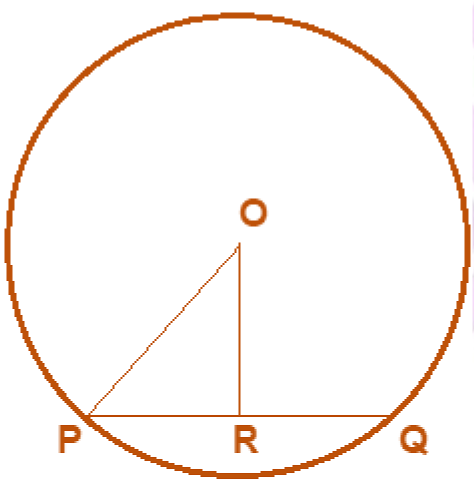
Consider the ΔPMO,

By using Pythagoras theorem, $OP^2 = PM^2 + OM^2$

$OP^2 = (3)^2 + (7.2)^2$

$OP^2 = 9 + 51.84$
$OP^2 = 60.84$
$OP = \sqrt{60.84}$
$OP = 7.8$ cm
Then, OP = OS = 7.8 cm because radii of same circle.
Now consider the ΔSLO
By using Pythagoras theorem, $OS^2 = SL^2 + LO^2$
$7.8^2 = (7.2)^2 + LO^2$
$LO^2 = 7.8^2 – 7.2^2$
$LO^2 = 60.84 – 51.84$
$LO^2 = 9$
$LO = \sqrt{9}$
LO = 3 cm
Therefore, distance of chord SR from the center is 3 cm.

5. A chord of length 8cm is drawn inside a circle of radius 6cm. Find the perpendicular distance of the chord from the center of the circle.
Solution:-



From the question it is given that, Length of chord is 8 cm.
Radius of circle is 6 cm.
From the figure we can say that, PR = RQ = 4 cm
Because, perpendicular from center to a chord bisects the chord.
Consider the ΔPRO,
By using Pythagoras theorem, $OP^2 = OR^2 + PR^2$
$6^2 = OR^2 + 4^2$
$OR^2 = 36-16$

$OR^2 = 20$

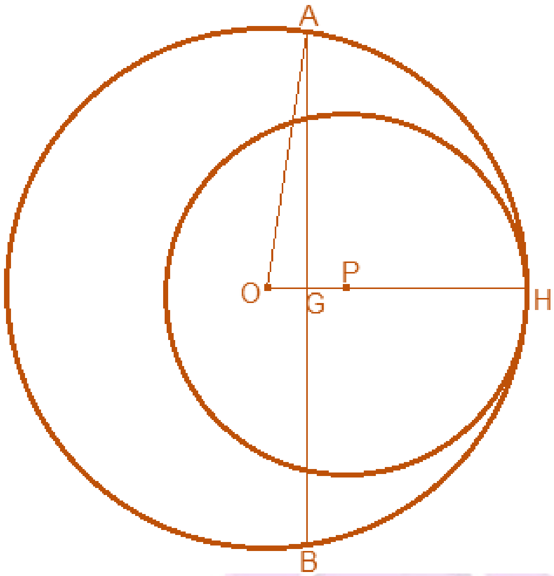
$OR = \sqrt{20}$

$OR = 2\sqrt{5}$ cm

Therefore, the perpendicular distance of the chord from the center of the circle is $2\sqrt{5}$ cm.

6. Two circles of radii 5cm and 3cm with centers O and P touch each other internally. If the perpendicular bisector of the line segment OP meets the circumference of the larger circle at A and B, find the length of AB.

