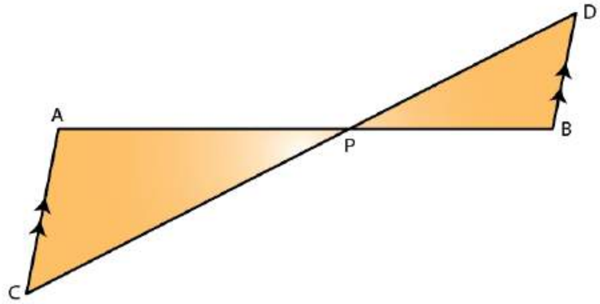
Exercise 15(A) Page No: 213

1. In the figure, given below, straight lines AB and CD intersect at P; and AC || BD. Prove that:
(i) ΔAPC and ΔBPD are similar.
(ii) If BD = 2.4 cm, AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm; find the lengths of PA and PC.

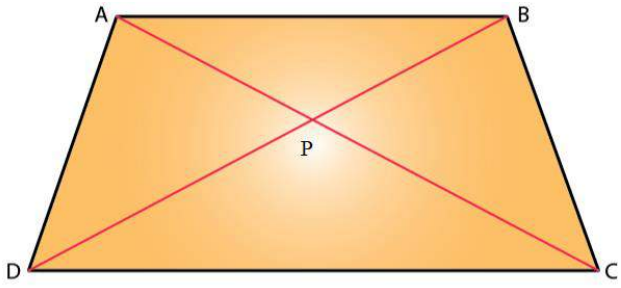


Solution:

(i) In ΔAPC and ΔBPD, we have
∠APC = ∠BPD [Vertically opposite angles]
∠ACP = ∠BDP [Alternate angles as, AC || BD]
Thus, ΔAPC ~ ΔBPD by AA similarity criterion

(ii) So, by corresponding parts of similar triangles, we have
PA/PB = PC/PD = AC/BD
Given, BD = 2.4 cm, AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm
PA/(3.2) = PC/4 = 3.6/2.4
PA/3.2 = 3.6/2.4 and PC/4 = 3.6/2.4
Thus,
PA = (3.6 x 3.2)/ 2.4 = 4.8 cm and
PC = (3.6 x 4)/ 2.4 = 6 cm

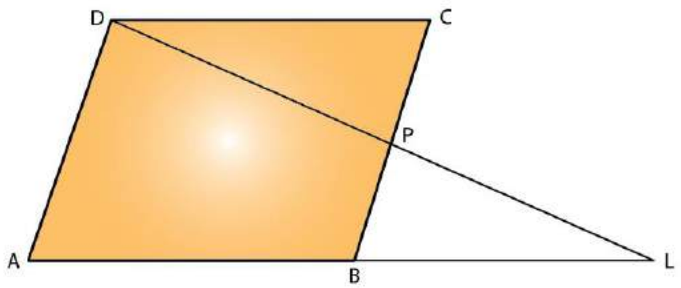
2. In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that:
(i) Δ APB is similar to Δ CPD.
(ii) PA x PD = PB x PC.
Solution:



(i) In ΔAPB and ΔCPD, we have
∠APB = ∠CPD [Vertically opposite angles]
∠ABP = ∠CDP [Alternate angles as, AB||DC]
Thus, ΔAPB ~ ΔCPD by AA similarity criterion.

(ii) As ΔAPB ~ ΔCPD
Since the corresponding sides of similar triangles are proportional, we have
PA/PC = PB/PD
Thus,
PA x PD = PB x PC

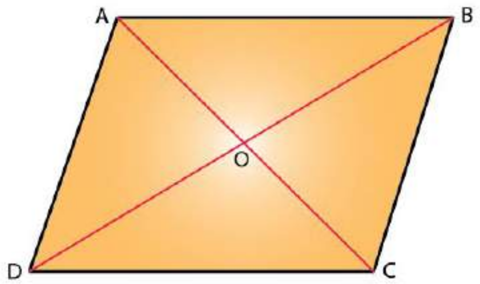
3. P is a point on side BC of a parallelogram ABCD. If DP produced meets AB produced at point L, prove that:
(i) DP : PL = DC : BL.
(ii) DL : DP = AL : DC.
Solution:



(i) As AD||BC, we have AD|| BP also.
So, by BPT
DP/PL = AB/BL
And, since ABCD is a parallelogram, AB = DC
Hence,
DP/PL = DC/BL
i.e., DP : PL = DC : BL

(ii) As AD||BC, we have AD|| BP also.
So, by BPT
DL/DP = AL/AB
And, since ABCD is a parallelogram, AB = DC
Hence,
DL/DP = AL/AB
i.e., DL : DP = AL : DC

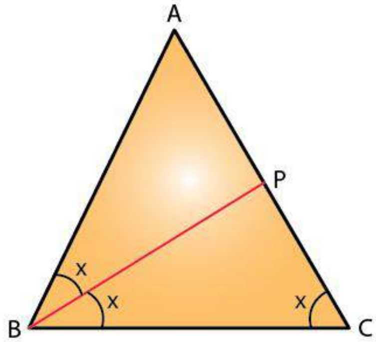
4. In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O. If AO = 2CO and BO = 2DO; show that:
(i) Δ AOB is similar to Δ COD.
(ii) OA x OD = OB x OC.
Solution:



(i) Given,
AO = 2CO and BO = 2DO,
AO/CO = 2/1 = BO/DO
And,
∠AOB = ∠DOC [Vertically opposite angles]
Hence, ΔAOB ~ ΔCOD [SAS criterion for similarity]

(ii) As, AO/CO = 2/1 = BO/DO [Given]
Thus,
OA x OD = OB x OC

5. In Δ ABC, angle ABC is equal to twice the angle ACB, and bisector of angle ABC meets the opposite side at point P. Show that :
(i) CB : BA = CP : PA
(ii) AB x BC = BP x CA
Solution:



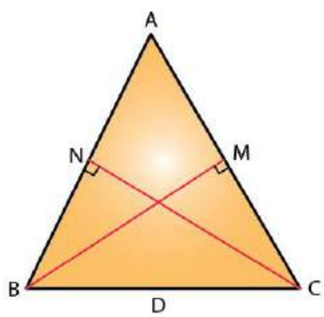
(i) In Δ ABC, we have
∠ABC = 2 ∠ACB [Given]
Now, let ∠ACB = x
So, ∠ABC = 2x
Also given, BP is bisector of ∠ABC
Thus, ∠ABP = ∠PBC = x
By using the angle bisector theorem,
i.e. the bisector of an angle divides the side opposite to it in the ratio of other two sides.
Therefore, CB: BA = CP: PA.

(ii) In Δ ABC and Δ APB,
∠ABC = ∠APB [Exterior angle property]
∠BCP = ∠ABP [Given]
Thus, ΔABC ~ ΔAPB by AA criterion for similarity
Now, since corresponding sides of similar triangles are proportional we have
CA/AB = BC/BP
Therefore, AB x BC = BP x CA

6. In Δ ABC, BM ⊥ AC and CN ⊥ AB; show that:

$$\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$$

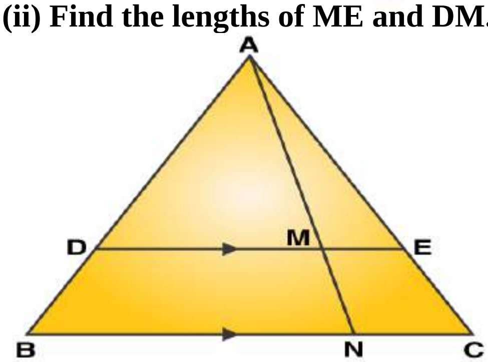
Solution:



In Δ ABM and Δ ACN,
∠AMB = ∠ANC [Since, BM ⊥ AC and CN ⊥ AB]
∠BAM = ∠CAN [Common angle]
Hence, ΔABM ~ ΔACN by AA criterion for similarity
So, by corresponding sides of similar triangles we have

$$\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$$

7. In the given figure, DE ∥ BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.
(i) Write all possible pairs of similar triangles.
(ii) Find the lengths of ME and DM.



