

1. Solve for x in the following in equations, if the replacement set is $N < 10$:

(i) $x + 5 > 11$

Solution:-

$$x + 5 > 11$$

By transposing we get,

$$x > 11 - 5$$

$$x > 6$$

As per the condition given in the question, $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{7, 8, 9\}$

(ii) $2x + 1 < 17$

Solution:-

$$2x + 1 < 17$$

By transposing we get,

$$2x < 17 - 1$$

$$x < 16/2$$

$$x < 8$$

As per the condition given in the question, $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{1, 2, 3, 4, 5, 6, 7\}$

(iii) $3x - 5 \leq 7$

Solution:-

$$3x - 5 \leq 7$$

By transposing we get,

$$3x \leq 7 + 5$$

$$x \leq 12/3$$

$$x \leq 4$$

As per the condition given in the question, $\{x : x \in N; N < 10\}$

Therefore, solution set $x = \{1, 2, 3, 4\}$

(iv) $8 - 3x \geq 2$

Solution:-

$$8 - 3x \geq 2$$

By transposing we get,

$$3x \geq 8 - 2$$

$$3x \geq 6$$

$$x \geq 6/3$$

$$x \geq 2$$

As per the condition given in the question, $\{x : x \in \mathbb{N}; N < 10\}$

Therefore, solution set $x = \{1, 2\}$

(v) $5 - 2x < 11$

Solution:-

$$5 - 2x < 11$$

By transposing we get,

$$2x > 5 - 11$$

$$2x > -6$$

$$x > -6/2$$

$$x > -3$$

As per the condition given in the question, $\{x : x \in \mathbb{N}; N < 10\}$

Therefore, solution set $x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

2. Solve for x in the following in-equations, if the replacement set is \mathbb{R} ;

(i) $3x > 12$

Solution:-

$$3x > 12$$

By cross multiplication we get,

$$x > 12/3$$

$$x > 4$$

As per the condition given in the question, the replacement set is \mathbb{R} .

Therefore, solution set $x = \{x : x \in \mathbb{R}; x > 4\}$

(ii) $2x - 3 > 7$

Solution:-

$$2x - 3 > 7$$

By transposing we get,

$$2x > 7 + 3$$

$$2x > 10$$

$$x > 10/2$$

$$x > 5$$

As per the condition given in the question, the replacement set is \mathbb{R} .

Therefore, solution set $x = \{x : x \in \mathbb{R}; x > 5\}$

(iii) $3x + 2 \leq 11$

Solution:-

$$3x + 2 \leq 11$$

By transposing we get,

$$3x \leq 11 - 2$$

$$3x \leq 9$$

$$x \leq 9/3$$

$$x \leq 3$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in R; x \leq 3\}$

(iv) $14 - 3x \geq 5$

Solution:-

$$14 - 3x \geq 5$$

By transposing we get,

$$3x \leq 14 - 5$$

$$3x \leq 9$$

$$x \leq 9/3$$

$$x \leq 3$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in R; x \leq 3\}$

(v) $7x + 11 > 16 - 3x$

Solution:-

$$7x + 11 > 16 - 3x$$

By transposing we get,

$$7x + 3x > 16 - 11$$

$$10x > 5$$

$$x > 5/10$$

$$x > \frac{1}{2}$$

$$x > 0.5$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in R; x > 0.5\}$

(vi) $3x + 25 > 8x - 10$

Solution:-

$$3x + 25 > 8x - 10$$

By transposing we get,

$$8x - 3x < 25 + 10$$

$$5x < 35$$

$$x < 35/5$$

$$x < 7$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in \mathbb{R}; x < 7\}$

(vii) $2(3x - 5) \leq 8$

Solution:-

$$2(3x - 5) \leq 8$$

$$6x - 10 \leq 8$$

By transposing we get,

$$6x \leq 8 + 10$$

$$6x \leq 18$$

$$x \leq 18/6$$

$$x \leq 3$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in \mathbb{R}; x \leq 3\}$

(viii) $x + 7 \geq 15 + 3x$

Solution:-

$$x + 7 \geq 15 + 3x$$

By transposing we get,

$$3x - x \leq 7 - 15$$

$$2x \leq -8$$

$$x \leq -8/2$$

$$x \leq -4$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in \mathbb{R}; x \leq -4\}$

(ix) $2x - 7 \geq 5x + 8$

Solution:-

$$2x - 7 \geq 5x + 8$$

By transposing we get,

$$5x - 2x \leq -8 - 7$$

$$3x \leq -15$$

$$x \leq -15/3$$

$$x \leq -5$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in \mathbb{R}; x \leq -5\}$

(x) $9 - 4x \leq 15 - 7x$

Solution:-

$$9 - 4x \leq 15 - 7x$$

By transposing we get,

$$7x - 4x \leq 15 - 9$$

$$3x \leq 6$$

$$x \leq 6/3$$

$$x \leq 2$$

As per the condition given in the question, the replacement set is R.

