KVPY-SX 2016 (PHYSICS)

PART-I

1. Physical processes are sometimes described visually by lines. Only the following can cross -

(a) Streamlines in fluid flow
(b) Lines of forces in electrostatics
(c) Rays in geometrical optics
(d) Lines of force in magnetism

2. A uniform ring of radius R is moving on a horizontal surface with speed v and then climbs up a ramp of inclination 30° to a height h. There is no slipping in the entire motion. Then h is

(a) $v^2/2g$
(b) v^2/g
(c) $3v^2/2g$
(d) $2v^2/g$

3. A gas at initial temperature T undergoes sudden expansion from volume V to 2V. Then -

(a) The process is adiabatic
(b) The process is isothermal
(c) The work done in this process is $nRT\lambda n_e(2)$ where n is the number of moles of the gas.
(d) The entropy in the process does not change

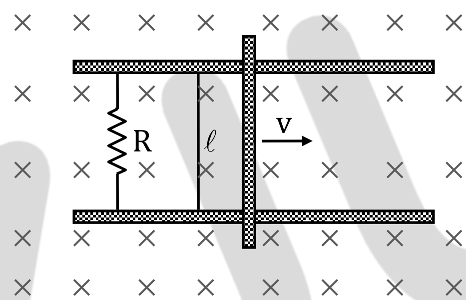
4. Photons of wavelength λ are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius R by a perpendicular magnetic field having a magnitude B. The work function of the metal is (Where symbols have their usual meanings) –

(a) $\frac{hc}{\lambda} - m_e + \frac{e^2B^2R^2}{2m_e}$
(b) $\frac{hc}{\lambda} + 2m_e\left(\frac{eBR}{2m_e}\right)^2$
(c) $\frac{hc}{\lambda} - m_eC^2 - \frac{e^2B^2R^2}{2m_e}$
(d) $\frac{hc}{\lambda} - 2m_e\left(\frac{eBR}{2m_e}\right)^2$

5. A container is divided into two equal part I and II by a partition with a small hole of diameter d. The two partitions are filled with same ideal gas, but held at temperatures T_I = 150 K and T_{II} = 300 K by connecting to heat reservoirs. Let λ_I and λ_{II} be the mean free paths of the gas particles in the two parts such that $d >> \lambda_I$ and $d >> \lambda_{II}$. Then λ_I/λ_{II} is close to -

(a) 0.25 (b) 0.5 (c) 0.7 (d) 1.0

6. A conducting bar of mass m and length ℓ moves on two frictionless parallel rails in the presence of a constant uniform magnetic field of magnitude B directed into the page as shown in the figure. The bar is given an initial velocity v_0 towards the right at t = 0. Then the



(a) Induced current in the circuit is in the clockwise direction

(b) Velocity of the bar decreases linearly with time

(c) Distance the bar travels before it comes to a complete stop is proportional to R

(d) Power generated across the resistance is proportional to ℓ

7. A particle with total mechanical energy, which is small and negative, is under the influence of a one dimensional potential $U(x) = x^4/4 - x^2/2$ J,where x is in meters. At time t = 0s, it is at x = –0.5 m. Then at a later time it can be found

(a) Anywhere on the x axis

(b) Between x = – 1.0 m to x = 1.0 m

(c) Between x = – 1.0 m to x = 0.0 m

(d) Between x = 0.0 m to x = 1.0 m

8. A nurse measures the blood pressure of a seated patient to be 190 mm of Hg -

(a) The blood pressure at the patient's feet is less than 190 mm of Hg

(b) The actual pressure is about 0.25 times the atmospheric pressure

(c) The blood pressure at the patient's neck is more than 190 mm of Hg

(d) The actual pressure is about 1.25 times the atmospheric pressure

9. A particle at a distance of 1 m from the origin starts moving such that $dr/d\theta = r$, where (r, θ) are polar coordinates. Then the angle between resultant velocity and tangential velocity component is

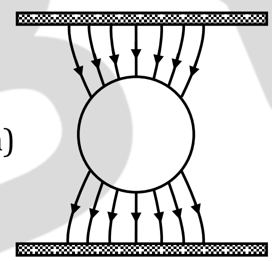
(a) 30 degrees | (b) 45 degrees

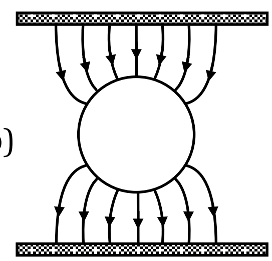
(c) 60 degrees | (d) Dependent on where the particle is