Solution:-



From the question it is given that,
Radius of bigger circle = 5 cm
Radius of smaller circle = 3 cm
Then,
OA = AH = 5 cm … [because both are radius of bigger circle]
PH = 3 cm … [because radius of smaller circle]
OP = 2 cm
So, perpendicular bisector of OP, i.e. AB meets OP at G.
OG = GP = ½ OP = 1 cm
Consider the ΔOGA,
By using Pythagoras theorem, $OA^2 = OG^2 + GA^2$

$5^2 = 1^2 + GA^2$

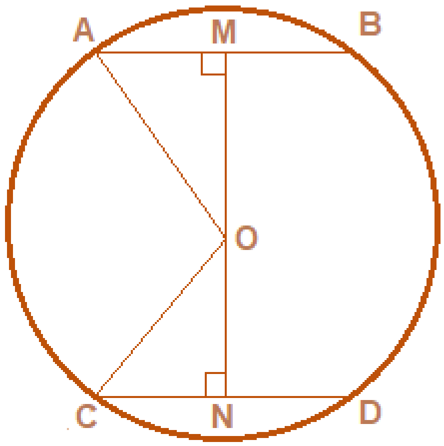
$GA^2 = 5^2 - 1^2$

$GA^2 = 25 - 1$

$GA^2 = 24$
$GA = \sqrt{24}$
$GA = 2\sqrt{6}$ cm

So, GA = GB = $2\sqrt{6}$ cm
AB = AG + GB
= $2\sqrt{6} + 2\sqrt{6}$
= $4\sqrt{6}$ cm
Therefore, the length of AB $4\sqrt{6}$ cm.

7. Two chords AB and CD of lengths 6cm and 12cm are drawn parallel inside the circle. If the distance between the chords of the circle is 3cm, find the radius of the circle.
Solution:-



From the figure we can say that, AM = MB = 3 cm and CN = ND = 6 cm
Because, perpendicular from center to a chord bisects the chord.
Let us assume, OA = OC = p
OM = y
ON = 3 - y
Consider the ΔCNO,
By using Pythagoras theorem, $OC^2 = ON^2 + CN^2$
$p^2 = (3 - y)^2 + (6)^2$ … [equation (i)]
Now consider the ΔOMA
By using Pythagoras theorem, $OA^2 = OM^2 + AM^2$
$p^2 = y^2 + 3^2$ … [equation (ii)]
Combining equation (i) and equation (ii) we get,
$(3 - y)^2 + 6^2 = y^2 + 3^2$
$9 + y^2 – 6y + 36 = y^2 + 9$

By transposing we get,

$9 - 9 + y^2 - y^2 - 6y + 36 = 0$

$-6y + 36 = 0$

$y = 36/6$

$y = 6$

Now substitute the value of y in equation (ii) to find out the value p,

$p^2 = y^2 + 3^2$

$p^2 = 6^2 + 3^2$

$p^2 = 36 + 9$

$p^2 = 45$

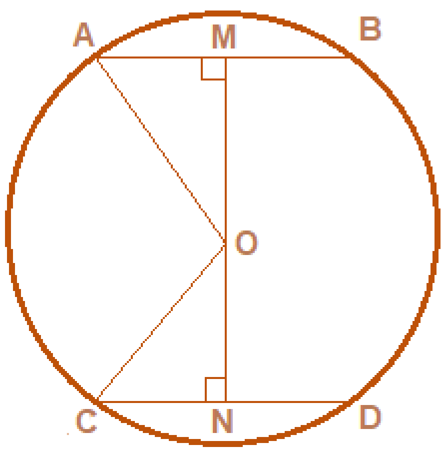
$p = \sqrt{45}$

$p = 3\sqrt{5}$

Therefore, the radius of circle is $3\sqrt{5}$ cm.

8. Two chords of lengths 10cm and 24cm are drawn parallel to each other in a circle. If they are on the same side of the center and the distance between them is 17cm, find the radius of the circle.

Solution:-



From the figure we can say that, AM = MB = 5 cm and CN = ND = 12 cm

Because, perpendicular from center to a chord bisects the chord.

