

Exercise 14(B)

Page No: 190

1. Find the slope of the line whose inclination is:

- (i) 0° (ii) 30°
(iii) $72^\circ 30'$ (iv) 46°

Solution:

We know that, the slope of a line is given by the tan of its inclination.

- (i) Slope = $\tan 0^\circ = 0$
(ii) Slope = $\tan 30^\circ = 1/\sqrt{3}$
(iii) Slope = $\tan 72^\circ 30' = 3.1716$
(iv) Slope = $\tan 46^\circ = 1.0355$

2. Find the inclination of the line whose slope is:

- (i) 0 (ii) $\sqrt{3}$
(iii) 0.7646 (iv) 1.0875

Solution:

- (i) Slope = $\tan \theta = 0$
 $\Rightarrow \theta = 0^\circ$
(ii) Slope = $\tan \theta = \sqrt{3}$
 $\Rightarrow \theta = 60^\circ$
(iii) Slope = $\tan \theta = 0.7646$
 $\Rightarrow \theta = 37^\circ 24'$
(iv) Slope = $\tan \theta = 1.0875$
 $\Rightarrow \theta = 47^\circ 24'$

3. Find the slope of the line passing through the following pairs of points:

- (i) $(-2, -3)$ and $(1, 2)$
(ii) $(-4, 0)$ and origin
(iii) $(a, -b)$ and $(b, -a)$

Solution:

We know that,

- Slope = $(y_2 - y_1)/(x_2 - x_1)$
(i) Slope = $(2 + 3)/(1 + 2) = 5/3$
(ii) Slope = $(0 - 0)/(0 + 4) = 0$
(iii) Slope = $(-a + b)/(b - a) = 1$

4. Find the slope of the line parallel to AB if:

- (i) $A = (-2, 4)$ and $B = (0, 6)$
(ii) $A = (0, -3)$ and $B = (-2, 5)$

Solution:

- (i) Slope of AB = $(6 - 4)/(0 + 2) = 2/2 = 1$
Hence, slope of the line parallel to AB = Slope of AB = 1

(ii) Slope of AB = $(5 + 3) / (-2 - 0) = 8 / -2 = -4$
Hence, slope of the line parallel to AB = Slope of AB = -4

5. Find the slope of the line perpendicular to AB if:

(i) A = (0, -5) and B = (-2, 4)

(ii) A = (3, -2) and B = (-1, 2)

Solution:

(i) Slope of AB = $(4 + 5) / (-2 - 0) = -9/2$
Slope of the line perpendicular to AB = $-1 / \text{Slope of AB} = -1 / (-9/2) = 2/9$
(ii) Slope of AB = $(2 + 2) / (-1 - 3) = 4 / -4 = -1$
Slope of the line perpendicular to AB = $-1 / \text{slope of AB} = -1 / -1 = 1$

6. The line passing through (0, 2) and (-3, -1) is parallel to the line passing through (-1, 5) and (4, a). Find a.

Solution:

Slope of the line passing through (0, 2) and (-3, -1) = $(-1 - 2) / (-3 - 0) = -3 / -3 = 1$
Slope of the line passing through (-1, 5) and (4, a) = $(a - 5) / (4 + 1) = (a - 5) / 5$
As, the lines are parallel.
The slopes must be equal.
 $1 = (a - 5) / 5$
 $a - 5 = 5$
 $a = 10$

7. The line passing through (-4, -2) and (2, -3) is perpendicular to the line passing through (a, 5) and (2, -1). Find a.

Solution:

Slope of the line passing through (-4, -2) and (2, -3) = $(-3 + 2) / (2 + 4) = -1/6$
Slope of the line passing through (a, 5) and (2, -1) = $(-1 - 5) / (2 - a) = -6 / (2 - a)$
As, the lines are perpendicular we have
 $-1/6 = -1 / (-6 / (2 - a))$
 $-1/6 = (2 - a) / 6$
 $2 - a = -1$
 $a = 3$

8. Without using the distance formula, show that the points A (4, -2), B (-4, 4) and C (10, 6) are the vertices of a right-angled triangle.