Therefore, solution set $x = \{x : x \in \mathbb{R}; x \leq 2\}$

3. Solve for x: $6 - 10x < 36$, $x \in \{-3, -2, -1, 0, 1, 2\}$

Solution:-

From the question it is given that,

$$6 - 10x < 36$$

So, by transposing we get,

$$-10x < 36 - 6$$

$$-10x < 30$$

$$10x > -30$$

$$x > -30/10$$

$$x > -3$$

As per the condition given in the question, $x \in \{-3, -2, -1, 0, 1, 2\}$.

Therefore, solution set $x = \{-2, -1, 0, 1, 2\}$

4. Solve for x: $3 - 2x \geq x - 12$, $x \in \mathbb{N}$

Solution:-

From the question it is given that,

$$3 - 2x \geq x - 12$$

So, by transposing we get,

$$2x + x \leq 12 + 3$$

$$3x \leq 15$$

$$3x \leq 15$$

$$x \leq 15/3$$

$$x \leq 5$$

As per the condition given in the question, $x \in \mathbb{N}$.

Therefore, solution set $x = \{1, 2, 3, 4, 5\}$

5. Solve for x : $5x - 9 \leq 15 - 7x$, $x \in \mathbb{W}$.

Solution:-

From the question it is given that,

$$5x - 9 \leq 15 - 7x$$

So, by transposing we get,

$$5x + 7x \leq 15 + 9$$

$$12x \leq 24$$

$$x \leq 24/12$$

$$x \leq 2$$

As per the condition given in the question, $x \in \mathbb{W}$.

Therefore, solution set $x = \{0, 1, 2\}$

6. Solve for x : $7 + 5x > x - 13$, where x is a negative integer.

Solution:-

From the question it is given that,

$$7 + 5x > x - 13$$

So, by transposing we get,

$$5x - x > -13 - 7$$

$$4x > -20$$

$$x > -20/4$$

$$x > -5$$

As per the condition given in the question, x is a negative integer.

Therefore, solution set $x = \{-4, -3, -2, -1\}$

7. Solve for x : $5x - 14 < 18 - 3x$, $x \in \mathbb{W}$.

Solution:-

From the question it is given that,

$$5x - 14 < 18 - 3x$$

So, by transposing we get,

$$5x + 3x < 18 + 14$$

$$8x < 32$$

$$x < 32/8$$

$$x < 4$$

As per the condition given in the question, x is $x \in W$.
Therefore, solution set $x = \{0, 1, 2, 3\}$

8. Solve for x : $2x + 7 \geq 5x - 14$, where x is a positive prime number.

Solution:-

From the question it is given that,

$$2x + 7 \geq 5x - 14$$

So, by transposing we get,

$$5x - 2x \leq 14 + 7$$

$$3x \leq 21$$

$$3x \leq 21$$

$$x \leq 21/3$$

$$x \leq 7$$

As per the condition given in the question, x is a positive prime number.
Therefore, solution set $x = \{2, 3, 5, 7\}$

9. Solve for x : $x/4 + 3 \leq x/3 + 4$, where x is a negative odd number.

Solution:-

From the question it is given that,

$$x/4 + 3 \leq x/3 + 4$$

So, by transposing we get,

$$x/4 - x/3 \leq 4 - 3$$

$$(3x - 4x)/12 \leq 1$$

$$-x \leq 12$$

$$x \geq -12$$

As per the condition given in the question, x is a negative odd number.
Therefore, solution set $x = \{-11, -9, -7, -5, -3, -1\}$

10. Solve for x : $(x + 3)/3 \leq (x + 8)/4$, where x is a positive even number.

Solution:-

From the question it is given that,

$$(x + 3)/3 \leq (x + 8)/4$$

So, by cross multiplication we get,

$$4(x + 3) \leq 3(x + 8)$$

$$4x + 12 \leq 3x + 24$$

Now, transposing we get

$$4x - 3x \leq 24 - 12$$

$$x \leq 12$$

As per the condition given in the question, x is a positive even number.

