

EXERCISE 18.1

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Find the maximum and the minimum values, if any, without using derivatives of the following functions:

1. $f(x) = 4x^2 - 4x + 4$ on \mathbb{R}

Solution:

Given $f(x) = 4x^2 - 4x + 4$ on \mathbb{R}

$$= 4x^2 - 4x + 1 + 3$$

By grouping the above equation we get,

$$= (2x - 1)^2 + 3$$

Since, $(2x - 1)^2 \geq 0$

$$= (2x - 1)^2 + 3 \geq 3$$

$$= f(x) \geq f\left(\frac{1}{2}\right)$$

Thus, the minimum value of $f(x)$ is 3 at $x = \frac{1}{2}$

Since, $f(x)$ can be made large. Therefore maximum value does not exist.

2. $f(x) = -(x - 1)^2 + 2$ on \mathbb{R}

Solution:

Given $f(x) = -(x - 1)^2 + 2$

It can be observed that $(x - 1)^2 \geq 0$ for every $x \in \mathbb{R}$

Therefore, $f(x) = -(x - 1)^2 + 2 \leq 2$ for every $x \in \mathbb{R}$

The maximum value of f is attained when $(x - 1) = 0$

$$(x - 1) = 0, x = 1$$

Since, Maximum value of $f = f(1) = -(1 - 1)^2 + 2 = 2$

Hence, function f does not have minimum value.

3. $f(x) = |x + 2|$ on \mathbb{R}

Solution:

Given $f(x) = |x + 2|$ on \mathbb{R}

$$\Rightarrow f(x) \geq 0 \text{ for all } x \in \mathbb{R}$$

So, the minimum value of $f(x)$ is 0, which attains at $x = -2$

Hence, $f(x) = |x + 2|$ does not have the maximum value.

4. $f(x) = \sin 2x + 5$ on \mathbb{R}

Solution:

Given $f(x) = \sin 2x + 5$ on \mathbb{R}

We know that $-1 \leq \sin 2x \leq 1$

$\Rightarrow -1 + 5 \leq \sin 2x + 5 \leq 1 + 5$

$\Rightarrow 4 \leq \sin 2x + 5 \leq 6$

Hence, the maximum value and minimum value of f are 6 and 4 respectively.

5. $f(x) = |\sin 4x + 3|$ on \mathbb{R}

Solution:

Given $f(x) = |\sin 4x + 3|$ on \mathbb{R}

We know that $-1 \leq \sin 4x \leq 1$

$\Rightarrow 2 \leq \sin 4x + 3 \leq 4$

$\Rightarrow 2 \leq |\sin 4x + 3| \leq 4$

Hence, the maximum value and minimum value of f are 4 and 2 respectively.