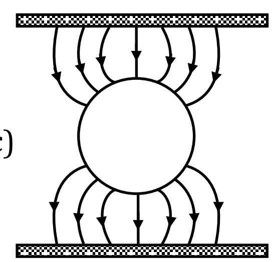
10. Electrons accelerated from rest by an electrostatic potential are collimated and sent through a Young's double slit setup. The fringe width is w. If the accelerating potential is doubled then the width is now close to -

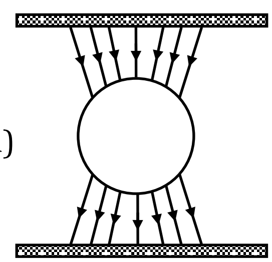
(a) 0.5 w (b) 0.7 w (c) 1.0 w (d) 2.0 w

11. A metallic sphere is kept in between two oppositely charged plates. The most appropriate representation of the field lines is –

(a)


(b)


(c)


(d)


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12. An electron with kinetic energy E collides with a hydrogen atom in the ground state. The collision will be elastic

(a) For all values of E (b) For E < 10.2 eV

(c) For 10.2 eV < E < 13.6 eV only (d) For 0 < E < 3.4 eV only

13. The continuous part of X-ray spectrum is a result of the

(a) Photoelectric effect (b)Raman effect

(c) Compton effect (d) Inverse photoelectric effect

14. Thermal expansion of a solid is due to the

(a) Symmetric characteristic of the inter atomic potential energy curve of the solid.

(b) Asymmetric characteristic of the inter atomic potential energy curve of the solid.

(c) Double well nature of the inter-atomic potential energy curve of the solid.

(d) Rotational motion of the atoms of the solid.

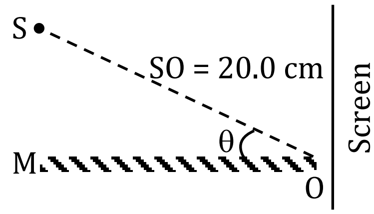
15. An electron and a photon have same wavelength of 10^{-9} m. If E is the energy of the photon and p is the momentum of the electron, the magnitude of E/p in SI units is

(a) 1.00×10^{-9} (b) 1.50×10^{8} (c) 3.00×10^{8} (d) 1.20×10^{7}

16. If one takes into account finite mass of the proton, the correction to the binding energy of the hydrogen atom is approximately (mass of proton = 1.60×10^{-27} kg, mass of electron = 9.10×10^{-31} kg)-

(a) 0.06% (b) 0.0006 % (c) 0.02 % (d) 0.00 %

17. A monochromatic light source S of wavelength 440 nm is placed slightly above a plane mirror M as shown. Image of S in M can be used as a virtual source to produce interference fringes on the screen. The distance of source S from O is 20.0 cm, and the distance of screen from O is 100.0 cm (figure is not to scale). If the angle $\theta = 0.50 \times 10^{-3}$ radians, the width of the interference fringes observed on the screen is –



(a) 2.20 mm (b) 2.64 mm (c) 1.10 mm (d) 0.55 mm

18. A nuclear fuel rod generates energy at a rate of 5×10^8 Watt/m^3. It is in the shape of a cylinder of radius 4.0 mm and length 0.20 m. A coolant of specific heat 4×10^3 J/(kg-K) flows past it at a rate of 0.2 kg/s. The temperature rise in this coolant is approximately -

(a) 2°C (b) 6°C (c) 12°C (d) 30°C

19. A hearing test is conducted on an aged person. It is found that her threshold of hearing is 20 decibels at 1 kHz and it rises linearly with frequency to 60 decibels at 9 kHz. The minimum intensity of sound that the person can hear at 5 kHz is-

(a) 10 times than that at 1 kHz (b) 100 times than that at 1 kHz

(c) 0.5 times than that at 9 kHz (d) 0.05 times than that at 9 kHz

20. Two infinitely long parallel wires carry currents of magnitude I_1 and I_2 and are at a distance 4 cm apart. The magnitude of the net magnetic field is found to reach a non-zero minimum value between the two wires and 1 cm away from the first wire. The ratio of the two currents and their mutual direction is

(a) $\frac{I_2}{I_1} = 9$, antiparallel (b) $\frac{I_2}{I_1} = 9$, parallel

(c) $\frac{I_2}{I_1} = 3$, antiparallel (d) $\frac{I_2}{I_1} = 3$, parallel

PART-II

21. A light balloon filled with helium of density ρ_{He} is tied to a long light string of length l and the string is attached to the ground. If the balloon is displaced slightly in the horizontal direction from the equilibrium and released then.