Solution:

(i) In Δ AME and Δ ANC,

∠AME = ∠ANC [Since DE || BC so, ME || NC]
∠MAE = ∠NAC [Common angle]
Hence, ΔAME ~ ΔANC by AA criterion for similarity

In Δ ADM and Δ ABN,
∠ADM = ∠ABN [Since DE || BC so, DM || BN]
∠DAM = ∠BAN [Common angle]
Hence, ΔADM ~ ΔABN by AA criterion for similarity

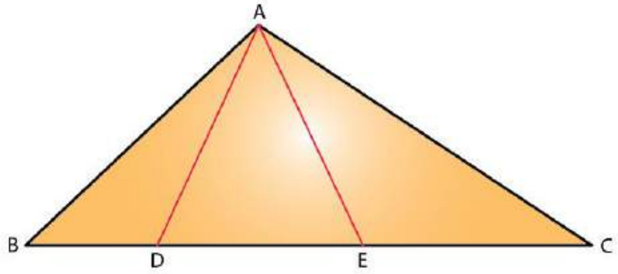
In Δ ADE and Δ ABC,
∠ADE = ∠ABC [Since DE || BC so, ME || NC]
∠AED = ∠ACB [Since DE || BC]
Hence, ΔADE ~ ΔABC by AA criterion for similarity

(ii) Proved above that, ΔAME ~ ΔANC
So as corresponding sides of similar triangles are proportional, we have
ME/NC = AE/AC
ME/ 6 = 15/ 24
ME = 3.75 cm

And, ΔADE ~ ΔABC [Proved above]
So as corresponding sides of similar triangles are proportional, we have
AD/AB = AE/AC = 15/24 …. (1)

Also, ΔADM ~ ΔABN [Proved above]
So as corresponding sides of similar triangles are proportional, we have
DM/BN = AD/AB = 15/24 …. From (1)
DM/ 24 = 15/ 24
DM = 15 cm

8. In the given figure, AD = AE and AD^2 = BD x EC. Prove that: triangles ABD and CAE are similar.

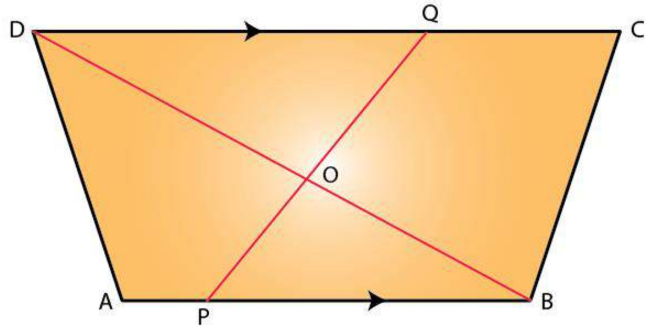


Solution:

In Δ ABD and Δ CAE,
∠ADE = ∠AED [Angles opposite to equal sides are equal.]
So, ∠ADB = ∠AEC [As ∠ADB + ∠ADE = 180^o and ∠AEC + ∠AED = 180^o]

And, $AD^2 = BD \times EC$ [Given]
AD/ BD = EC/ AD
AD/ BD = EC/ AE
Thus, ΔABD ~ ΔCAE by SAS criterion for similarity.

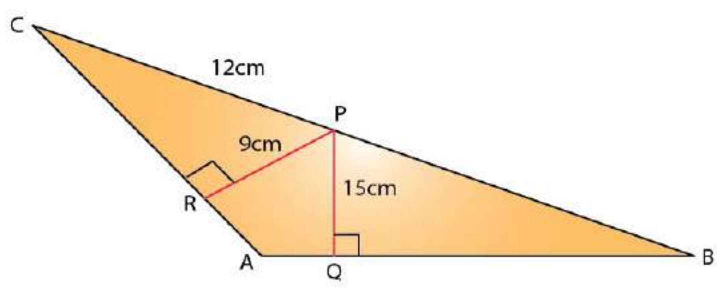
9. In the given figure, AB ∥ DC, BO = 6 cm and DQ = 8 cm; find: BP x DO.



