EXERCISE 19.5 PAGE NO: 19.33

1. $\int \frac{x+1}{\sqrt{2x+3}}\,dx$

Solution:

Given

$$\int \frac{x+1}{\sqrt{2x+3}}\,dx$$

In this type of questions, little manipulation makes the questions easier to solve

Here we have multiply and divide by 2 to given equation

$$\Rightarrow \frac{1}{2}\int \frac{2x+2}{\sqrt{2x+3}}\,dx$$

Add and subtract 1 from the numerator

$$\Rightarrow \frac{1}{2}\int \frac{2x+2+1-1}{\sqrt{2x+3}}\,dx$$

$$\Rightarrow \frac{1}{2}\int \frac{2x+3-1}{\sqrt{2x+3}}\,dx$$

Splitting the above equation we get

$$\Rightarrow \frac{1}{2}\int \frac{2x+3}{\sqrt{2x+3}}\,dx - \frac{1}{2}\int \frac{1}{\sqrt{2x+3}}\,dx$$

Taking ½ common from the above equation

$$\Rightarrow \frac{1}{2}\left(\int \sqrt{2x+3}\,dx - \int (2x+3)^{\frac{-1}{2}}\,dx\right)$$

Now by integrating the above equation we get

$$\Rightarrow \frac{1}{2}\times\frac{(2x+3)^{\frac{3}{2}}}{2\times\frac{3}{2}} - \frac{1}{2}\times\frac{(2x+3)^{\frac{1}{2}}}{2\times\frac{1}{2}} + c$$

$$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c$$

2. $\int x\sqrt{x+2}\,dx$

Solution:

Given

$$\int x\sqrt{x+2}\,dx$$

In this type of questions, little manipulation makes the questions easier to solve

Here add and subtract 2 from x in the given equation

We get

$$\Rightarrow \int (x+2-2)\sqrt{x+2}dx$$

$$\Rightarrow \int (x+2)^{\frac{3}{2}}dx - \int 2\sqrt{x+2}dx$$

On integrating we get

$$\Rightarrow \frac{2(x+2)^{\frac{5}{2}}}{5} - \frac{4(x+2)^{\frac{3}{2}}}{3} + c$$

3. $\int \frac{x-1}{\sqrt{x+4}}\,dx$

Solution:

Given

$$\int \frac{x-1}{\sqrt{x+4}}\,dx$$

In this type of questions, little manipulation makes the questions easier to solve

Add and subtract 5 from the numerator

$$\Rightarrow \int \frac{x+5-5-1}{\sqrt{x+4}}\,dx$$

$$\Rightarrow \int \frac{x+4-5}{\sqrt{x+4}}\,dx$$

By splitting the above equation

$$\Rightarrow \int \frac{x+4}{\sqrt{x+4}}\,dx - \int \frac{5}{\sqrt{x+4}}\,dx$$

$$\Rightarrow \left(\int \sqrt{x+4}\,dx - 5\int (x+4)^{\frac{-1}{2}}\,dx\right)$$

Now by integrating, we get

$$\Rightarrow \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5 \times \frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

By computing

$$\Rightarrow \frac{2(x+4)^{\frac{3}{2}}}{3} - 10(x+4)^{\frac{1}{2}} + c$$

4. $\int (x+2)\sqrt{3x+5}\,dx$

Solution:

Let

$$I = \int (x+2)\sqrt{3x+5}\,dx$$

Substitute 3x + 5 = t

$$\Rightarrow x = \frac{t-5}{3}$$

$$\Rightarrow 3dx = dt$$

$$\Rightarrow dx = \frac{dt}{3}$$

$$\therefore I = \int\left(\frac{t-5}{3}+2\right)\sqrt{t}\ \frac{dt}{3}$$

$$= \frac{1}{3}\int\left(\frac{t-5+6}{3}\right)\sqrt{t}\ dt$$

By taking 3 as common and multiplying, we get

$$= \frac{1}{9}\int\left(t^{\frac{3}{2}}+t^{\frac{1}{2}}\right)dt$$

On integrating we get

$$= \frac{1}{9}\left[\frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1}+\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]+C$$

On simplifying

$$= \frac{1}{9}\left[\frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}}\right] + C$$

By substituting the value of t

$$= \frac{1}{9}\left[\frac{2}{5}(3x+5)^{\frac{5}{2}} + \frac{2}{3}(3x+5)^{\frac{3}{2}}\right] + C$$

$$= \frac{2}{9}\left[(3x+5)^{\frac{3}{2}}\left\{\frac{3x+5}{5} + \frac{1}{3}\right\}\right] + C$$

$$= \frac{2}{9}\left[(3x+5)^{\frac{3}{2}}\left\{\frac{9x+15+5}{15}\right\}\right] + C$$

$$= \frac{2}{9}\left[(3x+5)^{\frac{3}{2}}\left\{\frac{9x+20}{15}\right\}\right] + C$$

$$= \frac{2}{135}(3x+5)^{\frac{3}{2}}(9x+20) + C$$

5. $\int \frac{2x+1}{\sqrt{3x+2}}\,dx$

Solution:

Given

$$\int \left(\frac{2x+1}{\sqrt{3x+2}}\right) dx$$

Multiply and divide by 3 in the above equation we get

$$= \frac{1}{3}\int \left(\frac{6x+3}{\sqrt{3x+2}}\right) dx$$

The above equation can be written as

$$= \frac{1}{3}\int\left(\frac{6x+4-1}{\sqrt{3x+2}}\right)dx$$

Taking 2 as common and subtracting

$$= \frac{1}{3}\int\left(\frac{2\,(3x+2)}{\sqrt{3x+2}} - \frac{1}{\sqrt{3x+2}}\right)dx$$

On simplifying

$$= \frac{1}{3}\int\left(2\sqrt{3x+2} - \frac{1}{\sqrt{3x+2}}\right)dx$$

By splitting the integral

$$= \frac{1}{3}\left[\int 2(3x+2)^{\frac{1}{2}}dx - \int (3x+2)^{-\frac{1}{2}}dx\right]$$

On integrating we get

$$= \frac{1}{3}\left[2\left\{\frac{(3x+2)^{\frac{1}{2}}+1}{3\left(\frac{1}{2}+1\right)}\right\} - \frac{(3x+2)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)\times 3}\right] + C$$

$$= \frac{1}{3}\left[\frac{4}{9}(3x+2)^{\frac{3}{2}} - \frac{2}{3}(3x+2)^{\frac{1}{2}}\right] + C$$

On simplifying we get

$$= \frac{4}{27}(3x+2)^{\frac{3}{2}} - \frac{2}{9}(3x+2)^{\frac{1}{2}} + C$$

$$= \sqrt{3x+2}\left(\frac{4}{27}(3x+2) - \frac{2}{9}\right) + C$$

$$= \sqrt{3x+2}\left(\frac{4\,(3x+2)-6}{27}\right) + C$$

$$= \sqrt{3x+2}\left(\frac{12x+8-6}{27}\right) + C$$

$$= \frac{2}{27}(6x+1)\sqrt{3x+2} + C$$