Solution:

Given points are A (4, -2), B (-4, 4) and C (10, 6).
Calculating the slopes, we have
Slope of AB = $(4 + 2) / (-4 - 4) = 6 / -8 = -3/4$
Slope of BC = $(6 - 4) / (10 + 4) = 2 / 14 = 1/7$
Slope of AC = $(6 + 2) / (10 - 4) = 8 / 6 = 4/3$
It's clearly seen that,

Slope of AB = -1/ Slope of AC

Thus, $AB \perp AC$.

Therefore, the given points are the vertices of a right-angled triangle.

9. Without using the distance formula, show that the points A (4, 5), B (1, 2), C (4, 3) and D (7, 6) are the vertices of a parallelogram.

Solution:

Given points are A (4, 5), B (1, 2), C (4, 3) and D (7, 6).

Slope of AB = $(2 - 5) / (1 - 4) = -3 / -3 = 1$

Slope of CD = $(6 - 3) / (7 - 4) = 3 / 3 = 1$

As the slope of AB = slope of CD

So, we can conclude $AB \parallel CD$

Now,

Slope of BC = $(3 - 2) / (4 - 1) = 1 / 3$

Slope of DA = $(6 - 5) / (7 - 4) = 1 / 3$

As, slope of BC = slope of DA

Hence, we can say $BC \parallel DA$

Therefore, ABCD is a parallelogram.

10. (-2, 4), (4, 8), (10, 7) and (11, -5) are the vertices of a quadrilateral. Show that the quadrilateral, obtained on joining the mid-points of its sides, is a parallelogram.

Solution:

Let the given points be A (-2, 4), B (4, 8), C (10, 7) and D (11, -5).

And, let P, Q, R and S be the mid-points of AB, BC, CD and DA respectively.

So,

Co-ordinates of P are

$$\left(\frac{-2+4}{2}, \frac{4+8}{2} \right) = (1, 6)$$

Co-ordinates of Q are

$$\left(\frac{4+10}{2}, \frac{8+7}{2} \right) = \left(7, \frac{15}{2} \right)$$

Co-ordinates of R are

$$\left(\frac{10+11}{2}, \frac{7-5}{2} \right) = \left(\frac{21}{2}, 1 \right)$$

Co-ordinates of S are

$$\left(\frac{11-2}{2}, \frac{-5+4}{2} \right) = \left(\frac{9}{2}, \frac{-1}{2} \right)$$

$$\text{Slope of PQ} = \frac{\frac{15}{2} - 6}{7 - 1} = \frac{\frac{15 - 12}{2}}{6} = \frac{3}{12} = \frac{1}{4}$$

$$\text{Slope of RS} = \frac{\frac{-1}{2} - 1}{\frac{9}{2} - \frac{21}{2}} = \frac{\frac{-1 - 2}{2}}{\frac{9 - 21}{2}} = \frac{-3}{-12} = \frac{1}{4}$$

As, slope of PQ = Slope of RS, we can say PQ \parallel RS.
Now,

$$\text{Slope of QR} = \frac{1 - \frac{15}{2}}{\frac{21}{2} - 7} = \frac{\frac{2 - 15}{2}}{\frac{21 - 14}{2}} = \frac{-13}{7}$$

$$\text{Slope of SP} = \frac{\frac{-1}{2} - 6}{\frac{9}{2} - 1} = \frac{\frac{-1 - 12}{2}}{\frac{9 - 2}{2}} = \frac{-13}{7}$$

As, slope of QR = Slope of SP, we can say QR \parallel SP.
Therefore, PQRS is a parallelogram.

11. Show that the points P (a, b + c), Q (b, c + a) and R (c, a + b) are collinear.

Solution:

We know that,

The points P, Q, R will be collinear if the slopes of PQ and QR are the same.

Calculating for the slopes, we get

$$\text{Slope of PQ} = \frac{(c + a - b - c)}{(b - a)} = \frac{(a - b)}{(b - a)} = -1$$

$$\text{Slope of QR} = \frac{(a + b - c - a)}{(c - b)} = \frac{(b - c)}{(c - b)} = -1$$

Therefore, the points P, Q, and R are collinear.