Therefore, solution set $x = \{2, 4, 6, 8, 10, 12\}$

11. If $x + 17 \leq 4x + 9$, find the smallest value of x , when:

(i) $x \in \mathbb{Z}$

Solution:-

From the question,

$$x + 17 \leq 4x + 9$$

So, by transposing we get,

$$4x - x \geq 17 - 9$$

$$3x \geq 8$$

$$x \geq 8/3$$

As per the condition given in the question, $x \in \mathbb{Z}$.

Therefore, smallest value of $x = \{3\}$

(ii) $x \in \mathbb{R}$

Solution:-

From the question,

$$x + 17 \leq 4x + 9$$

So, by transposing we get,

$$4x - x \geq 17 - 9$$

$$3x \geq 8$$

$$x \geq 8/3$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, smallest value of $x = \{2\frac{2}{3}\}$

12. If $(2x + 7)/3 \leq (5x + 1)/4$, find the smallest value of x , when:

(i) $x \in \mathbb{R}$

Solution:-

From the question,

$$(2x + 7)/3 \leq (5x + 1)/4$$

So, by cross multiplication we get,

$$4(2x + 7) \leq 3(5x + 1)$$

$$8x + 28 \leq 15x + 3$$

Now transposing we get,

$$15x - 8x \geq 28 - 3$$

$$7x \geq 25$$

$$x \geq 25/7$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, smallest value of $x = \{3\frac{4}{7}\}$

(ii) $x \in \mathbb{Z}$

Solution:-

From the question,

$$(2x + 7)/3 \leq (5x + 1)/4$$

So, by cross multiplication we get,

$$4(2x + 7) \leq 3(5x + 1)$$

$$8x + 28 \leq 15x + 3$$

Now transposing we get,

$$15x - 8x \geq 28 - 3$$

$$7x \geq 25$$

$$x \geq 25/7$$

As per the condition given in the question, $x \in \mathbb{Z}$.

Therefore, smallest value of $x = \{7\}$

13. Solve the following linear in-equations and graph the solution set on a real number line.

(i) $2x - 11 \leq 7 - 3x$, $x \in \mathbb{N}$.

Solution:-

$$2x - 11 \leq 7 - 3x$$

By transposing we get,

$$2x + 3x \leq 7 + 11$$

$$5x \leq 18$$

$$x \leq 18/5$$

$$x \leq 3.6$$

As per the condition given in the question, $x \in \mathbb{N}$.

Therefore, solution set $x = \{1, 2, 3\}$

Set can be represented in number line as,



(ii) $3(5x + 3) \geq 2(9x - 17)$, $x \in W$.

Solution:-

From the question it is given that,

$$3(5x + 3) \geq 2(9x - 17)$$

$$15x + 9 \geq 18x - 34$$

So, by transposing we get,

$$18x - 15x \leq 34 + 9$$

$$3x \leq 43$$

$$x \leq 43/3$$

As per the condition given in the question, $x \in W$.

Therefore, solution set $x \leq 43/3$

Set can be represented in number line as,



(iii) $2(3x - 5) > 5(13 - 2x)$, $x \in W$.

Solution:-

From the question it is given that,

$$2(3x - 5) > 5(13 - 2x)$$

$$6x - 10 > 65 - 10x$$

So, by transposing we get,

$$6x + 10x > 65 + 10$$

$$16x > 75$$

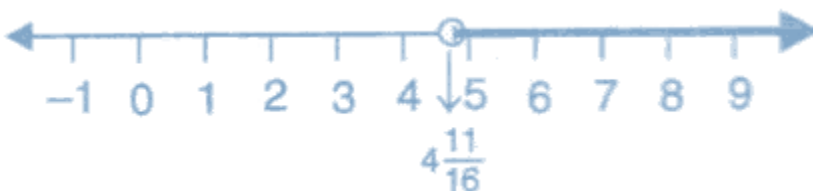
$$x > 75/16$$

$$x > 4\frac{11}{16}$$

As per the condition given in the question, $x \in W$.

Therefore, solution set $x > 4\frac{11}{16}$

Set can be represented in number line as,



(iv) $3x - 9 \leq 4x - 7 < 2x + 5$, $x \in R$.

