

EXERCISE 4.12

PAGE NO: 4.89

1. Evaluate: $\cos(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13})$
Solution:

 Given $\cos(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13})$

We know that,

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}]$$

By substituting this formula we get,

$$\begin{aligned} &= \cos\left(\sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]\right) \\ &= \cos\left(\sin^{-1}\left[\frac{3}{5} \times \frac{12}{13} + \frac{5}{13} \times \frac{4}{5}\right]\right) \\ &= \cos\left(\sin^{-1}\left[\frac{56}{65}\right]\right) \end{aligned}$$

Again, we know that

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2}$$

Now substituting, we get

$$\begin{aligned} &= \cos\left(\cos^{-1}\sqrt{1-\left(\frac{56}{65}\right)^2}\right) \\ &= \cos\left(\cos^{-1}\sqrt{\left(\frac{33}{65}\right)^2}\right) = \cos\left(\cos^{-1}\left(\frac{33}{65}\right)\right) \\ &= \frac{33}{65} \end{aligned}$$

$$\text{Hence, } \cos\left(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13}\right) = \frac{33}{65}$$