EXERCISE 16.1

1. Find the Slopes of the tangent and the normal to the following curves at the indicated points:

(i) $y = \sqrt{x^3}$ at $x = 4$

Solution:

Given $y = \sqrt{x^3}$ at $x = 4$

First, we have to find $\frac{dy}{dx}$ of given function, f(x) that is to find the derivative of f (x)

$y = \sqrt{x^3}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow y = (x^3)^{\frac{1}{2}}$

$\Rightarrow y = (x)^{\frac{3}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

We know that the Slope of the tangent is $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3}{2}(x)^{\frac{3}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}(x)^{\frac{1}{2}}$$

Since, x = 4

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2}(4)^{\frac{1}{2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times \sqrt{4}$$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times 2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = 3$

The Slope of the tangent at x = 4 is 3

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{x=4}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{3}$

(ii) y = √x at x = 9

Solution:

Given y = $\sqrt{x}$ at x = 9

First, we have to find $\frac{dy}{dx}$ of given function, f(x) that is to find the derivative of f(x)

$\Rightarrow y = \sqrt{x}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow y = (x)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = (x)^{\frac{1}{2}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{-1}{2}}$$

Since, x = 9

$$\left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2}(9)^{\frac{-1}{2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{(9)^{\frac{1}{2}}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{\sqrt{9}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{6}$$

The Slope of the tangent at x = 9 is $\frac{1}{6}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{x=9}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\frac{1}{6}}$

$\Rightarrow$ The Slope of the normal = – 6

(iii) y = x³ – x at x = 2

Solution:

First, we have to find $\frac{dy}{dx}$ of given function f(x) that is to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = x^3 - x$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}(x^3) + 3 \times \frac{dy}{dx}(x)$

$\Rightarrow \frac{dy}{dx} = 3.x^{3-1} - 1.x^{1-0}$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 1$

Since, x = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 3 \times (2)^2 - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = (3 \times 4) - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 12 - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 11$

∴The Slope of the tangent at x = 2 is 11

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{x=2}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{11}$

(iv) $y = 2x^2 + 3 \sin x$ at $x = 0$

Solution:

Given $y = 2x^2 + 3\sin x$ at $x = 0$

First, we have to find $\frac{dy}{dx}$ of given function f(x) that is to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n\, x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = 2x^2 + 3\sin x$

$\Rightarrow \frac{dy}{dx} = 2\, dy/dx\, (x^2) + 3\, dy/dx\, (\sin x)$

$\Rightarrow \frac{dy}{dx} = 2\,(2x^{2-1}) + 3\,(\cos x)$

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\Rightarrow \frac{dy}{dx} = 4x + 3\cos x$

Since, x = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 4\,(0) + 3\cos(0)$

We know cos (0) = 1

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 0 + 3\,(1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 3$

$\therefore$The Slope of the tangent at x = 0 is 3

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{x=0}}$

⇒ The Slope of the normal = $\frac{-1}{3}$

(v) x = a (θ - sin θ), y = a (1 + cos θ) at θ = -π /2

Solution:

Given x = a (θ - sin θ), y = a (1 + cos θ) at θ = -π /2

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get our desired $\frac{dy}{dx}$.

∴ $\frac{dy}{dx}(x^n) = n\,x^{n-1}$

⇒ x = a (θ – sin θ)

⇒ $\frac{dx}{d\theta} = a\,\{\frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin\theta)\}$

⇒ $\frac{dx}{d\theta} = a\,(1 - \cos\theta)$ ... (1)

∴ $\frac{d}{dx}$ (Sin x) = cos x

⇒ y = a (1 + cos θ)

⇒ $\frac{dy}{d\theta} = a\,[\frac{dx}{d\theta}(1) + \frac{dx}{d\theta}(\cos\theta)]$