(a) The balloon undergoes simple harmonic motion with period $2\pi\sqrt{\left(\frac{\rho_{air}}{\rho_{air}-\rho_{He}}\right)\frac{\ell}{g}}$

(b) The balloon undergoes simple harmonic motion with period $2\pi\sqrt{\left(\frac{\rho_{air}-\rho_{He}}{\rho_{air}}\right)\frac{\ell}{g}}$

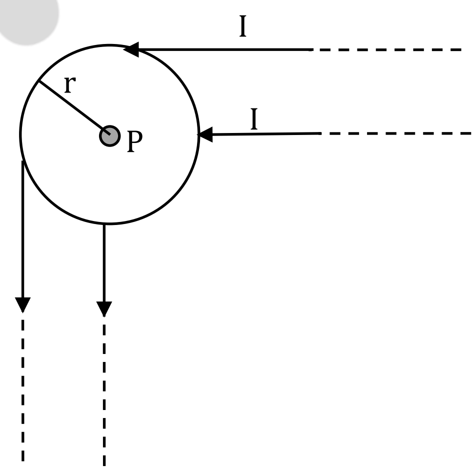
(c) The balloon undergoes simple harmonic motion with period $2\pi\sqrt{\left(\frac{\rho_{He}}{\rho_{air}-\rho_{He}}\right)\frac{\ell}{g}}$

(d) The balloon undergoes conical oscillations with period $2\pi\sqrt{\left(\frac{\rho_{air}+\rho_{He}}{\rho_{air}-\rho_{He}}\right)\frac{\ell}{g}}$

22. Consider a cube of uniform charge density ρ. The ratio of electrostatic potential at the centre of the cube to that at one of the corners of the cube is

(a) 2 (b) $\sqrt{3}/2$ (c) $\sqrt{2}$ (d) 1

23. Two infinitely long wires each carrying current I along the same direction are made into the geometry as shown in the figure. The magnetic field at the point P is



(a) $\frac{\mu_0 I}{\pi r}$ (b) $\frac{\mu_0 I}{r}\left(\frac{1}{\pi}+\frac{1}{4}\right)$ (c) Zero (d) $\frac{\mu_0 I}{2\pi r}$

24. A photon of wavelength λ is absorbed by an electron confined to a box of length $\sqrt{35h\lambda/8mc}$ .As a result, the electron makes a transition from state k = 1 to the state n. Subsequently the electron transits from the state n to the state m by emitting a photon of wavelength λ' = 1.85 λ. Then

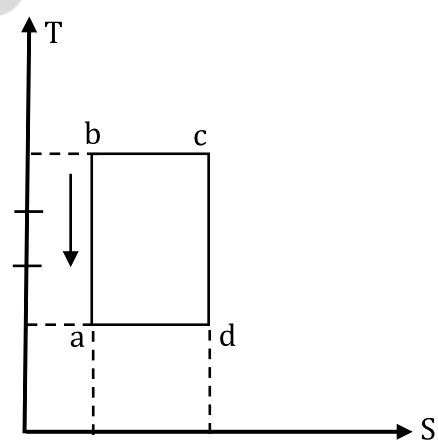
(a)n = 4; m = 2 (b) n = 5; m = 3 (c) n = 6; m = 4 (d) n = 3; m = 1

25. Consider two masses with $m_1 > m_2$ connected by a light inextensible string that passes over a pulley of radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley and the pulley turns without friction. The two masses are released from rest separated by a vertical distance 2h. When the two masses pass each other, the speed of the masses is proportional to

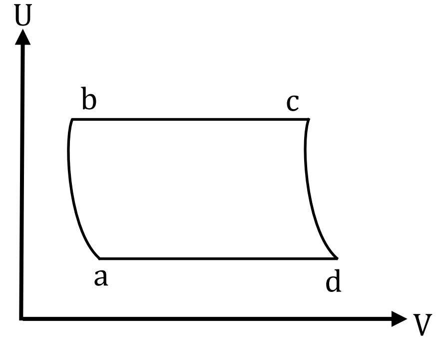
(a) $\sqrt{\dfrac{m_1-m_2}{m_1-m_2+\dfrac{1}{R^2}}}$ (b) $\sqrt{\dfrac{(m_1+m_2)(m_1-m_2)}{m_1-m_2+\dfrac{1}{R^2}}}$

