1. The sum of the square of the 2 consecutive natural numbers is 481. Find the numbers.

Solution:-

let us assume the two consecutive number be P and P + 1

As per the condition given in the question, sum of the square of the 2 consecutive natural numbers is 481, $P^2 + (P + 1)^2 = 481$

Then,

$P^2 + (P + 1)^2 = 481$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$P^2 + P^2 + 2P + 1 = 481$

$2P^2 + 2P + 1 = 481$

By transposing we get,

$2P^2 + 2P + 1 – 481 = 0$

$2P^2 + 2P – 480 = 0$

Divide by 2 for both side of each term we get,

$2P^2/2 + 2P/2 – 480/2 = 0/2$

$P^2 + P – 240 = 0$

$P^2 + 16P – 15P – 240 = 0$

Take out common in each terms,

$P(P + 16) – 15(P + 15) = 0$

$(P + 16)(P – 15) = 0$

Equate both to zero,

$P + 16 = 0$, $P – 15 = 0$

$P = -16$, $P = 15$

So, P = 15 … [because – 16 is not a natural number]

Then,

P = 15

P + 1 = 15 + 1 = 16

Therefore, the 2 consecutive numbers are 15 and 16.

2. The sum of 2 numbers is 18. If the sum of their reciprocals is ¼, find the numbers.

Solution:-

Let us assume the numbers be P and Q.

As per the condition given in the question,

The sum of 2 numbers is 18, P + Q = 18 … [equation i]

the sum of their reciprocals is ¼, 1/P + 1/Q = ¼ … [equation ii]

Consider equation (i), P + Q = 18

P = 18 – Q
Now, substitute the value of P in equation (ii) we get,
1/(18 - Q) + 1/Q = ¼
(Q +(18 - Q))/((18 - Q)Q) = ¼
By cross multiplication,
4(Q + 18 - Q) = (18 - Q)Q
$4(18) = 18Q – Q^2$
$72 = 18Q – Q^2$
Now, transposing we get,
$Q^2 – 18Q + 72 = 0$
$Q^2 – 12Q – 6Q + 72 = 0$
Take out common in each terms,
Q(Q - 12) – 6(Q - 12) = 0
(Q - 12) (Q - 6) = 0
Equate both to zero,
Q – 12 = 0, Q – 6 = 0
Q = 12, Q = 6
Therefore, the numbers are 6 and 12.

3. The sum of a number and its reciprocal is $2\frac{9}{40}$. Find the number.
Solution:-
Let us assume the number be B.
As per the condition given in the question, B + 1/B = $2\frac{9}{40}$
So, B + 1/B = 89/40
$(B^2 + 1)/B = 89/40$
Cross multiplication we get,
$B^2 + 1 = (89/40)B$
By transposing we get,
$B^2 + 1 – (89/40)B = 0$
Multiply by 40 for both side of each term we get,
$40B^2 + 40 – 89B = 0$
$40B^2 – 89B + 40 = 0$
$40B^2 – 64B – 25B + 40 = 0$
Take out common in each terms,
8B(5B - 8) – 5(5B - 8) = 0
(5B - 8) (8B - 5) = 0

Equate both to zero,
5B – 8 = 0, 8B – 5 = 0
5B = 8, 8B = 5
B = 8/5, B = 5/8
Therefore, the numbers are 5/8 and 8/5.

4. A two digit number is such that its product of its digit is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Solution:-

let us assume two digits be PQ.
As per the condition given in the question,
Product of 2 digits is 18, PQ = 18 … [equation (i)]
63 is subtracted from the number, the digits interchange their places,
PQ – 63 = QP … [equation (ii)]
Now, assume two-digit number be PQ, means P = 10p (as it comes in tens digit)
Then,
PQ – 63 = QP
10p + q – 63 = 10q + p
By transposing we get,
9p – 9q – 63 = 0
Divide both side by 9,
P – Q – 7 = 0 … [equation (iii)]
So, PQ =18
P = 18/Q
Substitute the value of P in equation (iii),
(18/Q) – Q – 7 = 0
$18 - Q^2 - 7Q = 0$
$Q^2 + 7Q - 18 = 0$
$Q^2 + 9Q - 2Q - 18 = 0$
Q(Q + 9) – 2(Q + 9) = 0
Q + 9 = 0, Q – 2 = 0
Q = -9, Q = 2
Therefore, Q = 2 … [because value of Q can’t be negative]
Then, P = 18/Q
P = 18/2
P = 9
Therefore, the number is 92.