∴ $\frac{d}{dx}$ (Cos x) = – sin x

∴ $\frac{d}{dx}$ (Constant) = 0

⇒ $\frac{dy}{d\theta} = a\,(0 + (-\sin\theta))$

⇒ $\frac{dy}{d\theta} = a\,(-\sin\theta)$

$\Rightarrow \frac{dy}{d\theta} = -a \sin\theta$ ... (2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{(1-\cos\theta)}$

The Slope of the tangent is $\frac{-\sin\theta}{(1-\cos\theta)}$

Since, $\theta = \frac{-\pi}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{-\sin\frac{-\pi}{2}}{(1-\cos\frac{-\pi}{2})}$

We know Cos (π /2) = 0 and sin (π /2) = 1

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{-(-1)}{(1-(-0))}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{1}{(1-0)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = 1$

∴The Slope of the tangent at x = $-\frac{\pi}{2}$ is 1

⇒ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}}}$

⇒ The Slope of the normal = $\frac{-1}{1}$

⇒ The Slope of the normal = – 1

(vi) $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ at $\theta = \pi /4$

Solution:

Given $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ at $\theta = \pi /4$

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n\, x^{n-1}$

$\Rightarrow x = a\cos^3 \theta$

$\Rightarrow \frac{dx}{d\theta} = a\left(\frac{dx}{d\theta}(\cos^3 \theta)\right)$

$\therefore \frac{d}{dx}(\text{Cos } x) = -\sin x$

$\Rightarrow \frac{dx}{d\theta} = a\,(3\cos^{3-1} \theta \times -\sin \theta)$

$\Rightarrow \frac{dx}{d\theta} = a\,(3\cos^2 \theta \times -\sin \theta)$

$\Rightarrow \frac{dx}{d\theta} = -3a\cos^2 \theta \sin \theta$... (1)

$\Rightarrow y = a\sin^3 \theta$

$\Rightarrow \frac{dy}{d\theta} = a\left(\frac{dy}{d\theta}(\sin^3 \theta)\right)$

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\Rightarrow \frac{dy}{d\theta} = a\,(3\sin^{3-1} \theta \cos \theta)$

$\Rightarrow \frac{dy}{d\theta} = a\,(3\sin^2 \theta \cos \theta)$

$\Rightarrow \frac{dy}{d\theta} = 3\,a \sin^2 \theta \cos \theta$... (2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-\cos\theta}{\sin\theta}$

$\Rightarrow \frac{dy}{dx} = -\tan\theta$

The Slope of the tangent is $-\tan\theta$

Since, $\theta = \pi/4$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -\tan(\pi/4)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -1$

We know $\tan(\pi/4) = 1$

The Slope of the tangent at $x = \pi/4$ is -1

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{-1}$

$\Rightarrow$ The Slope of the normal = 1

(vii) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = \pi/2$

Solution:

Given $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = \pi/2$

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n\, x^{n-1}$

$\Rightarrow x = a\,(\theta - \sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\,\{\frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin\theta)\}$

$\Rightarrow \frac{dx}{d\theta} = a\,(1 - \cos\theta)$ ... (1)

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\Rightarrow y = a\,(1 - \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a\,(\frac{dx}{d\theta}(1) - \frac{dx}{d\theta}(\cos\theta))$

$\therefore \frac{d}{dx}(\text{Cos } x) = -\sin x$

$\therefore \frac{d}{dx}(\text{Constant}) = 0$

$\Rightarrow \frac{dy}{d\theta} = a\,(\theta - (-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = a\sin\theta$ ... (2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{(1-\cos\theta)}$

The Slope of the tangent is $\frac{-\sin\theta}{(1-\cos\theta)}$

Since, $\theta = \frac{\pi}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{\sin\frac{\pi}{2}}{(1-\cos\frac{\pi}{2})}$

We know cos (π /2) = 0 and sin (π /2) = 1

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{(1)}{(1-(-0))}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{(1-0)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 1$

The Slope of the tangent at $x = \frac{\pi}{2}$ is 1

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{1}$

$\Rightarrow$ The Slope of the normal = – 1

(viii) y = (sin 2x + cot x + 2)^2 at x = π /2

Solution:

Given y = (sin 2x + cot x + 2)^2 at x = π /2

First, we have to find $\frac{dy}{dx}$ of given function f(x) that is to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = (\sin 2x + \cot x + 2)^2$

$\frac{dy}{dx} = 2 \times (\sin 2x + \cot x + 2)^{2-1}\{\frac{dy}{dx}(\sin 2x) + \frac{dy}{dx}(\cot x) + \frac{dy}{dx}(2)\}$

$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\ \{(\cos 2x) \times 2 + (-\operatorname{cosec}^2 x) + (0)\}$

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\therefore \frac{d}{dx}(\text{Cot } x) = -\operatorname{cosec}^2 x$

$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\ (2 \cos 2x - \operatorname{cosec}^2 x)$

Since, $x = \pi/2$

$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2 \times (\sin 2\ (\pi/2) + \cot(\pi/2) + 2)\ (2 \cos 2\ (\pi/2) - \operatorname{cosec}^2(\pi/2))$

$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2 \times (\sin(\pi) + \cot(\pi/2) + 2) \times (2\cos(\pi) - \operatorname{cosec}^2(\pi/2))$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2 \times (0 + 0 + 2) \times (2(-1) - 1)$

We know $\sin(\pi) = 0$, $\cos(\pi) = -1$

$\text{Cot}(\pi/2) = 0$, $\operatorname{cosec}(\pi/2) = 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2\ (2) \times (-2 - 1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 4 \times -3$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = -12$

The Slope of the tangent at $x = \frac{\pi}{2}$ is -12

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}}}$

⇒ The Slope of the normal = $\frac{-1}{-12}$

⇒ The Slope of the normal = $\frac{1}{12}$

(ix) $x^2 + 3y + y^2 = 5$ at (1, 1)

Solution:

Given $x^2 + 3y + y^2 = 5$ at (1, 1)

Here we have to differentiate the above equation with respect to x.

$\Rightarrow \frac{d}{dx}(x^2 + 3y + y^2) = \frac{d}{dx}(5)$

$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(3y) + \frac{d}{dx}(y^2) = \frac{d}{dx}(5)$

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

$\Rightarrow 2x + 3 \times \frac{dy}{dx} + 2y \times \frac{dy}{dx} = 0$

$\Rightarrow 2x + \frac{dy}{dx}(3 + 2y) = 0$

$\Rightarrow \frac{dy}{dx}(3 + 2y) = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{(3 + 2y)}$

The Slope of the tangent at (1, 1) is

$$\Rightarrow \frac{dy}{dx} = \frac{-2 \times 1}{(3 + 2 \times 1)}$$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{(3 + 2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{5}$

The Slope of the tangent at (1, 1) is $\frac{-2}{5}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\frac{-2}{5}}$

$\Rightarrow$ The Slope of the normal = $\frac{5}{2}$

(x) x y = 6 at (1, 6)

Solution:

Given xy = 6 at (1, 6)

Here we have to use the product rule for above equation, then we get

$\frac{d}{dx}(x\,y) = \frac{d}{dx}(6)$

$\Rightarrow x \times \frac{d}{dx}(y) + y\frac{d}{dx}(x) = \frac{d}{dx}(5)$

$\therefore \frac{d}{dx}$ (Constant) = 0

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow x\frac{dy}{dx} = -y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

The Slope of the tangent at (1, 6) is

$\Rightarrow \frac{dy}{dx} = \frac{-6}{1}$

$\Rightarrow \frac{dy}{dx} = -6$

The Slope of the tangent at (1, 6) is – 6

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{-6}$

$\Rightarrow$ The Slope of the normal = $\frac{1}{6}$

2. Find the values of a and b if the Slope of the tangent to the curve x y + a x + by = 2 at (1, 1) is 2.

Solution:

Given the Slope of the tangent to the curve xy + ax + by = 2 at (1, 1) is 2

First, we will find The Slope of tangent by using product rule, we get

$\Rightarrow x y + ax + by = 2$

$\Rightarrow x \frac{d}{dx}(y) + y\frac{d}{dx}(x) + a \frac{d}{dx}(x) + b \frac{d}{dx}(y) + = \frac{d}{dx}(2)$

$\Rightarrow x\frac{dy}{dx} + y + a + b\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(x + b) + y + a = 0$

$\Rightarrow \frac{dy}{dx}(x + b) = -(a + y)$

$\Rightarrow \frac{dy}{dx} = \frac{-(a+y)}{x+b}$

Since, the Slope of the tangent to the curve xy + ax + by = 2 at (1, 1) is 2 that is,

$\frac{dy}{dx} = 2$

$\Rightarrow \left\{\frac{-(a+y)}{x+b}\right\}_{(x=1,\, y=1)} = 2$

$\Rightarrow \frac{-(a+1)}{1+b} = 2$

⇒ – a – 1 = 2(1 + b)
⇒ – a – 1 = 2 + 2b
⇒ a + 2b = – 3 ... (1)
Also, the point (1, 1) lies on the curve xy + ax + by = 2, we have
1 × 1 + a × 1 + b × 1 = 2
⇒ 1 + a + b = 2
⇒ a + b = 1 ... (2)
From (1) & (2), we get b = -4
Substitute b = – 4 in a + b = 1
a – 4 = 1
⇒ a = 5
So the value of a = 5 & b = – 4

3. If the tangent to the curve $y = x^3 + a x + b$ at (1, – 6) is parallel to the line x – y + 5 = 0, find a and b

Solution:

Given the Slope of the tangent to the curve $y = x^3 + ax + b$ at (1, – 6)

First, we will find the slope of tangent

$y = x^3 + ax + b$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(ax) + \frac{d}{dx}(b)$

$\Rightarrow \frac{dy}{dx} = 3x^{3-1} + a\left(\frac{dx}{dx}\right) + 0$

$\Rightarrow \frac{dy}{dx} = 3x^2 + a$

The Slope of the tangent to the curve $y = x^3 + ax + b$ at (1, – 6) is

$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3(1)^2 + a$

$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3 + a$ ... (1)

The given line is x – y + 5 = 0
y = x + 5 is the form of equation of a straight line y = mx + c, where m is the Slope of the line.
So the slope of the line is y = 1 × x + 5
So the Slope is 1. ... (2)
Also the point (1, – 6) lie on the tangent, so
x = 1 & y = – 6 satisfies the equation, $y = x^3 + ax + b$
$-6 = 1^3 + a \times 1 + b$
⇒ – 6 = 1 + a + b
⇒ a + b = – 7 ... (3)
Since, the tangent is parallel to the line, from (1) & (2)
Hence, 3 + a = 1
⇒ a = – 2
From (3)
a + b = – 7
⇒ – 2 + b = – 7
⇒ b = – 5
So the value is a = – 2 & b = – 5

4. Find a point on the curve $y = x^3 – 3x$ where the tangent is parallel to the chord joining (1, – 2) and (2, 2).

Solution:
Given curve $y = x^3 – 3x$

First, we will find the Slope of the tangent

$y = x^3 – 3x$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x)$

$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 3(\frac{dx}{dx})$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 3$ ... (1)

The equation of line passing through (x_0, y_0) and The Slope m is $y - y_0 = m(x - x_0)$.

So the Slope, $m = \frac{y - y_0}{x - x_0}$

The Slope of the chord joining (1, – 2) & (2, 2)

$\Rightarrow \frac{dy}{dx} = \frac{2-(-2)}{2-1}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{1}$

$\Rightarrow \frac{dy}{dx} = 4$ ... (2)

From (1) & (2)

$3x^2 - 3 = 4$

$\Rightarrow 3x^2 = 7$

$\Rightarrow x^2 = \frac{7}{3}$

$\Rightarrow x = \pm\sqrt{\frac{7}{3}}$

$y = x^3 - 3x$

$\Rightarrow y = x(x^2 - 3)$

$\Rightarrow y = \pm\sqrt{\frac{7}{3}}((\pm\sqrt{\frac{7}{3}})^2 - 3)$

$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\left(\frac{7}{3} - 3\right)\right.$