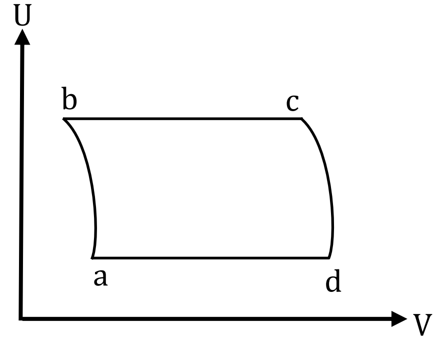
(c) $\sqrt{\dfrac{m_1+m_2+\dfrac{1}{R^2}}{m_1-m_2}}$ (d) $\sqrt{\dfrac{\dfrac{1}{R^2}}{m_1+m_2}}$

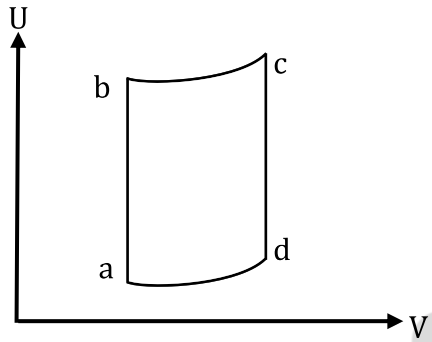
26. An ideal gas is taken reversibly around the cycle a-b-c-d-a as shown on the T (temperature) – S (entrophy) diagram

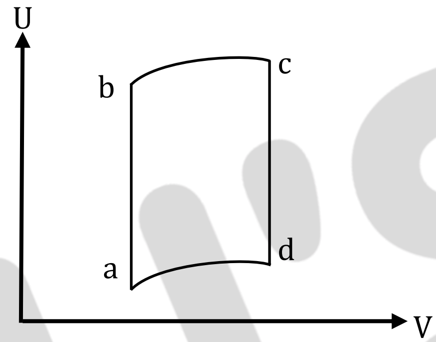


The most appropriate representation of above cycle on a U (internal energy) – V (volume) diagram is

(a) 

(b) 

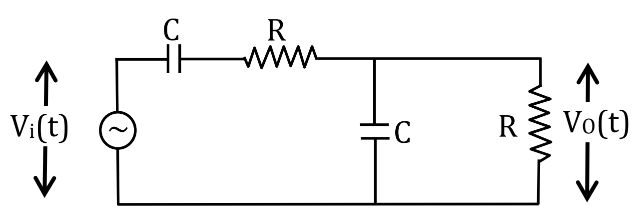
(c) 

(d) 

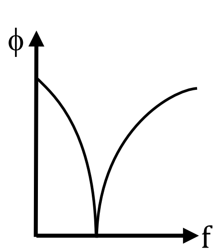
27. The heat capacity of one mole of an ideal is found to be $C_V = 3R(1 + aRT)/2$ where a is a constant. The equation obeyed by this gas during a reversible adiabatic expansion is -

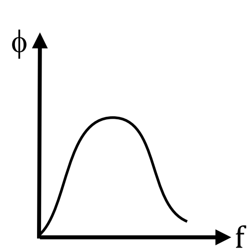
(a) $TV^{3/2} e^{aRT}$ = constant
(b) $TV^{3/2} e^{3aRT/2}$ =constant
(c) $TV^{3/2}$= constant
(d) $TV^{3/2} e^{2aRT/3}$= constant

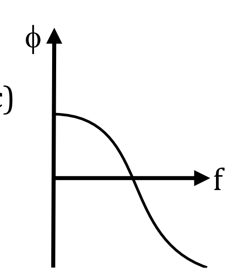
28. If the input voltage V_i to the circuit below is given by $V_i(t) = A\cos(2\pi ft)$, the output voltage is given by $V_O(t) = B\cos(2\pi ft + \phi)$ –

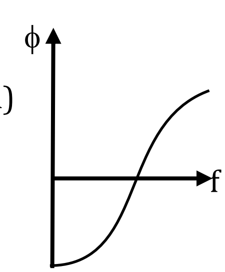


Which one of the following four graphs best depict the variation of ϕ vs f ?

