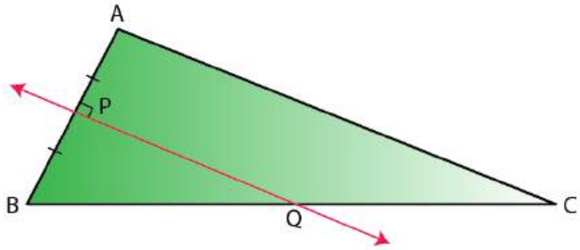
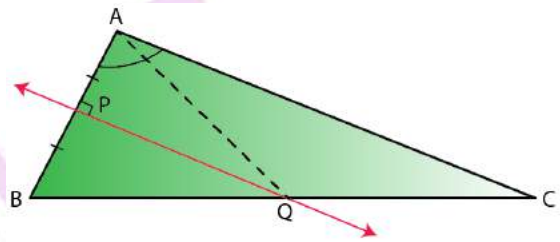
Exercise 16(A) Page No: 237

1. Given: PQ is a perpendicular bisector of side AB of the triangle ABC.

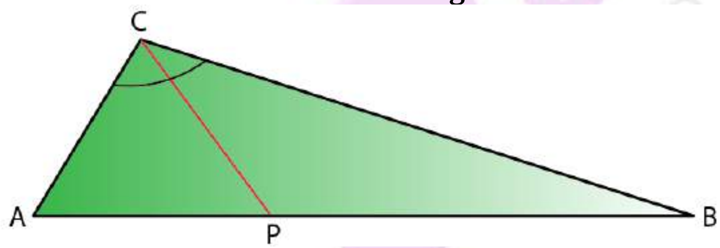


Prove: Q is equidistant from A and B.
Solution:

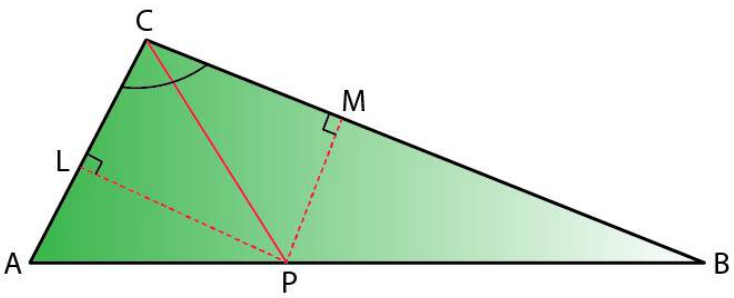


Construction: Join AQ
Proof:
In ΔAQP and ΔBQP,
AP = BP [Given]
∠QPA = ∠QPB [Each 90°]
PQ = PQ [Common]
Thus, by SAS criterion of congruence
ΔAQP ≅ ΔBQP
And, by Corresponding Parts of Congruent Triangle (CPCT)
AQ = BQ
Therefore, Q is equidistant from A and B.

2. Given: CP is the bisector of angle C of ΔABC



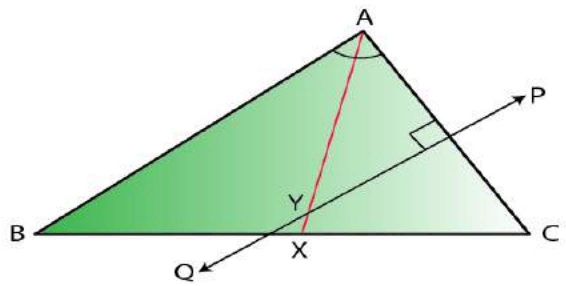
Prove: P is equidistant from AC and BC.
Solution:



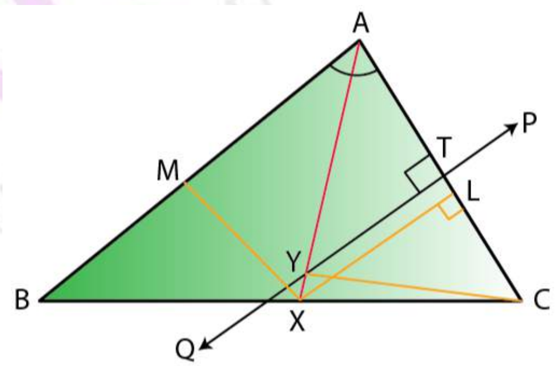
Construction: From P, draw PL ⊥ AC and PM ⊥ CB
Proof:
In ΔLPC and ΔMPC,
∠PLC = ∠PMC [Each 90°]
∠PCL = ∠MCP [Given]
PC = PC [Common]
Thus, by AAS criterion of congruence
ΔLPC ≅ ΔMPC
And, by Corresponding Parts of Congruent Triangle (CPCT)
PL = PM

Therefore, P is equidistant from AC and BC.