Let us assume, OA = OC = p

OM = y

ON = 17 - y

Consider the ΔCNO,

By using Pythagoras theorem, $OC^2 = ON^2 + CN^2$

$p^2 = (17 - y)^2 + (12)^2$ … [equation (i)]

Now consider the ΔOMA

By using Pythagoras theorem, $OA^2 = OM^2 + AM^2$

$p^2 = y^2 + 5^2$ … [equation (ii)]

Combining equation (i) and equation (ii) we get,

$(17 - y)^2 + 12^2 = y^2 + 5^2$

$289 + y^2 – 34y + 144 = y^2 + 25$

By transposing we get,

$289 + 144 – 25 + y^2 – y^2 – 34y = 0$

$408 – 34y = 0$

$y = 408/34$

$y = 12$

Now substitute the value of y in equation (ii) to find out the value p,

$p^2 = y^2 + 5^2$

$p^2 = 12^2 + 5^2$

$p^2 = 144 + 25$

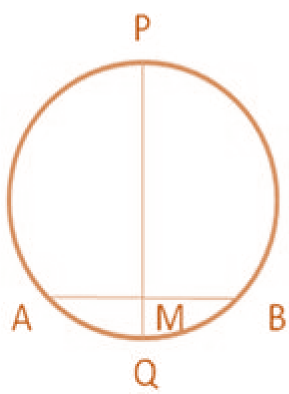
$p^2 = 169$

$p = \sqrt{169}$

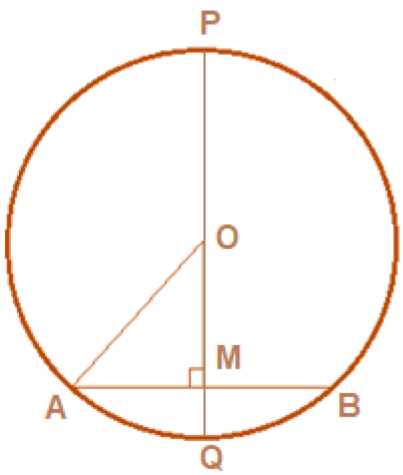
$p = 13$

Therefore, the radius of circle is 13.

9. In fig, AB a chord of the circle is of length 18 cm. It is perpendicularly bisected at M by PQ. If MQ = 3 cm, find the length of PQ.



Solution:-



From the question it is given that,
Length of chord AB = 18 cm
MQ = 3 cm
From the figure we can say that, AM = MB = 5 cm
Because, perpendicular from center to a chord bisects the chord.
Let us assume, OA = OQ = p
OM = (r - 3)
Consider the ΔOMA,
By using Pythagoras theorem, $OA^2 = OM^2 + MA^2$

$r^2 = (r - 3)^2 + 9^2$

$r^2 = r^2 + 9 – 6r + 81$

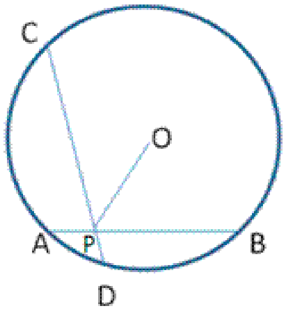
$r^2 – r^2 + 6r = 90$

$6r = 90$

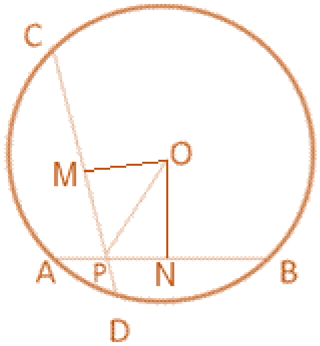
$r = 15$ cm

Then, PQ = 2r
= 2(15)
= 30 cm

10. AB and CD are two equal chords of a circle intersecting at P as shown in fig. P is joined to O, the center of the circle. Prove that OP bisects ∠CPB