5. A two digit number is such that the product of the digits is 14. When 45 is added to the number, then the digit are reversed. Find the number.

Solution:-

Let us assume the two digits be MN

As per the condition given in the question,

Product of 2 digits is 18, MN = 14 … [equation (i)]

45 is added to the number, then the digit are reversed,

MN + 45 = NM … [equation (ii)]

So, from equation (i) possible combinations are 27 and 72.

Then, from equation (ii) it is clear that number MN is less than NM, we are adding 45 to MN which makes it equal to NM.

Therefore, the number 27 is added by 45, we get a number 72.

72 is a number where the digits are interchanged.

Therefore, the number is 27.

6. Divide 25 into two parts such that twice the square of the larger part exceeds thrice the square of the smaller part by 29.

Solution:-

Let us assume the two numbers be A and B, B being the bigger number.

As per the condition given in the question,

A + B = 25 … [equation (i)]

$2B^2 = 3A^2 + 29$ … [equation (ii)]

Now, consider the equation (i), A + B = 25

B = 25 – A

Then, substitute the value of B is equation (ii),

$2(25 - A)^2 = 3A^2 + 29$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$

$2(25^2 - (2 \times 25 \times A) + A^2) = 3A^2 + 29$

$1250 + 2A^2 – 100A = 3A^2 + 29$

$A^2 + 100A – 1221 = 0$

$A^2 – 11A + 111A – 1221 = 0$

Take out common in each terms,

$A(A - 11) + 111(A - 11) = 0$

$(A - 11)(A + 111) = 0$

Equate both to zero,

$A – 11 = 0, A + 111 = 0$

A = 11, A = -111
Therefore, A = 11 … [because A can't be negative]
So, B = 25 – A
= 25 – 11
= 14

7. The sum of the square of 2 positive integers is 208. If the square of larger number is 18 times the smaller number, find the numbers.
Solution:-
Let us assume the two numbers be, P and Q.
As per the condition given in the question,
$P^2 + Q^2 = 208$ … [equation (i)]
$Q^2 = 18P$ … [equation (ii)]
Consider the equation (i), $P^2 + Q^2 = 208$
$Q^2 = 208 – P^2$
Now, substitute the value of Q^2 in equation (ii) we get,
$208 – P^2 = 18P$
By transposing we get,
$P^2 + 18P – 208 = 0$
$P^2 + 26P – 8P – 208 = 0$
Take out common in each terms,
$P(P + 26) – 8(P + 26) = 0$
$(P + 26) (P - 8) = 0$
Equate both to zero,
$P + 26 = 0$, $P – 8 = 0$
$P = - 26$, $P = 8$
Therefore, P = 8 … [because P can't be negative]
So, $Q^2 = 18P$
$Q^2 = 18 \times 8$
$Q^2 = 144$
$Q = \sqrt{144}$
$Q = 12$

8. The difference of the square of two natural numbers is 45. The square of the smaller number is 4 times the larger number. Determine the numbers.
Solution:-
Let us assume the two numbers be, P and Q, Q being the bigger number.

As per the condition given in the question,

$Q^2 - P^2 = 45$ … [equation (i)]

$P^2 = 4Q$ … [equation (ii)]

Consider the equation (i), $Q^2 - P^2 = 45$

$P^2 = Q^2 - 45$

Now, substitute the value of P^2 in equation (ii) we get,

$Q^2 – 45 = 4Q$

By transposing we get,

$Q^2 - 4Q – 45 = 0$

$Q^2 – 9Q + 5Q – 45 = 0$

Take out common in each terms,

$Q(Q - 9) + 5(Q - 9) = 0$

$(Q - 9) (Q + 5) = 0$

Equate both to zero,

$Q – 9 = 0, Q + 5 = 0$

$Q = 9, Q = -5$

Therefore, $Q = 9$ … [because Q can't be negative]

