EXERCISE 19.16 PAGE NO: 19.90

Evaluate the following integrals:

1. $\int \frac{\sec^2 x}{1-\tan^2 x} dx$

Solution:

Let $I = \int \frac{\sec^2 x}{1-\tan^2 x} dx$

Let tan x = t..... (i)

$\Rightarrow \sec^2 x$ dx = dt

So, substituting these values in given equation we get

$I = \int \frac{dt}{(1)^2 - t^2}$

$I = \frac{1}{2 \times 1} \log\left|\frac{1+t}{1-t}\right| + c$ $\left[\text{since, } \int \frac{1}{a^2-(x)^2} dx = \frac{1}{2 \times a} \log\left|\frac{a+x}{a-x}\right| + c\right]$

$I = \frac{1}{2} \log\left|\frac{1+\tan x}{1-\tan x}\right| + c$ [Using (i)]

2. $\int \frac{e^x}{1+e^{2x}} dx$

Solution:

Let $I = \int \frac{e^x}{1+e^{2x}} dx$

Let e^x = t..... (i)

$\Rightarrow e^x$ dx = dt

So, substituting these values in given equation we get

$I = \int \frac{dt}{(1)^2 + t^2}$

$I = \tan^{-1} t + c$

$[\text{since}, \int \frac{1}{1+(x)^2} dx = \tan^{-1} x + c]$

$I = \tan^{-1}(e^x) + c$ [Using (i)]

3. $\int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$

Solution:

Let $I = \int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$

Let sin x = t..... (i)

⇒ Cos x dx = dt

So, $I = \int \frac{dt}{t^2 + 4t + 5}$

Adding and subtracting 2^2 to the denominator we get

$= \int \frac{dt}{t^2 + (2t)(2) + 2^2 - 2^2 + 5}$

Above equation can be written as

$\int \frac{dt}{(t+2)^2 + 1}$

Again, let t + 2 = u .....(ii)

⇒ dt = du

$I = \int \frac{du}{u^2 + 1}$

$= \tan^{-1} u + c$

$[\text{since}, \int \frac{1}{1+(x)^2} dx = \tan^{-1} x + c]$

$= \tan^{-1}(\sin x + 2) + c$ [Using (i), (ii)]

4. $\int \frac{e^x}{e^{2x}+5e^x+6}\,dx$

Solution:

Let $I=\int \frac{e^x}{e^{2x}+5e^x+6}dx$

Let e^x = t..... (i)

$\Rightarrow e^x$ dx = dt

$$=\int \frac{1}{t^2+5t+6}dt$$

$$=\int \frac{1}{t^2+2t\times\frac{5}{2}+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+6}dt$$

$$=\int \frac{1}{\left(t+\frac{5}{2}\right)^2-\frac{1}{4}}dt$$

Let $t+\frac{5}{2}=u$ ..... (ii)

$\Rightarrow$ dt = du

So, substituting these values we get

$$I=\int \frac{1}{u^2-\left(\frac{1}{2}\right)^2}du$$

$$I=\frac{1}{2\times\frac{1}{2}}\log\left|\frac{u-\frac{1}{2}}{u+\frac{1}{2}}\right|+c$$

[since, $\int \frac{1}{x^2-(a)^2}dx=\frac{1}{2\times a}\log\left|\frac{x-a}{x+a}\right|+c$]

$I = \log\left|\frac{2u - 1}{2u + 1}\right| + c$

$I = \log\left|\frac{2(t+\frac{5}{2})-1}{2(t+\frac{5}{2})+1}\right| + c$ [Using (ii)]

$I = \log\left|\frac{e^x+2}{e^x+3}\right| + c$ [Using (i)]

5. $\int \frac{e^{3x}}{4e^{6x} - 9} dx$

Solution:

Let $I = \int \frac{e^{3x}}{4e^{6x}-9} dx$

Let $e^{3x} = t$..... (i)

$\Rightarrow 3e^{3x}\, dx = dt$

$I = \frac{1}{3}\int \frac{1}{4t^2 - 9} dt$

Taking (¼) as common we get

$= \frac{1}{12}\int \frac{1}{t^2 - \frac{9}{4}} dt$

The above equation can be written as

$I = \frac{1}{12}\int \frac{1}{t^2 - \left(\frac{3}{2}\right)^2} dt$

$I = \frac{1}{36}\log\left|\frac{t - \frac{3}{2}}{t + \frac{3}{2}}\right| + c$

[since, $\int \frac{1}{x^2 - (a)^2} dx = \frac{1}{2 \times a}\log\left|\frac{x - a}{x + a}\right| + c$]

$$I = 1/36 \log \left|\frac{2t-3}{2t+3}\right| + c$$

$$I = 1/36 \log \left|\frac{2e^{3x}-3}{2e^{3x}+3}\right| + c \quad \text{[Using (i)]}$$