Exercise 11(D)

1. Find the sum of G.P.:
(i) 1 + 3 + 9 + 27 + ………. to 12 terms
(ii) 0.3 + 0.03 + 0.003 + 0.0003 +….. to 8 terms.
(iii) 1 – 1/2 + 1/4 – 1/8 + …….. to 9 terms
(iv) 1 - 1/3 + $1/3^2$ – $1/3^3$ + ……… to n terms

(v) $\frac{x+y}{x-y}+1+\frac{x-y}{x+y}+\ldots..\ upto\ n\ terms$

(vi) $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots...to\ n\ terms.$

Solution:

(i) Given G.P: 1 + 3 + 9 + 27 + ………. to 12 terms
Here,
a = 1 and r = 3/1 = 3 (r > 1)
Number of terms, n = 12
Hence,
$S_n = a(r^n - 1)/ r – 1$
$\Rightarrow S_{12} = (1)((3)^{12} - 1)/ 3 – 1$
$= (3^{12} – 1)/ 2$
= (531441 – 1)/ 2
= 531440/2
= 265720

(ii) Given G.P: 0.3 + 0.03 + 0.003 + 0.0003 +….. to 8 terms
Here,
a = 0.3 and r = 0.03/0.3 = 0.1 (r < 1)
Number of terms, n = 8
Hence,
$S_n = a(1 - r^n)/ 1 - r$
$\Rightarrow S_8 = (0.3)(1 - 0.1^8)/ (1 – 0.1)$
$= 0.3(1 – 0.1^8)/ 0.9$
$= (1 – 0.1^8)/ 3$
$= 1/3(1 – (1/10)^8)$

(iii) Given G.P: 1 – 1/2 + 1/4 – 1/8 + …….. to 9 terms
Here,
a = 1 and r = (-1/2)/ 1 = -1/2 (| r | < 1)
Number of terms, n = 9
Hence,
$S_n = a(1 - r^n)/ 1 - r$
$\Rightarrow S_9 = (1)(1 - (-1/2)^9)/ (1 – (-1/2))$
$= (1 + (1/2)^9)/ (3/2)$
= 2/3 x (1 + 1/512)

= 2/3 x (513/512)
= 171/ 256

(iv) Given G.P: 1 - 1/3 + 1/3^2 – 1/3^3 + ……… to n terms
Here,
a = 1 and r = (-1/3)/ 1 = -1/3 (| r | < 1)
Number of terms is n
Hence,
S_n = a(1 - r^n)/ 1 - r

$$S_n = \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1+\frac{1}{3}}$$

$$= \frac{\left[1-\left(-\frac{1}{3}\right)^n\right]}{\frac{4}{3}}$$

$$= \frac{3}{4}\left[1-\left(-\frac{1}{3}\right)^n\right]$$

(v) Given G.P:

$$\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \ldots\ldots \ upto\ n\ terms$$

Here,
a = (x + y)/ (x – y) and r = 1/[(x + y)/ (x – y)] = (x - y)/ (x + y) (| r | < 1)
Number of terms = n
Hence,
S_n = a(1 - r^n)/ 1 - r

$$S_n = \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{1-\left(\frac{x-y}{x+y}\right)}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{x+y-x+y}{x+y}}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{2y}{x+y}}$$

$$= \frac{(x+y)^2\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{2y(x-y)}$$

(vi) Given G.P:

$$\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \text{......to } n \text{ terms.}$$

Here,

$a = \sqrt{3}$ and $r = 1/\sqrt{3}/\sqrt{3} = 1/3$ $(|r| < 1)$

Number of terms = n

Hence,

$S_n = a(1 - r^n)/ 1 - r$

$$S_n = \frac{\sqrt{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} = \frac{\sqrt{3}\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$$

$$= \frac{3\sqrt{3}}{2}\left(1-\frac{1}{3^n}\right)$$

2. How many terms of the geometric progression 1 + 4 + 16 + 64 + …….. must be added to get sum equal to 5461?

Solution:

Given G.P: 1 + 4 + 16 + 64 + ……..

Here,

$a = 1$ and $r = 4/1 = 4$ $(r > 1)$

And,

$S_n = 5461$
We know that,
$S_n = a(r^n - 1)/ r - 1$
$\Rightarrow S_n = (1)((4)^n - 1)/ 4 - 1$
$= (4^n - 1)/3$
$5461 = (4^n - 1)/3$
$16383 = 4^n - 1$
$4^n = 16384$
$4^n = 4^7$
$n = 7$
Therefore, 7 terms of the G.P must be added to get a sum of 5461.