3. Given: AX bisects angle BAC and PQ is perpendicular bisector of AC which meets AX at point Y.



Prove:
i) X is equidistant from AB and AC.
ii) Y is equidistant from A and C
Solution:

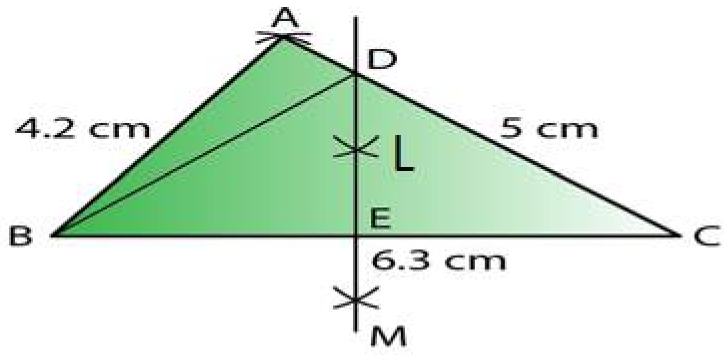


Construction: From X, draw XL ⊥ AC and XM ⊥ AB.
And, join YC
Proof:

(i) In ΔAXL and ΔAXM,
∠XAL = ∠XAM [Given]
AX = AX [Common]
∠XLA = ∠XMA [Each 90°]
Thus, by ASA criterion of congruence
ΔAXL ≅ ΔAXM
And, by Corresponding Parts of Congruent Triangle (CPCT)
XL = XM
Therefore, X is equidistant from AC and AB.

(ii) In ΔYTA and ΔYTC,
AT = CT [because PQ is perpendicular bisector of AC]
∠YTA = ∠YTC [Each 90°]
YT = YT [Common]
Thus, by SAS criterion of congruence
ΔYTA ≅ ΔYTC
And, by Corresponding Parts of Congruent Triangle (CPCT)
YA = YC
Therefore, Y is equidistant from A and C.

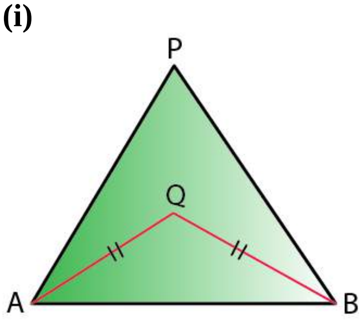
4. Construct a triangle ABC, in which AB = 4.2 cm, BC = 6.3 cm and AC = 5 cm. Draw perpendicular bisector of BC which meets AC at point D. Prove that D is equidistant from B and C.
Solution:



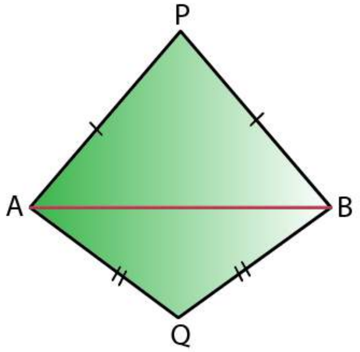
Given: In triangle ABC, AB = 4.2 cm, BC = 6.3 cm and AC = 5 cm
Steps of Construction:
i) Draw a line segment BC = 6.3 cm
ii) With centre B and radius 4.2 cm, draw an arc.
iii) With centre C and radius 5 cm, draw another arc which intersects the first arc at A.
iv) Join AB and AC.
Then, ΔABC is the required triangle.
v) Again with centre B and C and radius greater than ½ BC, draw arcs which intersects each other at L and M.
vi) Join LM intersecting AC at D and BC at E.
vii) Join DB.

Proof:
In ΔDBE and ΔDCE,
BE = EC [LM is the bisector of BC]
∠DEB = ∠DEC [Each 90º]
DE = DE [Common]
Thus, by SAS criterion of congruence
ΔDBE ≅ ΔDCE
And, by Corresponding Parts of Congruent Triangle (CPCT)
DB = DC
Therefore, D is equidistant from B and C.

