

Exercise 14(D)

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1. Find the slope and y-intercept of the line:

(i) $y = 4$

(ii) $ax - by = 0$

(iii) $3x - 4y = 5$

Solution:

(i) $y = 4$

On comparing the given equation with $y = mx + c$, we get

Slope = $m = 0$

y - intercept = $c = 4$

(ii) $ax - by = 0 \Rightarrow by = ax \Rightarrow y = (a/b)x$

On comparing the above equation with $y = mx + c$, we get

Slope = $m = a/b$

y - intercept = $c = 0$

(iii) $3x - 4y = 5 \Rightarrow 4y = 3x - 5 \Rightarrow y = \frac{3}{4}x - \frac{5}{4}$

On comparing the above equation with $y = mx + c$, we get

Slope = $m = \frac{3}{4}$

y-intercept = $c = -\frac{5}{4}$

2. The equation of a line $x - y = 4$. Find its slope and y-intercept. Also, find its inclination.

Solution:

Given equation of a line: $x - y = 4$

$\Rightarrow y = x - 4$

Comparing the above equation with $y = mx + c$, we get

Slope = $m = 1$

y - intercept = $c = -4$

Let the inclination be θ .

Slope = $1 = \tan \theta = \tan 45^\circ$

$\theta = 45^\circ$

3. (i) Is the line $3x + 4y + 7 = 0$ perpendicular to the line $28x - 21y + 50 = 0$?

(ii) Is the line $x - 3y = 4$ perpendicular to the line $3x - y = 7$?

(iii) Is the line $3x + 2y = 5$ parallel to the line $x + 2y = 1$?

(iv) Determine x so that the slope of the line through (1, 4) and (x, 2) is 2.

Solution:

(i) Given,

$3x + 4y + 7 = 0$

$\Rightarrow 4y = -3x - 7$

$\Rightarrow y = (-\frac{3}{4})x - \frac{7}{4}$

Slope of this line = $-\frac{3}{4}$

And, for

$$28x - 21y + 50 = 0$$

$$\Rightarrow 21y = 28x + 50$$

$$\Rightarrow y = (28/21)x + 50/21$$

$$\Rightarrow y = (4/3)x + 50/21$$

Slope of this line = $4/3$

As, the product of slopes of the two lines = $4/3 \times -3/4 = -1$

Therefore, the lines are perpendicular to each other.

(ii) Given,

$$x - 3y = 4$$

$$\Rightarrow 3y = x - 4$$

$$\Rightarrow y = (1/3)x - 4/3$$

So, the slope of this line = $1/3$

And, for

$$3x - y = 7$$

$$y = 3x - 7$$

So, the slope of this line = 3

Now, the slopes of the two lines = $1/3 \times 3 = 1$ and not equal to -1 .

Hence, the lines are not perpendicular to each other.

(iii) Given,

$$3x + 2y = 5$$

$$2y = -3x + 5$$

$$y = (-3/2)x + 5/2$$

So, the slope of this line = $-3/2$

And, for

$$x + 2y = 1$$

$$2y = -x + 1$$

$$y = -1/2x + 1/2$$

So, the slope of this line = $-1/2$

The slopes of the two lines are not equal.

Hence, the lines are not parallel to each other.

(iv) Given, the slope of the line through $(1, 4)$ and $(x, 2)$ is 2 .

So,

$$\frac{2 - 4}{x - 1} = 2$$

$$\frac{-2}{x - 1} = 2$$

$$\frac{-1}{x - 1} = 1$$

$$-1 = x - 1$$

$$x = 0$$

4. Find the slope of the line which is parallel to:

(i) $x + 2y + 3 = 0$ (ii) $x/2 - y/3 - 1 = 0$

Solution:

(i) $x + 2y + 3 = 0$

$$2y = -x - 3$$

$$y = -1/2x - 3/2$$

$$\text{Slope of this line} = -1/2$$

Thus, slope of the line which is parallel to the given line = slope of the given line = $-1/2$

(ii) $x/2 - y/3 - 1 = 0$

$$y/3 = x/2 - 1$$

$$y = (3/2)x - 3$$

$$\text{So, the slope of this line} = 3/2$$

Thus, slope of the line which is parallel to the given line = Slope of the given line = $3/2$

5. Find the slope of the line which is perpendicular to:

(i) $x - y/2 + 3 = 0$ (ii) $x/3 - 2y = 4$

Solution:

(i) $x - y/2 + 3 = 0$

$$y/2 = x + 3$$

$$y = 2x + 6$$

$$\text{So, the slope of this line} = 2$$

We know that,

$$\text{Slope of the line which is perpendicular to the given line} = -1/(\text{Slope of the given line}) = -1/2$$

(ii) $x/3 - 2y = 4$

$$2y = x/3 - 4$$

$$y = x/6 - 2$$

$$\text{So, the slope of this line} = 1/6$$

We know that,

$$\text{Slope of the line which is perpendicular to the given line} = -1/(\text{Slope of the given line}) = -1/(1/6) = -6$$

6. (i) Lines $2x - by + 3 = 0$ and $ax + 3y = 2$ are parallel to each other. Find the relation connecting a and b .

