Exercise 10(F)

1. The 6th term of an A.P. is 16 and the 14th term is 32. Determine the 36th term.
Solution:

Given,
$t_6 = 16$ and $t_{14} = 32$
Let's take 'a' to be the first term and 'd' to be the common difference of the given A.P.
We know that,
$t_n = a + (n - 1)d$
$\Rightarrow a + 5d = 16$ ….(1)
And,
$\Rightarrow a + 13d = 32$ ….(2)
Now, subtracting (1) from (2), we get
$8d = 16$
$d = 2$
Using d in (1) we get,
$a + 5(2) = 16$
$\Rightarrow a = 6$
Therefore, the 36th term = $t_{36} = a + 35d = 6 + 35(2) = 76$

2. If the third and the 9th term of an A.P. be 4 and -8 respectively, find which term is zero?
Solution:

Given,
$t_3 = 4$ and $t_9 = -8$
We know that,
$t_n = a + (n - 1)d$
So,
$4 = a + (3 - 1)d$
$a + 2d = 4$ ….. (1)
And,
$-8 = a + (9 - 1)d$
$a + 8d = -8$ ….. (2)
Subtracting (1) from (2), we have
$6d = -8 - 4$
$6d = -12$
$d = -2$
Using d in (1), we get
$a + 2(-2) = 4$
$a = 4 + 4 = 8$
Now,
$t_n = 8 + (n - 1)(-2)$
Let nth term of this A.P. be 0
$8 + (n - 1)(-2) = 0$
$8 - 2n + 2 = 0$

$10 – 2n = 0$
$2n = 10$
$n = 5$
Therefore, the 5^{th} term of an A.P. is zero.

3. An A.P. consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term of the A.P.
Solution:

Given,
Number of terms in an A.P, n = 50
And, $t_3 = 12$
We know that,
$t_n = a + (n - 1)d$
$\Rightarrow a + 2d = 12$ ….(1)
Last term, l = 106
$t_{50} = 106$
$a + 49d = 106$ ….(2)
Subtracting (1) from (2), we get
$47d = 94$
$d = 2$
Substituting the value of d in equation (1), we get
$a + 2(2) = 12$
$a = 8$
Therefore, the 29^{th} term is
$t_{29} = a + 28d = 8 + 28(2) = 8 + 56 = 64$

4. Find the arithmetic mean of:
(i) -5 and 41
(ii) 3x - 2y and 3x + 2y
(iii) $(m + n)^2$ and $(m - n)^2$
Solution:

(i) Arithmetic mean of -5 and 41 = (-5 + 41)/ 2 = 36/2 = 18
(ii) Arithmetic mean of (3x – 2y) and (3x + 2y) = [(3x – 2y) + (3x + 2y)]/ 2 = 6x/ 2 = 3x

(iii) Arithmetic mean of $(m + n)^2$ and $(m - n)^2$

$$= \frac{(m+n)^2 + (m-n)^2}{2}$$
$$= \frac{m^2 + n^2 + 2mn + m^2 + n^2 - 2mn}{2}$$
$$= \frac{2(m^2+n^2)}{2}$$
$$= m^2 + n^2$$

5. Find the sum of first 10 terms of the A.P.
4 + 6 + 8 + ……
Solution:

Given A.P. is 4 + 6 + 8 + ……
Here,
a = 4 and d = 6 - 4 = 2
And, n = 10
$S_n = n/2\ [2a + (n - 1)d]$
$S_{10} = 10/2\ [2a + (10 - 1)d]$
$= 5\ [2(4) + 9(2)]$
$= 5\ [8 + 18]$
$= 5 \times 26$
$= 130$

6. Find the sum of first 20 terms of an A.P. whose first term is 3 and the last term is 57.
Solution:

Given,
First term, a = 3 and last term, l = 57
And, n = 20
$S = n/2\ (a + l)$
$= 20/2\ (3 + 57)$
$= 10\ (60)$
$= 600$

7. How many terms of the series 18 + 15 + 12 +…….. when added together will give 45?
Solution:

Given series, 18 + 15 + 12 +……..
Here,
a = 18 and d = 15 – 18 = -3
Let's consider the number of terms to be added as 'n'.
So, we have
$S_n = n/2\ [2a + (n - 1)d]$
$45 = n/2\ [2(18) + (n - 1)(-3)]$
$90 = n[36 - 3n + 3]$
$90 = n[39 - 3n]$
$90 = 3n[13 - n]$
$30 = 13n - n^2$
$n^2 - 13n + 30 = 0$
$n^2 - 10n - 3n + 30 = 0$
$n(n - 10) - 3(n - 10) = 0$
$(n - 10)(n - 3) = 0$
$n - 10 = 0$ or $n - 3 = 0$
$n = 10$ or $n = 3$

Therefore, the required number of terms to be added is 3 or 10.

8. The n^{th} term of a sequence is 8 - 5n. Show that the sequence is an A.P.
Solution:

Given, $t_n = 8 - 5n$
Now, replacing n by (n + 1), we get
$t_{n+1} = 8 - 5(n + 1) = 8 - 5n - 5 = 3 - 5n$
Now,
$t_{n+1} - t_n = (3 - 5n) - (8 - 5n) = -5$
As, $(t_{n+1} - t_n)$ is independent of n and is thus a constant.
Therefore, the given sequence having n^{th} term (8 - 5n) is an A.P.

9. Find the general term (n^{th} term) and 23^{rd} term of the sequence 3, 1, -1, -3, ….. .
Solution:

Given sequence is 3, 1, -1, -3, …..
Now,
1 - 3 = -1 - 1 = -3 - (-1) = -2
Thus, the given sequence is an A.P. where a = 3 and d = -2.
So, the general term of an A.P is given by
$t_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$
Therefore, the 23^{rd} term $= t_{23} = 5 - 2(23) = 5 - 46 = -41$

10. Which term of the sequence 3, 8, 13, …….. is 78?
Solution:

The given sequence is 3, 8, 13, …..
Now,
8 - 3 = 13 - 8 = 5
Hence, the given sequence is an A.P. with first term a = 3 and common difference d = 5.
Let the n^{th} term of the given A.P. be 78.
78 = 3 + (n - 1)(5)
75 = 5n - 5
5n = 80
n = 16
Therefore, the 16^{th} term of the given sequence is 78.

11. Is -150 a term of 11, 8, 5, 2, ……. ?
Solution:

Given sequence is 11, 8, 5, 2, …..
It's seen that,

8 - 11 = 5 - 8 = 2 - 5 = -3
Thus, the given sequence is an A.P. with a = 11 and d = -3.
So, the general term of an A.P. is given by
$t_n = a + (n - 1)d$
$-150 = 11 + (n - 1)(-3)$
$-161 = -3n + 3$
$3n = 164$
n = 164/3 (which is a fraction)
As the number of terms cannot be a fraction.
Hence, clearly -150 is not a term of the given sequence.

12. How many two digit numbers are divisible by 3?
Solution:

The two-digit numbers divisible by 3 are given below:
12, 15, 18, 21, ….., 99
It's clear that the above sequence forms an A.P
Where,
a = 12, d = 3 and last term (l) = 99
And, the general term is given by
$t_n = a + (n - 1)d$
So,
$99 = 12 + (n - 1)3$
$99 = 12 + 3n – 3$
$99 = 9 + 3n$
$90 = 3n$
n = 90/3 = 30
Therefore, there are 30 two-digit numbers that are divisible by 3.