

## EXERCISE 4.11

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1. Prove the following results:

(i)  $\tan^{-1} (1/7) + \tan^{-1} (1/13) = \tan^{-1} (2/9)$

(ii)  $\sin^{-1} (12/13) + \cos^{-1} (4/5) + \tan^{-1} (63/16) = \pi$

(iii)  $\tan^{-1} (1/4) + \tan^{-1} (2/9) = \sin^{-1} (1/\sqrt{5})$

**Solution:**

(i) Given  $\tan^{-1} (1/7) + \tan^{-1} (1/13) = \tan^{-1} (2/9)$

Consider LHS

$$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$$

According to the formula, we can write as

$$= \tan^{-1} \left( \frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{7} \times \frac{1}{13}} \right)$$

$$= \tan^{-1} \left( \frac{\frac{13+7}{91}}{\frac{91-1}{91}} \right)$$

$$= \tan^{-1} \left( \frac{20}{90} \right)$$

$$= \tan^{-1} \left( \frac{2}{9} \right)$$

= RHS

Hence, proved.

(ii) Given  $\sin^{-1} (12/13) + \cos^{-1} (4/5) + \tan^{-1} (63/16) = \pi$

Consider LHS

$$\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\frac{4}{5} + \tan^{-1}\left(\frac{63}{16}\right)$$

We know that, Formula

$$\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\cos^{-1} x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Now, by substituting the formula we get,

$$\begin{aligned} & \tan^{-1}\left(\frac{\frac{12}{13}}{\sqrt{1-\left(\frac{12}{13}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right) + \tan^{-1}\left(\frac{63}{16}\right) \\ = & \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{63}{16}\right) \end{aligned}$$

Again we know that,

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}$$

Again by substituting, we get

$$\begin{aligned} = & \pi + \tan^{-1}\left(\frac{\frac{12}{5} + \frac{3}{4}}{1 - \frac{12}{5} \times \frac{3}{4}}\right) + \tan^{-1}\left(\frac{63}{16}\right) \\ = & \pi + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right) \end{aligned}$$

We know that,

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$= \pi - \tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \pi$$

$$\text{So, } \sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\frac{4}{5} + \tan^{-1}\left(\frac{63}{16}\right) = \pi$$

Hence, proved.

(iii) Given  $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \sin^{-1}(1/\sqrt{5})$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$$

We know that,

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}$$

By substituting this formula we get,

$$= \tan^{-1}\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}}$$

$$= \tan^{-1}\frac{\frac{17}{36}}{\frac{36}{36}}$$

$$= \tan^{-1}\frac{17}{36}$$

$$= \tan^{-1}\frac{1}{2}$$

Now let,  $\tan\theta = \frac{1}{2}$

Therefore,  $\sin\theta = \frac{1}{\sqrt{5}}$

So,  $\theta = \sin^{-1}\frac{1}{\sqrt{5}}$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \text{RHS}$$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Hence, Proved.

## 2. Find the value of $\tan^{-1}(x/y) - \tan^{-1}\{(x-y)/(x+y)\}$

**Solution:**

Given  $\tan^{-1}(x/y) - \tan^{-1}\{(x-y)/(x+y)\}$

We know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Now by substituting the formula, we get

$$= \tan^{-1}\frac{\frac{x}{y} - \left(\frac{x-y}{x+y}\right)}{1 + \frac{x}{y} \times \left(\frac{x-y}{x+y}\right)}$$

$$= \tan^{-1}\frac{x(x+y) - y(x-y)}{y(x+y) + x(x-y)}$$

$$= \tan^{-1}\frac{x^2 + y^2}{x^2 + y^2}$$

$$= \tan^{-1}1$$

$$= \frac{\pi}{4}$$

So,

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$$