(a) 

(b) 

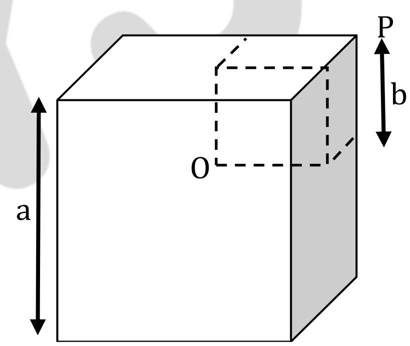
(c) 

(d) 

29. A glass prism has a right-triangular cross section ABC, with ∠A = 90º. A ray of light parallel to the hypotenuse BC and incident on the side AB emerges grazing the side AC. Another ray, again parallel to the hypotenuse BC, incident on the side AC suffers total internal reflection at the side AB. Which one of the following must be true about the refractive index μ of the material of the prism?

(a) $\sqrt{\frac{3}{2}} < \mu < \sqrt{2}$ (b) $\mu > \sqrt{3}$ (c) $\mu < \sqrt{\frac{3}{2}}$ (d) $\sqrt{2} < \mu < \sqrt{3}$

30. A smaller cube with side b (depicted by dashed lines) is excised from a bigger uniform cube with side a as shown below such that both cubes have a common vertex P. Let X = a/b. If the centre of mass of the remaining solid is at the vertex O of smaller cube then X satisfies.



(a) $X^3 - X^2 - X - 1 = 0$ (b) $X^2 - X - 1 = 0$

(c) $X^3 + X^2 - X - 1 = 0$ (d) $X^3 - X^2 - X + 1 = 0$

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ANSWER KEY

1. (c)	2. (b)	3. (a)	4. (d)	5. (c)
6. (c)	7. (c)	8. (d)	9. (b)	10. (b)
11. (b)	12. (b)	13. (d)	14. (b)	15. (c)
16. (a)	17. (b)	18. (b)	19. (b)	20. (a)
21. (c)	22. (a)	23. (d)	24. (c)	25. (c)
26. (a)	27. (G)	28. (c)	29. (a)	30. (a)

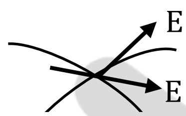
SOLUTIONS

PART-I

1. (c)

[a] Streamlines are always parallel to each other.

[b] If line of forces in electrostatics crosses each other then at the point of intersection there will be two direction of electric field intensity which is not possible.



[c] In case of reflection or refraction of light rays, the image formation is the meeting/crossing of light ray's through a point.

[d] If lines of force in magnetism cross each other then at the point of intersection there will be two direction of magnetic field intensity which is not possible.

2. (b)

As there is no slipping, energy will be conserved

$\therefore$ Initial kinetic energy = final potential energy

$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$ ....(1)

m = mass, h = height, I = moment or inertia = mR^2, ω (angular velocity) = $\frac{v}{R}$

$\therefore$ From equation (1)

$$\frac{1}{2}(mR^2)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mgh$$

$\Rightarrow h = \frac{v^2}{g}$

3. (a)

Whenever there is a sudden expansion or abrupt or rapid change of volume in a process then the process will be adiabatic,since there is no heat change.

For example: bursting of a tyre.

For adiabatic process: TV^{γ}= constant; where $\gamma = \frac{C_P}{C_V}$

From first law of thermodynamics dQ = dU+ dW

$\because$ dQ = 0

$\therefore$ dU = dW or –dU = dW (+ve because of expansion)

dU = –ve so dT = –ve ; dU = change in internal energy

i.e. Temperature will decrease so process is not isothermal

The given option (c) is applicable for isothermal process because for isothermal process work done $W_{isothermal} = nRT \log_e \frac{V_f}{V_i}$ so, it is incorrect.

Now, as entropy for system is given by

$dS = \frac{dQ}{dT}$ Substituting the value of dQ

$$dS = \frac{dU}{T} + \frac{dW}{T}$$

$$= nC_V \frac{dT}{T} + \frac{PdV}{T}$$

{Work done W = PdV

$$\int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} nC_V \frac{dT}{T} + \int \frac{nR}{V} dV$$

$$\left\{ \begin{array}{l} \because \text{from ideal gas equation} \\ \quad PV = nRT \\ \quad \Rightarrow \frac{P}{T} = \frac{nR}{V} \end{array} \right.$$

$$\Delta S = nC_V \log_e \frac{T_2}{T_1} + nR \log_e \frac{V_2}{V_1}$$

Since, entropy is not constant or not equal to zero therefore option (d) is also incorrect.