3. The first term of a G.P. is 27 and its 8th term is 1/81. Find the sum of its first 10 terms.
Solution:

Given,
First term (a) of a G.P = 27
And, 8th term = $t_8 = ar^{8-1} = 1/81$
$(27)r^7 = 1/81$
$r^7 = 1/(81 \times 27)$
$r^7 = (1/3)^7$
$r = 1/3$ $(r < 1)$
$S_n = a(1 - r^n)/ 1 - r$
Now,
Sum of first 10 terms = S_{10}

$$S_{10} = \frac{27\left(1-\left(\frac{1}{3}\right)^{10}\right)}{1-\frac{1}{3}} = \frac{27\left(1-\frac{1}{3^{10}}\right)}{\frac{2}{3}}$$

$$= \frac{81}{2}\left(1-\frac{1}{3^{10}}\right)$$

4. A boy spends Rs.10 on first day, Rs.20 on second day, Rs.40 on third day and so on. Find how much, in all, will he spend in 12 days?
Solution:

Given,
Amount spent on 1st day = Rs 10
Amount spent on 2nd day = Rs 20
And amount spent on 3rd day = Rs 40
It's seen that,
10, 20, 40, …… forms a G.P with first term, a = 10 and common ratio, r = 20/10 = 2 $(r > 1)$
The number of days, n = 12
Hence, the sum of money spend in 12 days is the sum of 12 terms of the G.P.

$S_n = a(r^n - 1)/ r – 1$
$S_{12} = (10)(2^{12} - 1)/ 2 – 1 = 10 (2^{12} - 1) = 10 (4096 - 1) = 10 \times 4095 = 40950$
Therefore, the amount spent by him in 12 days is Rs 40950

5. The 4th term and the 7th term of a G.P. are 1/27 and 1/729 respectively. Find the sum of n terms of the G.P.
Solution:

Given,
$t_4 = 1/27$ and $t_7 = 1/729$
We know that,
$t_n = ar^{n-1}$
So,
$t_4 = ar^{4-1} = ar^3 = 1/27$ …. (1)
$t_7 = ar^{7-1} = ar^6 = 1/729$ …. (2)
Dividing (2) by (1) we get,
$ar^6/ ar^3 = (1/729)/ (1/27)$
$r^3 = (1/3)^3$
$r = 1/3$ $(r < 1)$
In (1)
$a \times 1/27 = 1/27$
$a = 1$
Hence,
$S_n = a(1 - r^n)/ 1 - r$
$S_n = (1 – (1/3)^n)/ 1 – (1/3)$
$= (1 – (1/3)^n)/ (2/3)$
$= 3/2 (1 – (1/3)^n)$

6. A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728; find its first term.
Solution:

Given,
For a G.P.,
$r = 3$, $l = 486$ and $S_n = 728$
$$\frac{lr - a}{r - 1} = 728$$
$$\frac{486 \times 3 - a}{3 - 1} = 728$$
$$\frac{1458 - a}{2} = 728$$
$1458 – a = 728 \times 2 = 1456$
Thus, $a = 2$

7. Find the sum of G.P.: 3, 6, 12, …., 1536.

Solution:

Given G.P: 3, 6, 12, …., 1536
Here,
a = 3, l = 1536 and r = 6/3 = 2
So,
The sum of terms = (lr – a)/ (r - 1)
= (1536 x 2 – 3)/ (2 - 1)
= 3072 – 3
= 3069

8. How many terms of the series 2 + 6 + 18 + ….. must be taken to make the sum equal to 728?
Solution:

Given G.P: 2 + 6 + 18 + …..
Here,
a = 2 and r = 6/2 = 3
Also given,
$S_n = 728$
$S_n = a(r^n - 1)/ r – 1$
$728 = (2)(3^n - 1)/ 3 – 1 = 3^n – 1$
$729 = 3^n$
$3^6 = 3^n$
n = 6
Therefore, 6 terms must be taken to make the sum equal to 728.

9. In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125: 152.
Find its common ratio.
Solution:

Given,

$$\frac{a(r^3-1))}{r-1} : \frac{a(r^6-1))}{r-1} = 125 : 152$$

$$\frac{\frac{a(r^3-1))}{r-1}}{\frac{a(r^6-1))}{r-1}} = \frac{125}{152}$$

$$\frac{(r^3-1)}{(r^6-1)} = \frac{125}{152}$$

$$\frac{r^3-1}{(r^3)^2-(1)^2} = \frac{125}{152}$$

$$\frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$$

$$\frac{1}{r^3+1} = \frac{125}{152}$$

$$r^3+1 = \frac{152}{125}$$

$$r^3 = \frac{152}{125} - 1 = \frac{152-125}{125} = \frac{27}{125}$$

$$r^3 = \left(\frac{3}{5}\right)^3$$

$r = 3/5$

Therefore, the common ratio is 3/5.