Solution:-

From the question,

$$\text{Consider } 3x - 9 \leq 4x - 7$$

So, by transposing we get,

$$4x - 3x \geq -9 + 7$$

$$x \geq -2$$

Now, consider $4x - 7 < 2x + 5$

By transposing we get,

$$4x - 2x < 5 + 7$$

$$2x < 12$$

$$x < 12/2$$

$$x < 6$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, solution set = $[-2 \leq x < 6]$

Set can be represented in number line as,



(v) $2x - 7 < 5x + 2 \leq 3x + 14$, $x \in \mathbb{R}$.

Solution:-

From the question,

$$\text{Consider } 2x - 7 < 5x + 2$$

By transposing we get,

$$5x - 2x > -7 - 2$$

$$3x < -9$$

$$x < -9/3$$

$$x < -3$$

Now, consider $5x + 2 \leq 3x + 14$

So, by transposing we get,

$$5x - 3x \leq 14 - 2$$

$$2x \leq 12$$

$$x \leq 12/2$$

$$x \leq 6$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, solution set = $[-3 < x \leq 6]$

Set can be represented in number line as,



(vi) $-3 \leq \frac{1}{2} - \frac{2x}{3} \leq 2\frac{2}{3}$, $x \in \mathbb{N}$.

Solution:-

From the question,

Consider $-3 \leq \frac{1}{2} - \frac{2x}{3}$

$-3 \leq \frac{3 - 4x}{6}$

$-18 \leq (3 - 4x)$

So, by transposing we get,

$-18 - 3 \leq -4x$

$-21 \leq -4x$

$x \leq 21/4$

$x \leq 5\frac{1}{4}$

Now, consider $\frac{1}{2} - \frac{2x}{3} \leq 2\frac{2}{3}$

$\frac{3 - 4x}{6} \leq 8/3$

By cross multiplication we get,

$3(3 - 4x) \leq 48$

$9 - 12x \leq 48$

By transposing we get,

$-12x \leq 48 - 9$

$-12x \leq 39$

$12x \geq -39$

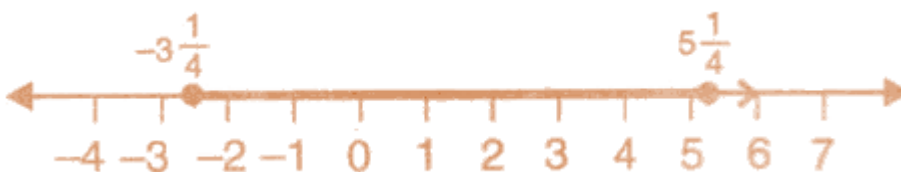
$x \geq -39/12$

$x \geq -3\frac{1}{4}$

As per the condition given in the question, $x \in \mathbb{N}$.

Therefore, solution set = $[-3\frac{1}{4} \leq x \leq 5\frac{1}{4}]$

Set can be represented in number line as



(vii) $4\frac{3}{4} \geq x + 5/6 > 1/3$, $x \in \mathbb{R}$

Solution:-

From the question,

Consider, $4\frac{3}{4} \geq x + 5/6$

$$19/4 \geq (6x + 5)/6$$

$$114 \geq 24x + 20$$

By transposing we get,

$$114 - 20 \geq 24x$$

$$94 \geq 24x$$

$$x \leq 94/24$$

$$x \leq 3\frac{11}{12}$$

Now, consider $x + 5/6 > 1/3$

$$(6x + 5)/6 > 1/3$$

$$18x + 15 > 6$$

By transposing we get,

$$18x > 6 - 15$$

$$18x > -9$$

$$x > -9/18$$

$$x > -\frac{1}{2}$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, solution set = $[-\frac{1}{2} < x \leq 3\frac{11}{12}]$

Set can be represented in number line as



(viii) $\frac{1}{3}(2x - 1) < \frac{1}{4}(x + 5) < \frac{1}{6}(3x + 4)$, $x \in \mathbb{R}$.

Solution:-

From the question it is given that,

Consider $\frac{1}{3}(2x - 1) < \frac{1}{4}(x + 5)$

By cross multiplication we get,

$$4(2x - 1) < 3(x + 5)$$

$$8x - 4 < 3x + 15$$

By transposing we get,

$$8x - 3x < 15 + 4$$

$$5x < 19$$

$$x < 19/5$$

$$x < 3\frac{4}{5}$$

Then, consider $\frac{1}{4}(x + 5) < \frac{1}{6}(3x + 4)$

$$6(x + 5) < 4(3x + 4)$$

$$6x + 30 < 12x + 16$$

By transposing we get,

$$6x - 12x < 16 - 30$$

$$-6x < -14$$

$$x > 2\frac{1}{3}$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, solution set = $[2\frac{1}{3} < x < 3\frac{4}{5}]$

Set can be represented in number line as



(ix) $\frac{1}{3}(5x - 8) \geq \frac{1}{2}(4x - 7)$, $x \in \mathbb{R}$.