$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\frac{-2}{3}\right)$

$\Rightarrow y = \mp\left(\frac{-2}{3}\right)\sqrt{\frac{7}{3}}$

Thus, the required point is $x = \pm\sqrt{\frac{7}{3}}$ & $y = \mp\left(\frac{-2}{3}\right)\sqrt{\frac{7}{3}}$

5. Find a point on the curve $y = x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to the line $y = 2x - 3$.

Solution:

Given the curve $y = x^3 - 2x^2 - 2x$ and a line $y = 2x - 3$

First, we will find the slope of tangent

$y = x^3 - 2x^2 - 2x$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(2x^2) - \frac{d}{dx}(2x)$

$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 2 \times 2\,(x^{2-1}) - 2 \times x^{1-1}$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x - 2$ ... (1)

$y = 2x - 3$ is the form of equation of a straight line $y = mx + c$, where m is the Slope of the line.

So the slope of the line is $y = 2 \times (x) - 3$

Thus, the Slope = 2. ... (2)

From (1) & (2)

$\Rightarrow 3x^2 - 4x - 2 = 2$

$\Rightarrow 3x^2 - 4x = 4$

$\Rightarrow 3x^2 - 4x - 4 = 0$

We will use factorization method to solve the above Quadratic equation.

$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$

$\Rightarrow 3\,x\,(x - 2) + 2\,(x - 2) = 0$

⇒ $(x – 2) (3x + 2) = 0$
⇒ $(x – 2) = 0$ & $(3x + 2) = 0$
⇒ $x = 2$ or
$x = -2/3$
Substitute $x = 2$ & $x = -2/3$ in $y = x^3 – 2x^2 – 2x$
When $x = 2$
⇒ $y = (2)^3 – 2 \times (2)^2 – 2 \times (2)$
⇒ $y = 8 – (2 \times 4) – 4$
⇒ $y = 8 – 8 – 4$
⇒ $y = – 4$

When $x = \frac{-2}{3}$

⇒ $y = (\frac{-2}{3})^3 - 2 \times (\frac{-2}{3})^2 - 2 \times (\frac{-2}{3})$

⇒ $y = (\frac{-8}{27}) - 2 \times (\frac{4}{9}) + (\frac{4}{3})$

⇒ $y = (\frac{-8}{27}) - (\frac{8}{9}) + (\frac{4}{3})$

Taking LCM

⇒ $y = \frac{(-8\times1)-(8\times3) + (4\times9)}{27}$

⇒ $y = \frac{-8-24+36}{27}$

⇒ $y = \frac{4}{27}$

Thus, the points are $(2, – 4)$ & $(\frac{-2}{3}, \frac{4}{27})$

6. Find a point on the curve $y^2 = 2x^3$ at which the Slope of the tangent is 3

Solution:
Given the curve $y^2 = 2x^3$ and the Slope of tangent is 3
$y^2 = 2x^3$
Differentiating the above with respect to x

$\Rightarrow 2y^{2-1}\frac{dy}{dx} = 2 \times 3x^{3-1}$

$\Rightarrow y\frac{dy}{dx} = 3x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{y}$

Since, The Slope of tangent is 3

$\frac{3x^2}{y} = 3$

$\Rightarrow \frac{x^2}{y} = 1$

$\Rightarrow x^2 = y$

Substituting $x^2 = y$ in $y^2 = 2x^3$,

$(x^2)^2 = 2x^3$

$x^4 - 2x^3 = 0$

$x^3(x - 2) = 0$

$x^3 = 0$ or $(x - 2) = 0$

$x = 0$ or $x = 2$

If $x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{3(0)^2}{y}$

dy/dx = 0 which is not possible.
So we take x = 2 and substitute it in $y^2 = 2x^3$, we get
$y^2 = 2(2)^3$
$y^2 = 2 \times 8$
$y^2 = 16$
$y = 4$
Thus, the required point is (2, 4)

7. Find a point on the curve x y + 4 = 0 at which the tangents are inclined at an angle of

45° with the x–axis.