Solution:

In Δ DOQ and Δ BOP,
∠QDO = ∠PBO [As AB || DC so, PB || DQ.]
So, ∠DOQ = ∠BOP [Vertically opposite angles]
Hence, ΔDOQ ~ ΔBOP by AA criterion for similarity
Since, corresponding sides of similar triangles are proportional we have
DO/BO = DQ/BP
DO/6 = 8/BP
BP x DO = 48 cm^2

10. Angle BAC of triangle ABC is obtuse and AB = AC. P is a point in BC such that PC = 12 cm. PQ and PR are perpendiculars to sides AB and AC respectively. If PQ = 15 cm and PR = 9 cm; find the length of PB.
Solution:



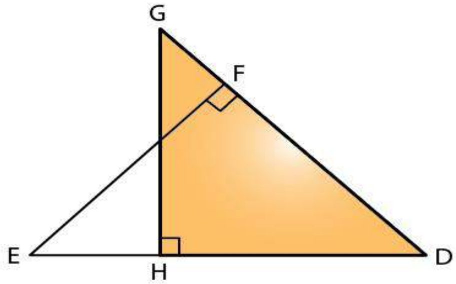
In Δ ABC,
AC = AB [Given]
So, ∠ABC = ∠ACB [Angles opposite to equal sides are equal.]
In Δ PRC and Δ PQB,

∠ABC = ∠ACB
∠PRC = ∠PQB [Both are right angles.]
Hence, ΔPRC ~ ΔPQB by AA criterion for similarity
Since, corresponding sides of similar triangles are proportional we have
PR/PQ = RC/QB = PC/PB
PR/PQ = PC/PB
9/15 = 12/PB
Thus,
PB = 20 cm

11. State, true or false:
(i) Two similar polygons are necessarily congruent.
(ii) Two congruent polygons are necessarily similar.
(iii) All equiangular triangles are similar.
(iv) All isosceles triangles are similar.
(v) Two isosceles-right triangles are similar.
(vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.
(vii) The diagonals of a trapezium, divide each other into proportional segments.
Solution:

(i) False
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) True

12. Given: ∠GHE = ∠DFE = 90º, DH = 8, DF = 12, DG = 3x - 1 and DE = 4x + 2.
Find: the lengths of segments DG and DE.

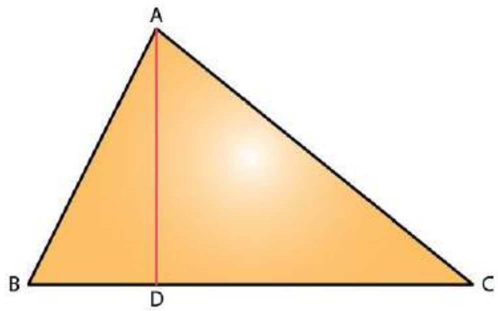


Solution:

In Δ DHG and Δ DFE,
∠GHD = ∠DFE = 90º
∠D = ∠D [Common]

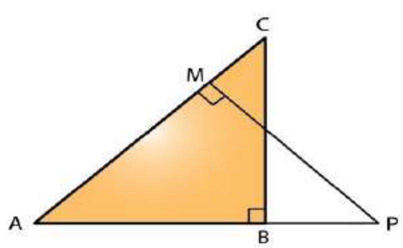
Thus, ΔDHG ~ ΔDFE by AA criterion for similarity
So, we have
DH/DF = DG/DE
8/12 = (3x - 1)/ (4x + 2)
32x + 16 = 36x – 12
28 = 4x
x = 7
Hence,
DG = 3 x 7 – 1 = 20
DE = 4 x 7 + 2 = 30

13. D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that: CA^2 = CB x CD.
Solution:



In Δ ADC and Δ BAC,
∠ADC = ∠BAC [Given]
∠ACD = ∠ACB [Common]
Thus, ΔADC ~ ΔBAC by AA criterion for similarity
So, we have
CA/CB = CD/CA
Therefore,
CA^2 = CB x CD

14. In the given figure, Δ ABC and Δ AMP are right angled at B and M respectively. Given AC = 10 cm, AP = 15 cm and PM = 12 cm.
(i) Δ ABC ~ Δ AMP.
(ii) Find AB and BC.
Solution:



(i) In Δ ABC and Δ AMP, we have
∠BAC = ∠PAM [Common]
∠ABC = ∠PMA [Each = 90^o]
Hence, ΔABC ~ ΔAMP by AA criterion for similarity

(ii) Now, in right triangle AMP
By using Pythagoras theorem, we have
$AM = \sqrt{(AP^2 – PM^2)} = \sqrt{(15^2 – 12^2)} = 9$
As ΔABC ~ ΔAMP,
AB/AM = BC/PM = AC/AP
AB/9 = BC/12 = 10/15
So,
AB/9 = 10/15
AB = (10 x 9)/ 15 = 6 cm
BC/12 = 10/ 15
BC = 8 cm