$\therefore$ The correct option is (a).

4. (d)

Let us suppose, the energy of the photon is $h\nu$,ϕ is work function of the metal, $(KE)_{max}$ is the maximum kinetic energy of the ejected electron.

Now from Einstein photo electric equation,

$h\nu - \phi = (K.E.)_{max.}$

$\Rightarrow \phi = \frac{hc}{\lambda} - (K.E.)_{max.}$ ....(1) $\left\{\because \nu = \frac{c}{\lambda}\right\}$

Given that electron bent into a circular arc of radius R by a perpendicular magnetic field having a magnitude B, then it will experience a force perpendicular to the direction of velocity of electron and it is equal to centripetal force.

So, $qVB = \frac{m_e v^2}{R}$, m_e = mass of the electron, q = charge of the electron = e

$v = \frac{eBR}{m_e}$

$\because$ K.E. = $\frac{1}{2}mv^2$

Substituting the value of v

$(K.E.)_{max.} = \frac{1}{2}m_e\left(\frac{eBR}{m_e}\right)^2$

$\therefore$ From equation (1)

$$\phi = \frac{hc}{\lambda} - \frac{1}{2}m_e\left(\frac{eBR}{m_e}\right)^2$$

Or $\phi = \frac{hc}{\lambda} - 2m_e\left(\frac{eBR}{2m_e}\right)^2$

So the correct option is [D]

5. (c)

As partition has hole of diameter d, mean pressure between both sections is equal.

$\because$ Mean force path (λ) is given by = $\dfrac{1}{\sqrt{2}\pi r^2 n_v}$

r = radius of the gas particle

n_V = number density of gas i.e. = $\dfrac{N}{V}$ $\dfrac{\text{(Total no. of particle)}}{\text{(Volume)}}$

$\because$ From ideal gas equation PV = nRT = $\dfrac{N}{N_A}$ RT = NK_B T

N_A = Avogadro number; K_B = Boltzmann constant

$\Rightarrow n_V = \dfrac{N}{V} = \dfrac{P}{k_B T}$

$\therefore\ \lambda = \dfrac{1}{\sqrt{2}\pi r^2}\dfrac{K_B T}{P}$

$$\frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2} \times \frac{P_2}{P_1} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2} \times \frac{\sqrt{T_2}}{\sqrt{T_1}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{150}{300}} = 0.7$$

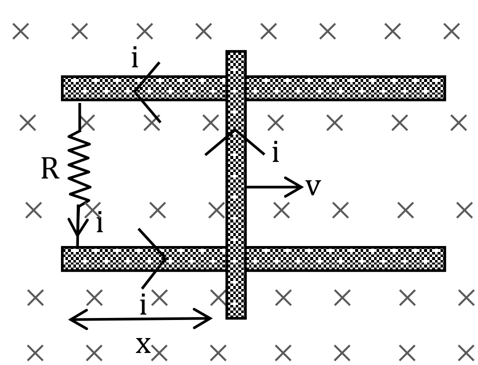
So the correct option is (c)

6. (c)

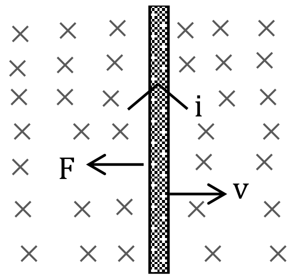
Given mass of the bar = m, length of the conducting bar = ℓ, magnitude of magnetic field = B

When bar moves on frictionless parallel rails then area of left side will increase therefore, flux will also increase.

Since, magnetic field in inward the plane so according to Lenz law it will try to reduce the magnetic flux and magnetic field will be outside the plane so current will be in anti clockwise direction.



So, option (a) is incorrect.

(c) 

$\because$ Magnetic field is inward and current is in anticlockwise direction and bar is moving towards right with velocity v so, the force that acts on it will be towards left.

i.e. $F = Bi\ell \sin\theta \begin{Bmatrix} \because \theta = 90^\circ \\ \sin 90^\circ = 1 \end{Bmatrix} \begin{Bmatrix} \text{Force on a conduction} \\ \text{in magnetic field} \end{Bmatrix}$

$F = Bi\ell$

So, acceleration will be

$a = \dfrac{F}{m} = \dfrac{Bi\ell}{m}$ ......(1)

$\because$ fluxϕ = B. A.

