EXERCISE 19.6

1. $\int \sin^2(2x+5)\,dx$

Solution:

We know that

$\sin^2 x = \frac{1-\cos 2x}{2}$

By substituting the above formula

∴ The given equation becomes,

$\Rightarrow \int \frac{1-\cos 2(2x+5)}{2}\,dx$

We know $\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

$\Rightarrow \frac{1}{2}\int dx - \frac{1}{2}\int \cos(4x+10)\,dx$

On integrating we get

$\Rightarrow \frac{x}{2} - \frac{1}{8}\sin(4x+10) + c$

2. $\int \sin^3(2x+1)\,dx$

Solution:

We know that $\sin 3x = -4\sin^3 x + 3\sin x$

The above formula can be written as

$\Rightarrow 4\sin^3 x = 3\sin x - \sin 3x$

The above equation becomes

$\Rightarrow \sin^3 x = \frac{3\sin x - \sin 3x}{4}$

Now applying above formula to the given question we get

$\Rightarrow \int \sin^3(2x+1)dx = \int \frac{3\sin(2x+1) - \sin 3(2x+1)}{4} dx$

We know $\int \sin ax\, dx = \frac{-1}{a}\cos ax + c$

By substituting the above formula we get

$\Rightarrow \frac{3}{8}\int \sin(2x+1)dx - \frac{1}{4}\int \sin(6x+3)dx$

On integrating we get

$\Rightarrow \frac{-3}{8}\cos(2x+1) + \frac{1}{24}\cos(6x+3) + c$.

3. $\int \cos^4 2x\, dx$

Solution:

Consider,

$\cos^4 2x = (\cos^2 2x)^2$

We know that

$\Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$

The above equation

$\Rightarrow (\cos^2 2x)^2 = \left(\frac{1+\cos 4x}{2}\right)^2$

$\Rightarrow \left(\frac{1+\cos 4x}{2}\right)^2 = \left(\frac{1+2\cos 4x+\cos^2 4x}{4}\right)$

$\Rightarrow \cos^2 4x = \frac{1+\cos 8x}{2}$

$\Rightarrow \left(\frac{1+2\cos 4x+\cos^2 4x}{4} = \frac{1}{4} + \frac{\cos 4x}{2} + \frac{1+\cos 8x}{8}\right)$

Now the question becomes,

$\Rightarrow \frac{1}{4}\int dx + \frac{1}{2}\int \cos 4x\, dx + \frac{1}{8}\int dx + \frac{1}{8}\int \cos 8x\, dx$

We know $\int \cos ax\, dx = \frac{1}{a}\sin ax + c$

$\Rightarrow \frac{x}{4} + \frac{1}{8}\sin 4x + \frac{x}{8} + \frac{\sin 8x}{64} + c$

$\Rightarrow \frac{24x+8\sin 4x+\sin 8x}{64} + c$

4. $\int \sin^2 bx\, dx$

Solution:

We know that

$\sin^2 x = \frac{1-\cos 2x}{2}$

By substituting this formula,

∴ The given equation becomes,

$\Rightarrow \int \frac{1-\cos 2bx}{2}\, dx$

We know $\int \cos ax\, dx = \frac{1}{a}\sin ax + c$

$\Rightarrow \frac{1}{2}\int dx - \frac{1}{2}\int \cos(2bx)\, dx$

On integration

$\Rightarrow \frac{x}{2} - \frac{1}{4b}\sin(2bx) + c$