

Exercise 10(D)

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1. Find three numbers in A.P. whose sum is 24 and whose product is 440.

Solution:

Let's assume the terms in the A.P to be $(a - d)$, a , $(a + d)$ with common difference as d .

Given conditions,

$$S_n = 24$$

$$(a - d) + a + (a + d) = 3a = 24$$

$$a = 24/3 = 8$$

And,

$$\text{Product of terms} = 440$$

$$(a - d) \times a \times (a + d) = 440$$

$$a(a^2 - d^2) = 440$$

$$8(64 - d^2) = 440$$

$$(64 - d^2) = 55$$

$$d^2 = 9$$

$$d = \pm 3$$

Hence,

When $a = 8$ and $d = 3$, we have

$$\text{A.P.} = 5, 8, 11$$

And, when $a = 8$ and $d = -3$ we have

$$\text{A.P.} = 11, 8, 5$$

2. The sum of three consecutive terms of an A.P. is 21 and the sum of their squares is 165. Find these terms.

Solution:

Let the three consecutive terms of an A.P. be $(a - d)$, a , $(a + d)$

Given conditions,

$$\text{Sum of the three consecutive terms} = 21$$

$$(a - d) + a + (a + d) = 21$$

$$3a = 21$$

$$a = 7$$

And,

$$\text{Sum of squares} = (a - d)^2 + a^2 + (a + d)^2 = 165$$

$$a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 165$$

$$3a^2 + 2d^2 = 165$$

$$3(49) + 2d^2 = 165$$

$$2d^2 = 165 - 147 = 18$$

$$d^2 = 9$$

$$d = \pm 3$$

Hence,

When $a = 7$ and $d = 3$, we have

$$\text{A.P.} = 4, 7, 10$$

And, when $a = 7$ and $d = -3$ we have

A.P. = 10, 7, 4

3. The angles of a quadrilateral are in A.P. with common difference 20° . Find its angles.

Solution:

Given, the angles of a quadrilateral are in A.P. with common difference 20° .

So, let the angles be taken as x , $x + 20^\circ$, $x + 40^\circ$ and $x + 60^\circ$.

We know that,

Sum of all the interior angles of a quadrilateral is 360°

$$x + x + 20^\circ + x + 40^\circ + x + 60^\circ = 360^\circ$$

$$4x + 120^\circ = 360^\circ$$

$$4x = 360^\circ - 120^\circ = 240^\circ$$

$$x = 240^\circ / 4 = 60^\circ$$

Hence, the angles are

60° , $(60^\circ + 20^\circ)$, $(60^\circ + 40^\circ)$ and $(60^\circ + 60^\circ)$

i.e. 60° , 80° , 100° and 120°

4. Divide 96 into four parts which are in A.P and the ratio between product of their means to product of their extremes is 15: 7.

Solution:

Let 96 be divided into 4 parts as $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$ which are in A.P with common difference of $2d$.

Given,

$$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 96$$

$$4a = 96$$

$$\text{So, } a = 24$$

And, given that,

$$(a - d)(a + d) / (a - 3d)(a + 3d) = 15/7$$

$$[a^2 - d^2] / [a^2 - 9d^2] = 15/7$$

Substituting the value of a , we get,

$$(576 - d^2) / (576 - 9d^2) = 15 / 7$$

On cross multiplication, we get,

$$4032 - 7d^2 = 8640 - 135d^2$$

$$128d^2 = 4608$$

$$d^2 = 36$$

$$d = +6 \text{ or } -6$$

Hence,

When $a = 24$ and $d = 6$

The parts are 6, 18, 30 and 42

And, when $a = 24$ and $d = -6$

The parts are 42, 30, 18 and 6

5. Find five numbers in A.P. whose sum is 12.5 and the ratio of the first to the last terms is 2: 3.

Solution:

Let the five numbers in A.P be taken as $(a - 2d)$, $(a - d)$, a , $(a + d)$ and $(a + 2d)$.

Given conditions,

$$(a - 2d) + (a - d) + a + (a + d) + (a + 2d) = 12.5$$

$$5a = 12.5$$

$$a = 12.5/5 = 2.5$$

And,

$$(a - 2d)/(a + 2d) = 2/3$$

$$3(a - 2d) = 2(a + 2d)$$

$$3a - 6d = 2a + 4d$$

$$a = 10d$$

$$2.5 = 10d$$

$$d = 0.25$$

Therefore, the five numbers in A.P are $(2.5 - 2(0.25))$, $(2.5 - 0.25)$, 2.5 , $(2.5 + 0.25)$ and $(2.5 + 2(0.25))$

i.e. 2, 2.25, 2.5, 2.75 and 3.