Exercise 11(C)

1. Find the seventh term from the end of the series: $\sqrt{2}, 2, 2\sqrt{2}, \ldots\ldots , 32$
Solution:

Given series: $\sqrt{2}, 2, 2\sqrt{2}, \ldots\ldots , 32$
Here,
$a = \sqrt{2}$
$r = 2/\sqrt{2} = \sqrt{2}$
And, the last term (l) = 32
$l = t_n = ar^{n-1} = 32$
$(\sqrt{2})(\sqrt{2})^{n-1} = 32$
$(\sqrt{2})^n = 32$
$(\sqrt{2})^n = (2)^5 = (\sqrt{2})^{10}$
Equating the exponents, we have
n = 10
So, the 7^{th} term from the end is $(10 - 7 + 1)^{th}$ term.
i.e. 4^{th} term of the G.P
Hence,
$t_4 = (\sqrt{2})(\sqrt{2})^{4-1} = (\sqrt{2})(\sqrt{2})^3 = (\sqrt{2}) \times 2\sqrt{2} = 4$

2. Find the third term from the end of the G.P.
2/27, 2/9, 2/3, ……., 162
Solution:

Given series: 2/27, 2/9, 2/3, ……., 162
Here,
a = 2/27
r = (2/9) / (2/27)
r = 3
And, the last term (l) = 162
$l = t_n = ar^{n-1} = 162$
$(2/27)(3)^{n-1} = 162$
$(3)^{n-1} = 162 \times (27/2)$
$(3)^{n-1} = 2187$
$(3)^{n-1} = (3)^7$
n - 1 = 7
n = 7+1
n = 8
So, the third term from the end is $(8 - 3 + 1)^{th}$ term
i.e 6^{th} term of the G.P. = t_6
Hence,
$t_6 = ar^{6-1}$
$t_6 = (2/27)(3)^{6-1}$
$t_6 = (2/27)(3)^5$
$t_6 = 2 \times 3^2$

$t_6 = 18$

3. Find the G.P. 1/27, 1/9, 1/3, ……, 81; find the product of fourth term from the beginning and the fourth term from the end.
Solution:

Given G.P. 1/27, 1/9, 1/3, ……, 81
Here, a = 1/27, common ratio (r) = (1/9)/ (1/27) = 3 and l = 81
We know that,
$l = t_n = ar^{n-1} = 81$
$(1/27)(3)^{n-1} = 81$
$3^{n-1} = 81 \times 27 = 2187$
$3^{n-1} = 3^7$
$n - 1 = 7$
$n = 8$
Hence, there are 8 terms in the given G.P.
Now,
4^{th} term from the beginning is t_4 and the 4^{th} term from the end is $(8 - 4 + 1) = 5^{th}$ term (t_5)
Thus,
the product of t_4 and $t_5 = ar^{4-1} \times ar^{5-1} = ar^3 \times ar^4 = a^2r^7 = (1/27)^2(3)^7 = 3$

4. If for a G.P., p^{th}, q^{th} and r^{th} terms are a, b and c respectively; prove that:
(q - r) log a + (r - p) log b + (p - q) log c = 0
Solution:

Let's take the first term of the G.P. be A and its common ratio be R.
Then,
p^{th} term = a ⇒ $AR^{p-1} = a$
q^{th} term = b ⇒ $AR^{q-1} = b$
r^{th} term = c ⇒ $AR^{r-1} = c$
Now,

$$\begin{aligned} a^{q-r} \times b^{r-p} \times c^{p-q} &= (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q} \\ &= A^{q-r}.R^{(p-1)(q-r)} \times A^{r-p}.R^{(q-1)(r-p)} \times A^{p-q}.R^{(r-1)(p-q)} \\ &= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)} \\ &= A^0 \times R^0 \\ &= 1 \end{aligned}$$

On taking log on both the sides, we get
$\log(a^{q-r} \times b^{r-p} \times c^{p-q}) = \log 1$
⇒ (q - r)log a + (r - p)log b + (p - q)log c = 0
- Hence Proved