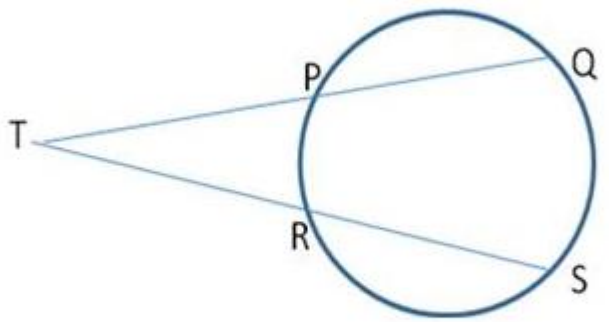


Solution:-

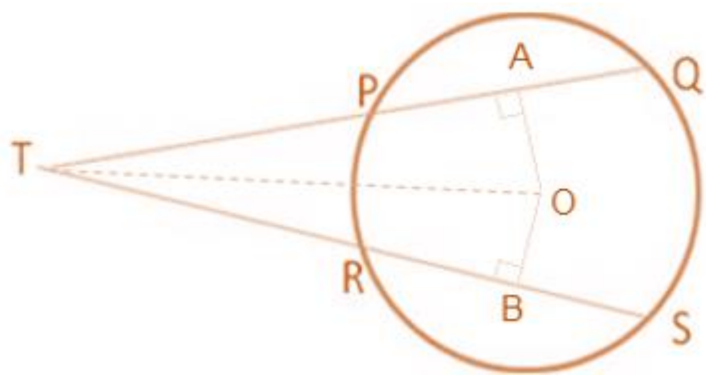


Construction: Draw perpendiculars OM and ON to AB and CD respectively.
Now, consider the ΔOMP and ΔONP,
OP = OP … [common side for both triangles]
OM = ON … [distance of equal chords from the center are equal]
∠PMO = ∠PNO … [both angles are equal to 90^o]
Therefore, ΔOMP ≅ΔONP
So, ∠MPO = ∠NPO
Hence it is proved that, OP bisects ∠CPB.

11. In fig, the center is O. PQ and RS are two equal chords of the circle which, when produced, meet at T outside the circle. Prove that (a) TP = TR, (B) TQ = TS

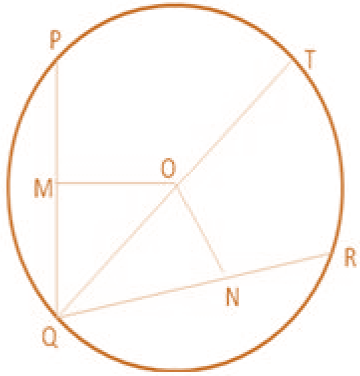


Solution:-



From the question it is given that PQ = RS
We have to prove that, TP = TR and TQ = TS
Then, Draw OA ⊥ PQ and OB ⊥ RS,
Since equal chords are equidistant from the circle therefore,
PQ = RS ⇒ OA = OB … [equation (i)]
So, PA = AQ = ½ PQ and RB = BS = ½ RS
Given PQ = RS, we get
PA = RB … [equation (ii)]
And, AQ = BS … [equation (iii)]
Now, consider the ΔTAO and ΔTBO,
TO = TO [common side for both triangles]
AO = BO [from equation (i)]
∠TAO = ∠TBO [both angles are equal to 90^o]
Therefore, ΔTAO ≅ ΔTBO [By RHS]
So TA = TB [By CPCT] … [equation (iv)]
By subtracting equation (ii) from equation (iv), we get
TA – PA = TB – RB
⇒ TP = TR
Now adding equation (iii) and equation (iv), we get
TA + TQ = TB + BS
Therefore, TQ = TS
Hence proved

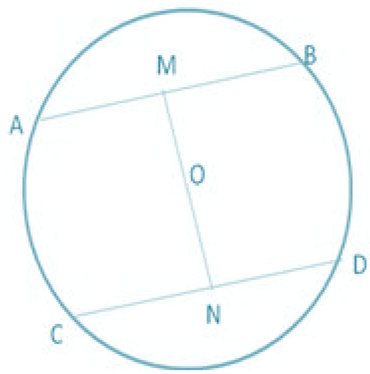
12. PQ and QR are two equal chords of a circle. A diameter of the circle is drawn through Q. Prove that the diameter bisect ∠PQR.
Solution:-



let us assume that QT be the diameter of ∠PQR
Then, from question it is given that PQ = QR