$\therefore \phi = B\,\ell\,x$

$\therefore$ differentiating with respect to t

$$\frac{d\phi}{dt} = B\,\ell\left(\frac{dx}{dt}\right)$$

$\because$ emf $\varepsilon = \dfrac{d\phi}{dt}$

$\therefore\ \varepsilon = B\,\ell v \left\{\because v = \dfrac{dx}{dt}\right\}$

So, current i will be

$i = \dfrac{\varepsilon}{R} = \dfrac{B\ell v}{R}$ .....(2)

So, from equation (1) and (2)

$$a = \frac{B}{m}\left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2 v}{mR}$$

$$v\frac{dv}{dx} = \frac{B^2\ell^2 v}{mR}$$

$$\int_{v_o}^{0} dv = \int_{0}^{x} \frac{B^2\ell^2}{mR} dx$$

$$-v_0 = \frac{B^2\ell^2}{mR} x$$

$$\Rightarrow \quad x = \frac{mv_0 R}{B^2\ell^2} x$$

So, distance that the bar travels before it comes to a complete stop is proportional to R.

So, option (c) is correct.

(d) $\because$ Power $P = i^2R$

Substituting the value of i

$$P = \frac{B^2\ell^2v^2}{R^2} . R$$

$$P = \frac{B^2\ell^2v^2}{R}$$

$P \propto \ell^2$

So, option (d) is incorrect.

(b)From equation (1) $a = \dfrac{Bi\ell}{m}$

$$\frac{dv}{dt} = \frac{Bi\ell}{m}$$

Substituting the value of i from equation (1)

$$\frac{dv}{dt} = \frac{B^2\ell^2v}{mR}$$

$$\frac{dv}{v} = \frac{B^2\ell^2dt}{mR}$$

Integrating both sides

$$\int \frac{dv}{v} = \int \frac{B^2\ell^2dt}{mR}$$

The result will come out in exponential form so velocity of the bar decreases exponentially not linearly so option (b) is also incorrect.

7. (c)

Given, one dimensional potential of particle $U(x) = \dfrac{x^4}{4} - \dfrac{x^2}{2}$ J .....(1)

At t = 0, x = –0.5

Substituting the value of t and x in equation (1)

$U = \frac{(-0.5)^4}{4} - \frac{(-0.5)^2}{2}$

$= \frac{1}{4}\times\frac{1}{16} - \frac{1}{2}\times\frac{1}{4} = \left|\frac{7}{4}\right|$

Differentiating equation (1) with respect to x

$F = \frac{dU}{dx} = \frac{4x^3}{4} - \frac{2x}{2}$

$\frac{dU}{dx} = x^3 - x$

$\frac{dU}{dx} = x(x^2-1)$ .....(2)

At mean position F = 0

$\frac{dU}{dx} = 0$ at point of maxima and minima

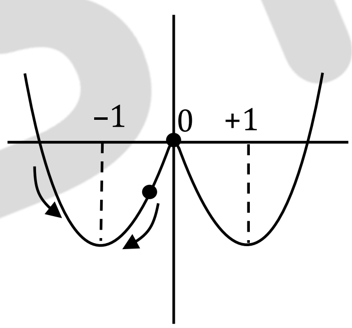
$x = 0 ; x = \pm 1$

Differentiating equation (2) with respect to x

$\frac{d^2U}{dx^2} = 2x^2 - 1$

$\left(\frac{d^2U}{dx^2}\right)_{x=0} = -1$ point of maxima

$\left(\frac{d^2U}{dx^2}\right)_{x=\pm1} = 2$ point of minima



Particle will be found between (–1,0)

8. (d)

$\because$ Blood pressure is gauge pressure = 190 mm Hg

We know atmospheric pressure = 760 mm Hg.

So, actual pressure will be = gauge pressure + atmospheric pressure

= 190 + 760

= 950 mm Hg

Now $\frac{P_{actual}}{P_{atm}} = \frac{950}{760} = 1.25$

$\Rightarrow P_{actual} = 1.25\ P_{atm}$

So, the actual pressure is about 1.25 times the atmospheric pressure.