5. In each of the given figures: PA = PB and QA = QB
(i)

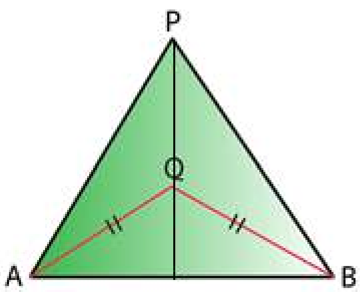


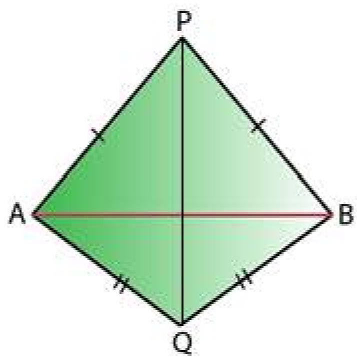
(ii)



Prove in each case, that PQ (produced, if required) is perpendicular bisector of AB. Hence, state the locus of the points equidistant from two given fixed points.
Solution:

Construction: Join PQ which meets AB in D.

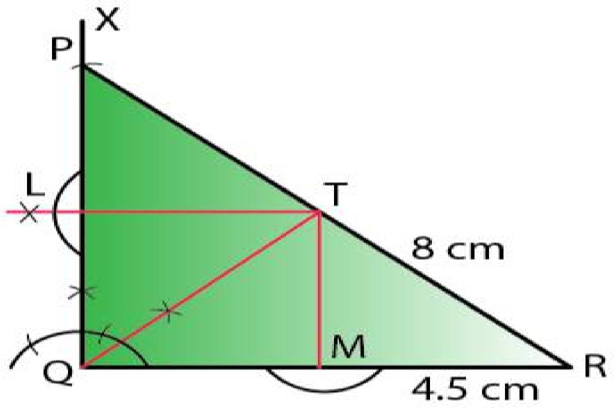




Proof:
As P is equidistant from A and B.
Thus, P lies on the perpendicular bisector of AB.
Similarly, as Q is equidistant from A and B.
Q lies on perpendicular bisector of AB.
Thus, both P and Q lie on the perpendicular bisector of AB.
Therefore, PQ is the perpendicular bisector of AB.

Hence, locus of the points which are equidistant from two fixed points, is a perpendicular bisector of the line joining the fixed points.

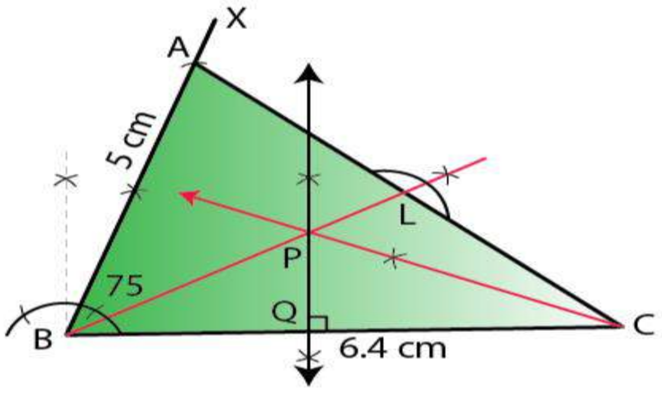
6. Construct a right angled triangle PQR, in which ∠Q = 90º, hypotenuse PR = 8 cm and QR = 4.5 cm. Draw bisector of angle PQR and let it meet PR at point T. Prove that T is equidistant from PQ and QR.
Solution:



Steps of Construction:
i) Draw a line segment QR = 4.5 cm
ii) At Q, draw a ray QX making an angle of 90°
iii) With centre R and radius 8 cm, draw an arc which intersects QX at P.
iv) Join RP.
ΔPQR is the required triangle.
v) Draw the bisector of ∠PQR which meets PR at T.
vi) From T, draw perpendicular TL and TM respectively on PQ and QR.

Proof:
In ΔLTQ and ΔMTQ
∠TLQ = ∠TMQ [Each 90°]
∠LQT = ∠TQM [QT is angle bisector]
QT = QT [Common]
Thus, by AAS criterion of congruence
ΔLTQ ≅ ΔMTQ
And, by Corresponding Parts of Congruent Triangle (CPCT)
TL = TM
Therefore, T is equidistant from PQ and QR.

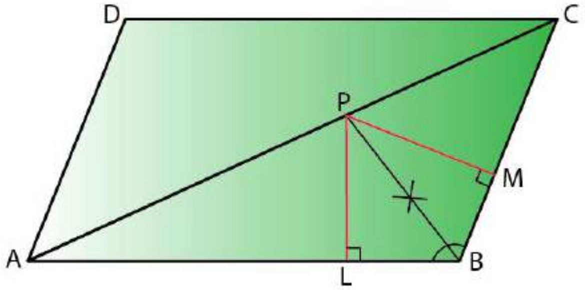
7. Construct a triangle ABC in which angle ABC = 75°, AB = 5cm and BC = 6.4 cm. Draw perpendicular bisector of side BC and also the bisector of angle ACB. If these bisectors intersect each other at point P; prove that P is equidistant from B and C; and also from AC and BC.
Solution:



Steps of Construction:
i) Draw a line segment BC = 6.4 cm
ii) At B, draw a ray BX making an angle of 75° with BC and cut off BA = 5 cm.
iii) Join AC.
ΔABC is the required triangle.
iv) Draw the perpendicular bisector of BC.
v) Draw the angle bisector of angle ACB which intersects the perpendicular bisector of BC at P.
vi) Join PB and draw PL ⊥ AC.

