EXERCISE 19.10 PAGE NO: 19.65

1. $\int x^2\sqrt{x+2}\,dx$

Solution:

Let $I = \int x^2\sqrt{x+2}dx$

Substituting, $x + 2 = t \Rightarrow dx = dt$,

$$I = \int (t-2)^2\sqrt{t}dt$$

$$\Rightarrow I = \int (t^2 - 4t + 4)\sqrt{t}dt$$

$$\Rightarrow I = \int \left(t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + 4t^{\frac{1}{2}}\right)dt$$

$$\Rightarrow I = \frac{2}{7}t^{\frac{7}{2}} - \frac{8}{5}t^{\frac{5}{2}} + \frac{8}{3}t^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$

Therefore, $\int x^2\sqrt{x+2}dx = \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$

2. $\int \frac{x^2}{\sqrt{x-1}}\,dx$

Solution:

Let $I = \int \frac{x^2}{\sqrt{x-1}}dx$

Substituting $x - 1 = t \Rightarrow dx = dt$,

Now substituting the values we get

$$\Rightarrow I = \int \frac{(t + 1)^2}{\sqrt{t}} dt$$

Expanding using $(a + b)^2$ formula

$$\Rightarrow I = \int \frac{t^2 + 2t + 1}{\sqrt{t}} dt$$

On simplification

$$\Rightarrow I = \int \left(t^{\frac{3}{2}} + 2t^{\frac{1}{2}} + t^{-\frac{1}{2}}\right) dt$$

On integrating we get

$$\Rightarrow I = \frac{2}{5} t^{\frac{5}{2}} + 2t^{\frac{1}{2}} + \frac{4}{3} t^{\frac{3}{2}} + c$$

Again taking LCM

$$\Rightarrow I = \frac{\left(6t^{\frac{5}{2}} + 30t^{\frac{1}{2}} + 20t^{\frac{3}{2}}\right)}{15} + c$$

$$\Rightarrow I = \frac{2}{15} t^{\frac{1}{2}} (3t^2 + 15 + 10t) + c$$

Substituting the value of t we get

$$\Rightarrow I = \frac{2}{15} (x - 1)^{\frac{1}{2}} (3(x - 1)^2 + 15 + 10(x - 1)) + c$$

$$\Rightarrow I = \frac{2}{15} (x - 1)^{\frac{1}{2}} (3(x^2 - 2x + 1) + 15 + 10x - 10) + c$$

By simplifying we get

$$\Rightarrow I = \frac{2}{15} (x - 1)^{\frac{1}{2}} (3x^2 + 4x + 8) + c$$

Therefore, $\int \frac{x^2}{\sqrt{x - 1}} dx = \frac{2}{15} (x - 1)^{\frac{1}{2}} (3x^2 + 4x + 8) + c$

3. $\int \frac{x^2}{\sqrt{3x+4}}\, dx$

Solution:

Let $I = \int \frac{x^2}{\sqrt{3x + 4}} dx$

Substituting 3 x + 4 = t ⇒ 3 dx = dt,

Substituting the values of x

$$\Rightarrow I = \int \frac{\left(\frac{t-4}{3}\right)^2}{3\sqrt{t}} dt$$

Expanding the above given function using $(a - b)^2$ formula

$$\Rightarrow I = \frac{1}{27}\int \frac{t^2 + 16 - 8t}{\sqrt{t}} dt$$

On simplifying, we get

$$\Rightarrow I = \frac{1}{27}\int \left(t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + 16t^{-\frac{1}{2}}\right) dt$$

On integrating, we get

$$\Rightarrow I = \frac{1}{27}\left[\frac{2}{5}t^{\frac{5}{2}} - \frac{16}{3}t^{\frac{3}{2}} + 32t^{\frac{1}{2}}\right] + c$$

$$\Rightarrow I = \frac{1}{27}\left[\frac{2}{5}(3x + 4)^{\frac{5}{2}} - \frac{16}{3}(3x + 4)^{\frac{3}{2}} + 32(3x + 4)^{\frac{1}{2}}\right] + c$$

$$\Rightarrow I = \frac{2}{135}(3x + 4)^{\frac{5}{2}} - \frac{16}{81}(3x + 4)^{\frac{3}{2}} + \frac{32}{27}(3x + 4)^{\frac{1}{2}} + c$$

Therefore, $\int \frac{x^2}{\sqrt{3x + 4}} dx$

$$= \frac{2}{135}(3x + 4)^{\frac{5}{2}} - \frac{16}{81}(3x + 4)^{\frac{3}{2}} + \frac{32}{27}(3x + 4)^{\frac{1}{2}} + c$$

4. $\int \frac{2x-1}{(x-1)^2}\, dx$

Solution:

$$\text{Let } I = \int \frac{2x-1}{(x-1)^2} dx$$

Substituting x - 1 = t ⇒dx = dt

Substituting the values of x

$$\Rightarrow I = \int \frac{2(t+1)-1}{t^2} dt$$

Multiplying and simplifying we get

$$\Rightarrow I = \int \frac{2t+1}{t^2} dt$$

$$\Rightarrow I = \int \left(\frac{2}{t} + \frac{1}{t^2}\right) dt$$

On integration

$$\Rightarrow I = 2\log|t| - \frac{1}{t} + c$$

$$\Rightarrow I = 2\log|x-1| - \frac{1}{x-1} + c$$

$$\text{Therefore, } \int \frac{2x-1}{(x-1)^2} dx = 2\log|x-1| - \frac{1}{x-1} + c$$

5. $\int (2x^2+3)\sqrt{x+2}\, dx$

Solution:

$$\text{Let } I = \int (2x^2+3)\sqrt{x+2}\, dx$$

Substituting x + 2 = t ⇒ dx = dt

Substituting the values of x in given equation, we get

$$\Rightarrow I = \int [2(t-2)^2 + 3]\sqrt{t}dt$$

Expanding above equation using $(a-b)^2$ formula

$$\Rightarrow I = \int [2t^2 - 8t + 8 + 3]\sqrt{t}dt$$

On simplification

$$\Rightarrow I = \int \left[2t^{\frac{5}{2}} - 8t^{\frac{3}{2}} + 11t^{\frac{1}{2}}\right]dt$$

On integrating we get

$$\Rightarrow I = \frac{4}{7}t^{\frac{7}{2}} - \frac{16}{5}t^{\frac{5}{2}} + \frac{22}{3}t^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+2)^{\frac{5}{2}} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$

$$\therefore \int (2x^2 + 3)\sqrt{x+2}dx = \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+2)^{\frac{5}{2}} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$