

EXERCISE 19.7

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Integrate the following integrals:

1. $\int \sin 4x \cos 7x \, dx$

Solution:

Given

$$\int \sin 4x \cos 7x \, dx$$

We know that $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

Now by substituting this formula in given question we get

$$\therefore \sin 4x \cos 7x = \frac{\sin 11x + \sin(-3x)}{2}$$

We know $\sin(-\theta) = -\sin \theta$

Hence $\sin(-3x) = -\sin 3x$

\therefore the above equation becomes

$$\Rightarrow \int \frac{1}{2} (\sin 11x - \sin 3x) \, dx$$

$$\Rightarrow \frac{1}{2} (\int \sin 11x \, dx - \int \sin 3x \, dx)$$

We know $\int \sin ax \, dx = \frac{-1}{a} \cos ax + c$

$$\Rightarrow \frac{1}{2} \left(\frac{-1}{11} \cos 11x + \frac{1}{3} \cos 3x \right)$$

$$= -\frac{1}{22} \cos 11x + \frac{1}{6} \cos 3x + c$$

2. $\int \cos 3x \cos 4x \, dx$

Solution:

Given

$$\int \cos 4x \cos 3x \, dx$$

Multiply and divide the given equation by 2

$$= \frac{1}{2} \int 2 \cos 4x \cos 3x \, dx$$

We know that $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

$$= \frac{1}{2} \int [\cos (4x + 3x) + \cos (4x - 3x)] \, dx$$

Now by simplifying we get

$$= \frac{1}{2} \int (\cos (7x) + \cos x) \, dx$$

On integration we get

$$= \frac{1}{2} \left[\frac{\sin 7x}{7} + \sin x \right] + C$$

$$= \frac{1}{14} \sin 7x + \frac{1}{2} \sin x + C$$

3. $\int \cos mx \cos nx \, dx, m \neq n$

Solution:

Given

$$\int \cos mx \cos nx \, dx, m \neq n$$

We know $2 \cos A \cos B = \cos (A - B) + \cos (A + B)$

Now substituting the above formula we get,

$$\therefore \cos mx \cos nx = \frac{\cos(m-n)x + \cos(m+n)x}{2}$$

∴ The above equation becomes

$$\Rightarrow \int \frac{1}{2}(\cos(m-n)x + \cos(m+n)x)dx$$

We know $\int \cos ax dx = \frac{1}{a} \sin ax + c$

Applying the above

$$\Rightarrow \frac{1}{2} \left(\frac{1}{m-n} \sin(m-n)x + \frac{1}{m+n} \sin(m+n)x \right)$$

$$\Rightarrow \frac{1}{2} \left(\frac{(m+n)\sin(m-n)x + (m-n)\sin(m+n)x}{m^2-n^2} \right) + c$$

We know that $a^2 - b^2 = (a+b)(a-b)$

By substituting the above formula and simplifying we get

$$\frac{1}{2} \left\{ \frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n} \right\} + c$$