(ii) Lines $mx + 3y + 7 = 0$ and $5x - ny - 3 = 0$ are perpendicular to each other. Find the relation connecting m and n .

Solution:

(i) We know that, if two lines are parallel then, the slopes of the two lines must be equal.

$$\text{For, } 2x - by + 3 = 0$$

$$by = 2x + 3$$

$$y = (2/b)x + 3/b$$

$$\text{So, the slope of this line} = 2/b$$

And, $ax + 3y = 2$

$$3y = -ax + 2$$

$$y = (-a/3)x + 2/3$$

So, the slope of this line = $-a/3$

Now, equating the slopes we get

$$2/b = -a/3$$

$$ab = -6$$

- (ii) We know that, if two lines are perpendicular to each other then, the product of their slopes = -1 .

For, $mx + 3y + 7 = 0$

$$3y = -mx - 7$$

$$y = -m/3x - 7/3$$

Slope of this line = $-m/3$

And, $5x - ny - 3 = 0$

$$ny = 5x - 3$$

$$y = (5/n)x - 3/n$$

Slope of this line = $5/n$

Products of slopes is

$$(-m/3) \times (5/n) = -1$$

$$5m = 3n$$

7. Find the value of p if the lines, whose equations are $2x - y + 5 = 0$ and $px + 3y = 4$ are perpendicular to each other.

Solution:

$$2x - y + 5 = 0 \dots (1)$$

$$px + 3y = 4 \dots (2)$$

$$y = 2x + 5$$

Now,

$$\text{Slope of line} = 2$$

$$px + 3y = 4$$

The above equation can be rewritten as

$$3y = -px + 4$$

$$y = (-p/3)x + 4/3$$

So, the slope of this line = $-p/3$

For 2 lines to be perpendicular to each other, the product of their slopes must be -1 .

So,

$$(2) \times (-p/3) = -1$$

$$2p/3 = 1$$

$$p = 3/2$$

8. The equation of a line AB is $2x - 2y + 3 = 0$.

(i) Find the slope of the line AB.

(ii) Calculate the angle that the line AB makes with the positive direction of the x-axis.

Solution:

- (i) Given, equation of the line
 $2x - 2y + 3 = 0$
 $2y = 2x + 3$
 $y = x + (3/2)$
So, the slope of the line $AB = 1$
- (ii) Required to find the angle of the line $AB = \theta$
We have,
 $\text{Slope} = \tan \theta = 1$
And, $\tan 45^\circ = 1$
Hence, $\theta = 45^\circ$

9. The lines represented by $4x + 3y = 9$ and $px - 6y + 3 = 0$ are parallel. Find the value of p .

Solution:

$$\begin{aligned}4x + 3y &= 9 \\3y &= -4x + 9 \\y &= (-4/3)x + 3 \\\text{Slope of this line} &= -4/3 \\\text{And,} \\px - 6y + 3 &= 0 \\6y &= px + 3 \\y &= (p/6)x + 1/2 \\\text{Slope of this line} &= p/6 \\\text{For two lines to be parallel, their slopes must be equal.} \\-4/3 &= p/6 \\-4 &= p/2 \\p &= -8\end{aligned}$$

10. If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p .

Solution:

$$\begin{aligned}y &= 3x + 7 \\\text{Slope of this line} &= 3 \\\text{And,} \\2y + px &= 3 \\2y &= -px + 3 \\y &= (-p/2)x + 3 \\\text{So, the slope of this line} &= -p/2 \\\text{If two lines are perpendicular to each other, then the product of their slopes is } -1. \\(3) \times (-p/2) &= -1 \\3p/2 &= 1 \\p &= 2/3\end{aligned}$$

11. The line through $A(-2,3)$ and $B(4,b)$ is perpendicular to the line $2x - 4y = 5$. Find the value of b .

Solution:

Given,

Points A (-2, 3) and B (4, b)

And, line equation: $2x - 4y = 5$

$$4y = 2x - 5$$

$$y = (1/2)x - 5/4$$

So, the slope of this line = $1/2$

From the question, it's said that

The line through A and B is perpendicular to above given line.

We know that, when two lines are perpendicular their product of slopes is -1

Hence, the slope of the line through A and B must be -2.

Now,

The slope of the line through A and B is given by,

$$\begin{aligned}\text{Slope of AB} &= (b - 3) / (4 - (-2)) \\ &= (b - 3) / 6\end{aligned}$$

Thus,

$$(b - 3) / 6 = -2$$

$$b - 3 = -12$$

$$b = -9$$

12. Find the equation of the line through (-5, 7) and parallel to:

(i) x-axis (ii) y-axis

Solution:

(i) We know that, the slope of a line parallel to x-axis is 0.

Here, $(x_1, y_1) = (-5, 7)$ and $m = 0$

So, the required line equation is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 0(x + 5)$$

$$y = 7$$

(ii) We know that, the slope of a line parallel to y-axis is not defined. ($\tan 90^\circ$)

So, the given line is parallel to y-axis.

Here, $(x_1, y_1) = (-5, 7)$

So, the required equation of the line is

$$x - x_1 = 0$$

$$x + 5 = 0$$