So, $P^2 = 4Q$

$P^2 = 4 \times 9$

$P^2 = 36$

$P = \sqrt{36}$

$P = 6$

9. A two digit number is four times the sum and 3 times the product of its digits, find the number.

Solution:-

Let us assume the two numbers be, P and Q,

Then, $P = 10P$ … [because it comes in tens digit]

As per the condition given in the question,

$PQ = 4(P + Q)$

$4(P + Q) = 10P + Q$ … [equation (i)]

$PQ = 3(PQ) = 10P + Q$ … [equation (ii)]

Consider the equation (i), $4(P + Q) = 10P + Q$

$4P + 4Q = 10P + Q$

$10P – 4P = 4Q – Q$

$6P = 3Q$

$P = 3Q/6$

$P = Q/2$

Now, substitute the value of P in equation (ii) we get,

$3(PQ) = 10P + Q$

$(3Q/2) \times Q = (10Q/2) + Q$

$3Q^2/2 = 5Q + Q$

$3Q^2/2 = 6Q$

By cross multiplication we get,

$3Q^2 = 12Q$

By simplification,

$Q = 4$

So, $P = Q/2 = 2$

Therefore, the number is 24.

10. Two natural numbers differ by 4. If the sum of their square is 656, find the numbers.

Solution:-

Let us assume the two numbers be, P and Q.

As per the condition given in the question,

$P^2 + Q^2 = 656$ … [equation (i)]

$P - Q = 4$

$P = 4 + Q$ … [equation (ii)]

Now, substitute the equation (ii) in equation (i) we get,

$(4 + Q)^2 + Q^2 = 656$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$(4^2 + (2 \times 4 \times Q) + Q^2) + Q^2 = 656$

$16 + 8Q + Q^2 + Q^2 = 656$

$16 + 8Q + 2Q^2 = 656$

By transposing we get,

$2Q^2 + 8Q + 16 – 656 = 0$

$2Q^2 + 8Q – 640 = 0$

Divide both side by 2 we get,

$2Q^2/2 + 8Q/2 – 640/2 = 0/2$

$Q^2 + 4Q – 320 = 0$

$Q^2 + 20Q – 16Q – 320 = 0$

Take out common in each terms,

$Q(Q + 20) – 16(Q + 20) = 0$

$(Q + 20) (Q - 16) = 0$

Equate both to zero,

Q + 20 = 0, Q – 16 = 0

Q = - 20, Q = 16

Q = 16 … [because Q can't be negative]

So, P = 4 + Q

P = 4 + 16

P = 20

Therefore, the numbers are 16 and 20.

11. The sum of the square of 2 consecutive odd positive integers is 290. Find them.

Solution:-

let us assume the two consecutive odd positive number be P and P + 2

As per the condition given in the question, $P^2 + (P + 2)^2 = 290$

Then,

$P^2 + (P + 2)^2 = 290$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$P^2 + P^2 + 4P + 4 = 290$

$2P^2 + 4P + 4 = 290$

By transposing we get,

$2P^2 + 4P + 4 – 290 = 0$

$2P^2 + 4P – 286 = 0$

Divide by 2 for both side of each term we get,

$2P^2/2 + 4P/2 – 286/2 = 0/2$

$P^2 + 2P – 143 = 0$

$P^2 + 13P – 11P – 143 = 0$

Take out common in each terms,

P(P + 13) – 11(P + 13) = 0

(P + 13) (P – 11) = 0

Equate both to zero,

P + 13 = 0, P – 11 = 0

P = - 13, P = 11

So, P = 11 … [because – 16 is a negative even integer]

Then,

P = 11

P + 2 = 11 + 2 = 11

Therefore, the 2 consecutive positive odd integers are 11 and 13.

12. Three consecutive natural numbers are such that the square of the first increased by the product of other two gives 154. Find the numbers.