Solution:

Given the curve is xy + 4 = 0

If a tangent line to the curve y = f(x) makes an angle θ with x – axis in the positive direction, then

$\frac{dy}{dx}$ = The Slope of the tangent = tan θ

xy + 4 = 0

Differentiating the above with respect to x

$\Rightarrow x\frac{d}{dx}(y) + y\frac{d}{dx}(x) + \frac{d}{dx}(4) = 0$

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow x\frac{dy}{dx} = -y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$ ... (1)

Also, $\frac{dy}{dx} = \tan 45° = 1$ ... (2)

From (1) & (2), we get,

$\Rightarrow \frac{-y}{x} = 1$

$\Rightarrow x = -y$

Substitute in xy + 4 = 0, we get

$\Rightarrow x(-x) + 4 = 0$

$\Rightarrow -x^2 + 4 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

So when x = 2, y = – 2

And when x = – 2, y = 2

Thus, the points are (2, – 2) & (– 2, 2)

8. Find a point on the curve $y = x^2$ where the Slope of the tangent is equal to the x – coordinate of the point.

Solution:

Given the curve is $y = x^2$

$y = x^2$

Differentiating the above with respect to x

$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$

$\Rightarrow \frac{dy}{dx} = 2x$ ... (1)

Also given the Slope of the tangent is equal to the x – coordinate,

$\frac{dy}{dx} = x$ ... (2)

From (1) & (2), we get,

$2x = x$

$\Rightarrow x = 0.$

Substituting this in $y = x^2$, we get,

$y = 0^2$

$\Rightarrow y = 0$

Thus, the required point is (0, 0)

9. At what point on the circle $x^2 + y^2 – 2x – 4y + 1 = 0$, the tangent is parallel to x – axis.

Solution:

Given the curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Differentiating the above with respect to x

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$

$\Rightarrow 2x^{2-1} + 2y^{2-1} \times \frac{dy}{dx} - 2 - 4 \times \frac{dy}{dx} + 0 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(2y - 4) = -2x + 2$

$\Rightarrow \frac{dy}{dx} = \frac{-2(x-1)}{2(y-2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-(x-1)}{(y-2)}$ ... (1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = tan θ

Since, the tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx}$ = tan (0) = 0 ... (2)

Because tan (0) = 0

From (1) & (2), we get,

$\Rightarrow \frac{-(x-1)}{(y-2)} = 0$

$\Rightarrow -(x - 1) = 0$

$\Rightarrow x = 1$

Substituting x = 1 in $x^2 + y^2 - 2x - 4y + 1 = 0$, we get,

$\Rightarrow 1^2 + y^2 - 2(1) - 4y + 1 = 0$

$\Rightarrow 1 - y^2 - 2 - 4y + 1 = 0$

$\Rightarrow y^2 - 4y = 0$

$\Rightarrow y (y - 4) = 0$

$\Rightarrow$ y = 0 and y = 4

Thus, the required point is (1, 0) and (1, 4)

10. At what point of the curve $y = x^2$ does the tangent make an angle of 45° with the x–axis?

Solution:

Given the curve is $y = x^2$

Differentiating the above with respect to x

$\Rightarrow y = x^2$

$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$

$\Rightarrow \frac{dy}{dx} = 2x$ ... (1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = tan θ

Since, the tangent make an angle of 45° with x – axis

$\Rightarrow \frac{dy}{dx} = \tan(45°) = 1$ ... (2)

Because tan (45°) = 1

From (1) & (2), we get,

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ in $y = x^2$, we get,

$\Rightarrow y = (\frac{1}{2})^2$

$\Rightarrow y = \frac{1}{4}$

Thus, the required point is $(\frac{1}{2}, \frac{1}{4})$