Proof:
In ΔPBQ and ΔPCQ,
PQ = PQ [Common]
∠PQB = ∠PQC [Each 90°]
BQ = QC [PQ is the perpendicular bisector of BC]
Thus, by SAS criterion of congruence
ΔPBQ ≅ ΔPCQ
And, by Corresponding Parts of Congruent Triangle (CPCT)
PB = PC
Therefore, P is equidistant from B and C.
Also,
In ΔPQC and ΔPLC,
∠PQC = ∠PLC [Each 90°]
∠PCQ = ∠PCL [Given]
PC = PC [Common]
Thus, by AAS criterion of congruence
ΔPQC ≅ ΔPLC
And, by Corresponding Parts of Congruent Triangle (CPCT)
PQ = PL
Therefore, P is equidistant from AC and BC.

8. In parallelogram ABCD, side AB is greater than side BC and P is a point in AC such that PB bisects angle B. Prove that P is equidistant from AB and BC.
Solution:

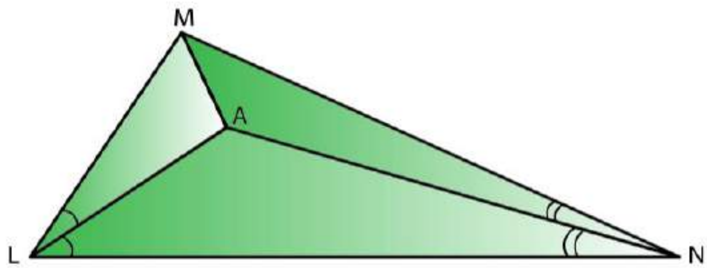


Construction: From P, draw PL ⊥ AB and PM ⊥ BC
Proof:

In ΔPLB and ΔPMB,
∠PLB = ∠PMB [Each 90°]

∠PBL = ∠PBM [Given]
PB = PB [Common]
Thus, by AAS criterion of congruence
ΔPLB ≅ ΔPMB
And, by Corresponding Parts of Congruent Triangle (CPCT)
PL = PM
Therefore, P is equidistant from AB and BC.

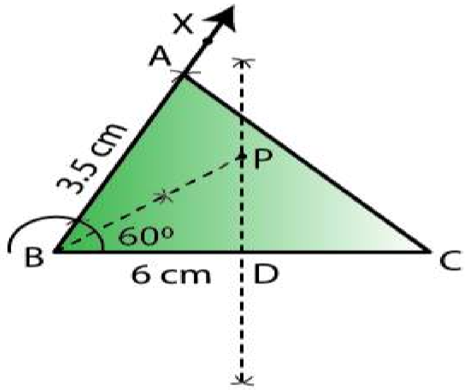
9. In triangle LMN, bisectors of interior angles at L and N intersect each other at point A. Prove that:
i) point A is equidistant from all the three sides of the triangle.
ii) AM bisects angle LMN.
Solution:



Construction: Join AM
Proof:
(i) Since, A lies on bisector of ∠N
So, A is equidistant from MN and LN.
Again, as A lies on the bisector of ∠L
A is equidistant from LN and LM.
Therefore, A is equidistant from all three sides of the triangle LMN.

And,
(ii) As A lies on the bisector of ∠M.
Hence, AM bisects ∠LMN

10. Use ruler and compasses only for this question.
(i) Construct ΔABC, where AB = 3.5 cm, BC = 6 cm and ∠ABC = 60º
(ii) Construct the locus of points inside the triangle which are equidistant from BA and BC.
(iii) Construct the locus of points inside the triangle which are equidistant from B and C.
(iv) Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB.
Solution:



Steps of construction:

(i) Draw line BC = 6 cm and construct angle CBX = 60°. Cut off AB = 3.5. Join AC, triangle ABC is the required triangle.

(ii) Draw the perpendicular bisector of BC and bisector of angle B.

(iii) Now, bisector of angle B meets bisector of BC at P.

Hence, BP is the required length, where PB = 3.5 cm.

(iv) P is the point which is equidistant from BA and BC, which is also equidistant from B and C.

PB = 3.5 cm