Solution:-

let us assume the three consecutive natural number be P – 1, P and P + 1

As per the condition given in the question, $(P - 1)^2 + (P)(P + 1) = 154$

Then,

$(P - 1)^2 + P^2 + P = 154$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$

$P^2 - 2P + 1 + P^2 + P = 154$

$2P^2 - P + 1 = 154$

By transposing we get,

$2P^2 - P + 1 - 154 = 0$

$2P^2 - P - 153 = 0$

$2P^2 - 18P - 17P - 153 = 0$

Take out common in each terms,

$2P(P - 9) - 17(P - 9) = 0$

$(P - 9)(2P - 17) = 0$

Equate both to zero,

$P - 9 = 0$, $2P - 17 = 0$

$P = 9$, $2P = 17$

$P = 9$, $P = 17/2$

So, $P = 9$ … [because 9 is a natural number]

Then,

$P = 9$

$P - 1 = 9 - 1 = 8$

$P + 1 = 9 + 1 = 10$

Therefore, the three consecutive natural number are 8, 9 and 10.

13. The sum of the square of two numbers is 233. If one of the numbers is 3 less than twice the other number. Find the numbers.

Solution:-

let us assume the two numbers be y, 2y - 3

As per the condition given in the question, $y^2 + (2y - 3)^2 = 233$

Then,

$y^2 + (2y - 3)^2 = 233$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$

$y^2 + 4y^2 - 12y + 9 = 233$

$5y^2 – 12y + 9 = 233$
By transposing we get,
$5y^2 – 12y + 9 - 233= 0$
$5y^2 – 12y - 224 = 0$
$5y^2 – 40y + 28y – 224 = 0$
Take out common in each terms,
$5y(y - 8) + 28(y - 8) = 0$
$(y - 8) (5y + 28) = 0$
Equate both to zero,
$y – 8 = 0, 5y + 28 = 0$
$y = 8, 5y = - 28$
$y = 8, y = - 28/5$
So, $y = 8$ … [because 8 is a natural number]
Then,
$y = 8$
$2y - 3 = 2(8) – 3$
$= 16 – 3$
$= 13$
Therefore, the 2 numbers are 8 and 13.

14. Find two natural numbers which differ by 3 and whose squares have the sum of 117.
Solution:-
let us assume the two natural numbers be y, y - 3
As per the condition given in the question, $y^2 + (y - 3)^2 = 117$
Then,
$y^2 + (y - 3)^2 = 117$
We know that, $(a - b)^2 = a^2 - 2ab + b^2$
$y^2 + y^2 – 6y + 9 = 117$
$2y^2 – 6y + 9 = 117$
By transposing we get,
$2y^2 – 6y + 9 – 117 = 0$
$2y^2 – 6y - 108 = 0$
Divide by 2 for both side of each term we get,
$2y^2/2 – 6y/2 – 108/2 = 0/2$
$y^2 – 3y – 54 = 0$
$y^2 – 9y + 6y – 54 = 0$

Take out common in each terms,
$y(y - 9) + 6(y - 9) = 0$
$(y - 9)(y + 6) = 0$
Equate both to zero,
$y - 9 = 0, y + 6 = 0$
$y = 9, y = -6$
$y = 9, y = -6$
So, $y = 9$ … [because 9 is a natural number]
Then,
$y = 9$
$y - 3 = 9 - 3 = 6$
Therefore, the 2 natural numbers are 6 and 9.

15. A two digit number is such that the product of its digit is 8. When 18 is subtracted from the number, the digits interchange its place. Find the numbers.
Solution:-
Let us assume the two numbers be, P and Q,
Then, $P = 10P$ … [because it comes in tens digit]
As per the condition given in the question,
$PQ = 8$ … [equation (i)]
$10P + Q - 18 = 10Q + P$
$10P - P + Q - 10Q - 18 = 0$
$9P - 9Q - 18 = 0$
Divide by 2 for both side of each term we get,
$9P/9 - 9Q/9 - 18/9 = 0/9$
$P - Q - 2 = 0$ … [equation (ii)]
Consider the equation (i), $PQ = 8$
$P = 8/Q$

Now, substitute the value of P in equation (ii) we get,
$8/Q - Q - 2 = 0$
By simplification,
$8 - Q^2 - 2Q = 0$
$Q^2 + 2Q - 8 = 0$
$Q^2 + 4Q - 2Q - 8 = 0$
Take out common in each terms,
$Q(Q + 4) - 2(Q + 4) = 0$

(Q + 4) (Q - 2)
Equate both to zero,
Q + 4 = 0, Q – 2= 0
Q = - 4, Q = 2
So, Q = 2
From equation (i), PQ = 8
2P = 8
P = 8/2
P = 4
Therefore, the number is 42.