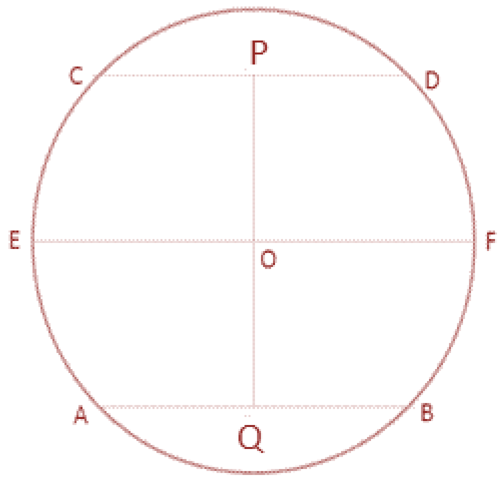
So, OM = ON
Now, consider the ΔOMQ and ΔONQ,
∠OMQ = ∠ONQ ...[both angles are equal to 90^o]
OM = ON ... [from question it is given that equal chords]
OQ = OQ ... [common side for both triangles]
Therefore, ΔOMQ ≅ ΔONQ [By RHS]
So, ∠OQM = ∠OQN [CPCT]
Hence QT i.e. diameter of the circle bisects ∠PQR

13. M and N are the midpoints of chords AB and CD. The line MN passes through the center O. Prove that AB ∥ CD.
Solution:-



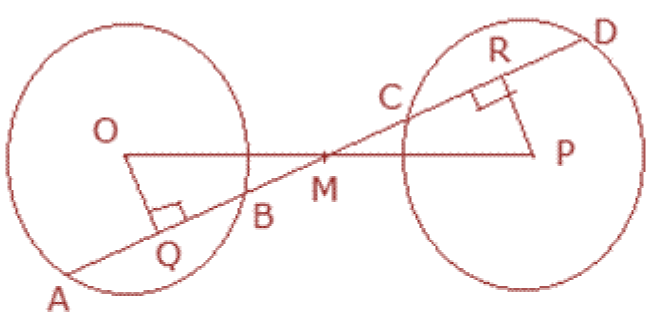
From the figure,
AM = MB
CN = ND
Therefore, OM ⊥ AB
Then, ON ⊥ CD
Because, a line bisecting the chord and passing through the centre of the circle is perpendicular to the chord.
So, ∠OMA = ∠OND ...[both angles are equal to 90^o]
Therefore, AB ∥CD

14. Prove that the line segment joining the midpoints of two parallel chords of a circle passes through its centre.
Solution:-



Let us assume AB and CD be two parallel chords having Q and P as their mid-points, respectively.
Let O be the centre of the circle.
Join OP and OQ and draw EF || AB | | CD. Since, P is the mid-point of CD.
OP ⊥ CD
$\angle CPO = \angle DPO = 90^o$
But OF || CD
$\therefore \angle POF = \angle CPO$ … [alternate interior angle]
$\angle POF = 90^o$
Similarly, $\angle FOQ = 90^o$
Now, $\angle POF + \angle FOQ = 90^o + 90^o = 180^o$
Therefore, POQ is a straight line.
Hence proved

15. Two congruent circles have their centers at O and P. M is the midpoint of the line segment OP. A straight line is drawn through M cutting the two circles at the points A, B, C and D. Prove that the chords AB and CD are equal.
Solution:-



From the question it is given that, two congruent circles have their centers at O and P.
M is the midpoint of the line segment OP.
We have to prove that, chord AB and CD are equal.
Then, draw OQ ⊥ AB and PR ⊥ CD.
Now consider the ΔOQM and ΔPRM

OM = MP	… [because M is the midpoint]
∠OQM = ∠PRC	…[both angles are equal to 90^o]
∠OMQ = ∠PMR	…[vertically opposite angles are equal]
Therefore, ΔOQM ≅ ΔPRM	[By AAS]
OQ = PR	[By CPCT]

The perpendicular distances of two chords in two congruent circles are equal,
Therefore, chords are also equal
So, AB = CD
Hence proved