Solution:-

From the question it is given that,

$$\frac{1}{3}(5x - 8) \geq \frac{1}{2}(4x - 7)$$

By cross multiplication we get,

$$2(5x - 8) \geq 3(4x - 7)$$

$$10x - 16 \geq 12x - 21$$

Transposing we get,

$$12x - 10x \leq 21 - 16$$

$$2x \leq 5$$

$$x \leq \frac{5}{2}$$

$$x \leq 2\frac{1}{2}$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, solution set = $\{x < -8\}$

Set can be represented in number line as



(x) $\frac{5}{4}x > 1 + \frac{1}{3}(4x - 1)$, $x \in \mathbb{R}$.

Solution:-

From the question,

Consider, $(\frac{5}{4})x > 1 + \frac{1}{3}(4x - 1)$

$$(5/4)x > (3 + (4x - 1)/3)$$

$$15x > 12 + 16x - 4$$

By transposing we get,

$$15x - 16x > 8$$

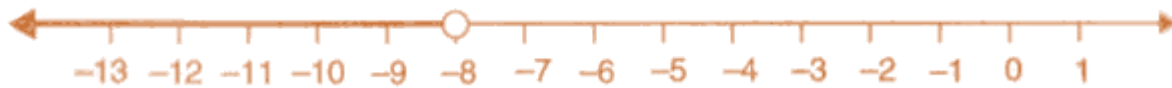
$$-x > 8$$

$$x < -8$$

As per the condition given in the question, $x \in \mathbb{R}$.

Therefore, solution set = $\{x < -8\}$

Set can be represented in number line as,



14. If $P = \{x : -3 < x \leq 7, x \in \mathbb{R}\}$ and $Q = \{x : -7 \leq x < 3, x \in \mathbb{R}\}$, represent the following solution set on the different number lines:

(i) $P \cap Q$

(ii) $Q' \cap P$

(iii) $P - Q$

Solution:-

As per the condition given in the question,

$$P = \{x : -3 < x \leq 7, x \in \mathbb{R}\}$$

$$\text{So, } P = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$$

$$\text{Then, } Q = \{x : -7 \leq x < 3, x \in \mathbb{R}\}$$

$$Q = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}$$

$$\begin{aligned} \text{(i) } P \cap Q &= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\} \cap \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2\} \\ &= \{-2, -1, 0, 1, 2\} \end{aligned}$$



(ii) $Q' \cap P$

$$Q' = \{3, 4, 5, 6, 7\}$$

$$\begin{aligned} Q' \cap P &= \{3, 4, 5, 6, 7\} \cap \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\} \\ &= \{3, 4, 5, 6, 7\} \end{aligned}$$



(iii) $P - Q$

$$P - Q = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\} - \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}$$

$$= \{3, 4, 5, 6, 7\}$$



15. If $P = \{x : 7x - 4 > 5x + 2, x \in \mathbb{R}\}$ and $Q = \{x : x - 19 \geq 1 - 3x, x \in \mathbb{R}\}$, represent the following solution set on the different number lines:

(i) $P \cap Q$

(ii) $P' \cap Q$

Solution:-

As per the condition given in the question,

$$P = \{x : 7x - 4 > 5x + 2, x \in \mathbb{R}\}$$

$$7x - 4 > 5x + 2$$

By transposing we get,

$$7x - 5x > 4 + 2$$

$$2x > 6$$

$$x > 6/2$$

$$x > 3$$

$$\text{Therefore, } P = \{4, 5, 6, 7, \dots\}$$

$$Q = \{x : x - 19 \geq 1 - 3x, x \in \mathbb{R}\}$$

$$x - 19 \geq 1 - 3x$$

By transposing we get,

$$x + 3x \geq 1 + 19$$

$$4x \geq 20$$

$$x \geq 20/4$$

$$x \geq 5$$

$$Q = \{5, 6, 7, 8, \dots\}$$

Then,

$$(i) P \cap Q = \{2, 3, 4, 5, \dots\} \cap \{5, 6, 7, 8, \dots\}$$

$$= \{5, 6, 7, 8, \dots\}$$



$$(ii) P' \cap Q = \{\emptyset\}$$

