EXERCISE 17.1 PAGE NO: 17.10

1. Prove that the function f(x) = $\log_e x$ is increasing on (0, ∞).

Solution:
Let $x_1, x_2 \in (0, \infty)$
We have, $x_1 < x_2$
$\Rightarrow \log_e x_1 < \log_e x_2$
$\Rightarrow f(x_1) < f(x_2)$
So, f(x) is increasing in (0, ∞)

2. Prove that the function f(x) = $\log_a x$ is increasing on (0, ∞) if a > 1 and decreasing on (0, ∞), if 0 < a < 1.

Solution:
Case I

When a > 1

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in (0, ∞)

Case II

When 0 < a < 1

$f(x) = \log_a x = \frac{\log x}{\log a}$

When $a < 1 \Rightarrow \log a < 0$

Let $x_1 < x_2$

$\Rightarrow \log x_1 < \log x_2$

$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a}$ [$\because \log a < 0$]

$\Rightarrow f(x_1) > f(x_2)$

So, f(x) is decreasing in $(0, \infty)$

3. Prove that f(x) = ax + b, where a, b are constants and a > 0 is an increasing function on R.

Solution:
Given,
$f(x) = ax + b, a > 0$
Let $x_1, x_2 \in R$ and $x_1 > x_2$
$\Rightarrow ax_1 > ax_2$ for some $a > 0$
$\Rightarrow ax_1 + b > ax_2 + b$ for some b
$\Rightarrow f(x_1) > f(x_2)$
Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$
So, f(x) is increasing function of R

4. Prove that f(x) = ax + b, where a, b are constants and a < 0 is a decreasing function on R.

Solution:
Given,
$f(x) = ax + b, a < 0$
Let $x_1, x_2 \in R$ and $x_1 > x_2$
$\Rightarrow ax_1 < ax_2$ for some $a > 0$
$\Rightarrow ax_1 + b < ax_2 + b$ for some b
$\Rightarrow f(x_1) < f(x_2)$
Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$
So, f(x) is decreasing function of R