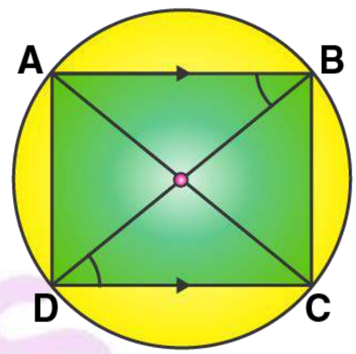
Exercise 17(B) Page No: 265

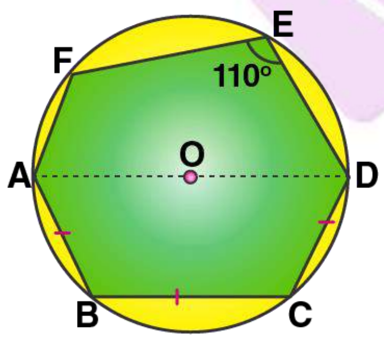
1. In a cyclic-trapezium, the non-parallel sides are equal and the diagonals are also equal. Prove it.

Solution:



Let ABCD be the cyclic trapezium in which AB || DC, AC and BD are the diagonals.
Required to prove:
(i) AD = BC
(ii) AC = BD
Proof:
It's seen that chord AD subtends ∠ABD and chord BC subtends ∠BDC at the circumference of the circle.
But, ∠ABD = ∠BDC [Alternate angles, as AB || DC with BD as the transversal]
So, Chord AD must be equal to chord BC
AD = BC
Now, in ΔADC and ΔBCD
DC = DC [Common]
∠CAD = ∠CBD [Angles in the same segment are equal]
AD = BC [Proved above]
Hence, by SAS criterion of congruence
ΔADC ≅ ΔBCD
Therefore, by CPCT
AC = BD

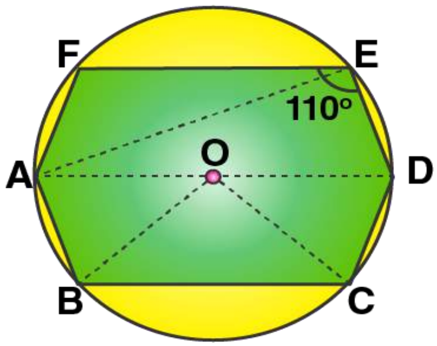
2. In the following figure, AD is the diameter of the circle with centre O. Chords AB, BC and CD are equal. If ∠DEF = 110°, calculate:
(i) ∠AFE, (ii) ∠FAB.



Solution:

Join AE, OB and OC.
(i) As AOD is the diameter
∠AED = 90° [Angle in a semi-circle is a right angle]
But, given ∠DEF = 110°

So,
∠AEF = ∠DEF - ∠AED = 110° – 90° = 20°

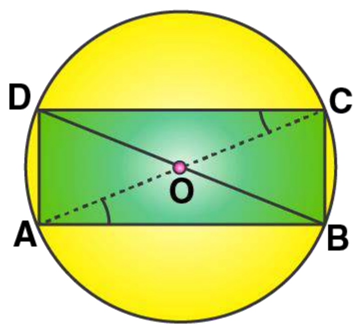


(ii) Also given, Chord AB = Chord BC = Chord CD
So,
∠AOB = ∠BOC = ∠COD [Equal chords subtends equal angles at the centre]
But,
∠AOB + ∠BOC + ∠COD = 180° [Since, AOD is a straight line]
Thus,
∠AOB = ∠BOC = ∠COD = 60°
Now, in ΔOAB we have
OA = OB [Radii of same circle]

So, ∠OAB = ∠OBA [Angles opposite to equal sides]
But, by angle sum property of ΔOAB
∠OAB + ∠OBA = 180° - ∠AOB
= 180° – 60°
= 120°
Therefore, ∠OAB = ∠OBA = 60°
Now, in cyclic quadrilateral ADEF
∠DEF + ∠DAF = 180°
∠DAF = 180° - ∠DEF
= 180° – 110°
= 70°
Thus,
∠FAB = ∠DAF + ∠OAB
= 70° + 60° = 130°

3. If two sides of a cycli-quadrilateral are parallel; prove that:
(i) its other two sides are equal.
(ii) its diagonals are equal.
Solution:

Let ABCD is a cyclic quadrilateral in which AB || DC. AC and BD are its diagonals.
Required to prove:
(i) AD = BC
(ii) AC = BD
Proof:

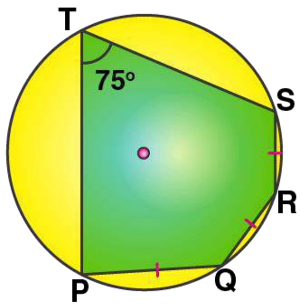


(i) As AB || DC (given)
∠DCA = ∠CAB [Alternate angles]
Now, chord AD subtends ∠DCA and chord BC subtends ∠CAB at the circumference of the circle.
So,
∠DCA = ∠CAB

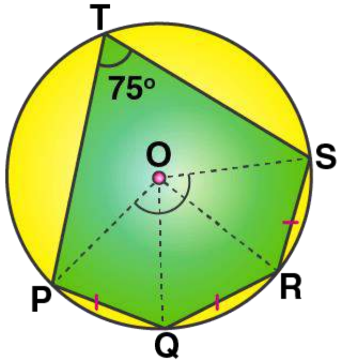
Hence, chord AD = chord BC or AD = BC.

(ii) Now, in ΔABC and ΔADB
AB = AB [Common]
∠ACB = ∠ADB [Angles in the same segment are equal]
BC = AD [Proved above]
Hence, by SAS criterion of congruence
ΔACB ≅ ΔADB
Therefore, by CPCT
AC = BD

4. The given figure show a circle with centre O. Also, PQ = QR = RS and ∠PTS = 75°.



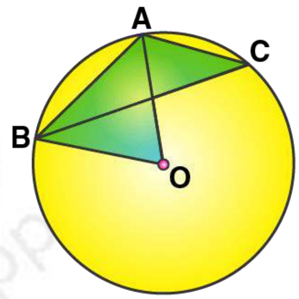
Calculate:
(i) ∠POS,
(ii) ∠QOR,
(iii) ∠PQR.
Solution:



Join OP, OQ, OR and OS.
Given, PQ = QR = RS
So, ∠POQ = ∠QOR = ∠ROS [Equal chords subtends equal angles at the centre]
Arc PQRS subtends ∠POS at the centre and ∠PTS at the remaining part of the circle.
Thus,
$\angle POS = 2 \times \angle PTS = 2 \times 75^o = 150^o$
$\angle POQ + \angle QOR + \angle ROS = 150^o$
$\angle POQ = \angle QOR = \angle ROS = 150^o/ 3 = 50^o$
In ΔOPQ we have,
OP = OQ [Radii of the same circle]
So, ∠OPQ = ∠OQP [Angles opposite to equal sides are equal]
But, by angle sum property of ΔOPQ
$\angle OPQ + \angle OQP + \angle POQ = 180^o$
$\angle OPQ + \angle OQP + 50^o = 180^o$
$\angle OPQ + \angle OQP = 130^o$
$2 \angle OPQ = 130^o$
$\angle OPQ = \angle OPQ = 130^o/ 2 = 65^o$

Similarly, we can prove that
In ΔOQR,
$\angle OQR = \angle ORQ = 65^o$
And in ΔORS,
$\angle ORS = \angle OSR = 65^o$
Hence,
(i) $\angle POS = 150^o$
(ii) $\angle QOR = 50^o$ and
(iii) $\angle PQR = \angle PQO + \angle OQR = 65^o + 65^o = 130^o$

5. In the given figure, AB is a side of a regular six-sided polygon and AC is a side of a regular eight-sided polygon inscribed in the circle with centre O. calculate the sizes of:
(i) ∠AOB,
(ii) ∠ACB,
(iii) ∠ABC.



Solution:

(i) Arc AB subtends ∠AOB at the centre and ∠ACB at the remaining part of the circle.
$\angle ACB = \frac{1}{2} \angle AOB$
And as AB is the side of a regular hexagon, we have
$\angle AOB = 60^o$

(ii) Now,
$\angle ACB = \frac{1}{2} (60^o) = 30^o$

(iii) Since AC is the side of a regular octagon,
$\angle AOC = 360^o / 8 = 45^o$
Again, arc AC subtends ∠AOC at the centre and ∠ABC at the remaining part of the circle.
$\angle ABC = \frac{1}{2} \angle AOC$
$\angle ABC = 45^o / 2 = 22.5^o$