9. (b)

Radius r = 1m

Velocity is given by

$\vec{V} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$; r, θ are polar coordinates

Now, $\frac{dr}{d\theta} = r$ (given)

$\Rightarrow \frac{dr}{dt}.\frac{dt}{d\theta} = r$

$\Rightarrow \dot{r}\cdot\frac{1}{\dot{\theta}} = r$

$\Rightarrow \dot{r} = r\dot{\theta}$

$\therefore \vec{V} = r\dot{\theta}\hat{r} + r\dot{\theta}\hat{\theta}$

$\Rightarrow \tan\theta = \frac{\text{component of } y}{\text{component of } x} = \frac{\text{component of } \hat{\theta}}{\text{component of } \hat{r}}$

$\Rightarrow \tan\theta = \frac{r\dot{\theta}}{r\dot{\theta}}$

$\Rightarrow \theta = \tan^{-1}(1)$

$\Rightarrow \theta = 45°$

10. (b)

In Young's double slit experiment fringe width is given by $\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \lambda$ ....(1)

Where λ = wavelength of the particle (electron)

D = Distance between the slit and screen

d = distance between the slits

Let, initial potential = ΔV_i

Final potential = ΔV_f

So, according to the question $\Delta V_f = 2\Delta V_i$

From de Broglie's wavelength $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2qm\Delta V}}$

$\Rightarrow \lambda \propto \frac{1}{\sqrt{\Delta V}}$

$\Rightarrow$from equation (1) $\beta \propto \frac{1}{\sqrt{\Delta V}}$

So $\beta_f = \sqrt{\Delta V_f} = \beta_i \sqrt{\Delta V_i}$

$\because \beta_i = w$ (given)

$\therefore$ Substituting the values of β_i, ΔV_i&ΔV_f , we get

$\beta_f = w.\ \frac{1}{\sqrt{2}}$

$\beta_f = 0.7\ w$

11. (b)

Electric field lines should be perpendicular to surface of metal so the correct option is [b].

12. (b)

The hydrogen atom in ground state will only absorb energy greater than 10.2 eV. When electron with kinetic energy, EeV collides with hydrogen atom in ground state, if energy of an electron is absorbed then collision will be inelastic. If there is no absorption of energy (i.e. E < 10.2 eV) then collision will be elastic.

13. (d)

The process of x-ray emission by incoming electron is also known as inverse photoelectric effect.

The continuous part of x-ray spectrum is result of the inverse photo electric effect.

14. (b)

Thermal expansion is a tendency of matter to change its shape, area and volume in response to change in temperature. Temperature is a monotonic function of average kinetic energy of a substance. When a substance is heated, the kinetic energy of its molecules increase. Thus, the molecules begin vibrating/moving more and usually maintain a greater average separation. Asymmetric characteristic of the inter atomic potential energy curve of a solid is the reason for thermal expansion.

15. (c)

Given,

Wavelength of photon and electron are equal

$\Rightarrow \lambda_p = \lambda_e = 10^{-9}$m

E = energy of photon

P = momentum of electron

From plank's equation

(Energy) $E = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E}$; h = plank's constant,c = speed of light (3×10^8 m/s)

From de-Broglie's equation,

$\lambda_e = \frac{h}{p}$

Since,$\lambda_p = \lambda_e$

$\Rightarrow \frac{hc}{E} = \frac{h}{p}$

$\Rightarrow \frac{E}{p} = c = 3 \times 10^8$ m/s

16. (a)

We know that from Einstein-mass relation is given by $E = mc^2$

Relative error: $\frac{\Delta E}{E} = \frac{\Delta m}{m}$; c = constant

Percentage error: $\frac{\Delta E}{E} \times 100 = \frac{\Delta m}{m} \times 100$

$$= \frac{9.1 \times 10^{-31}}{1.6 \times 10^{-27}} \times 100$$

$$= \frac{9.1}{1.6} \times 10^{-2}$$

$$= 6 \times 10^{-2}$$

$$= 0.06\%$$

So, the correct option is (a)

17. (b)

Given wavelength, $\lambda = 440 \text{ nm} = 440 \times 10^{-9} \text{ m} = 440 \times 10^{-7}\text{cm}$

$SO = 20 \text{ cm} = 20 \times 10^{-2}\text{m}$

$\theta = 0.5 \times 10^{-2}$ radians (very small)

To find width of fringes

S and S_I are source of Young's double slit experiment (YDSE)

$D = SO \cos\theta + 100 = 20 \times 1 + 100 = 120\text{cm}$

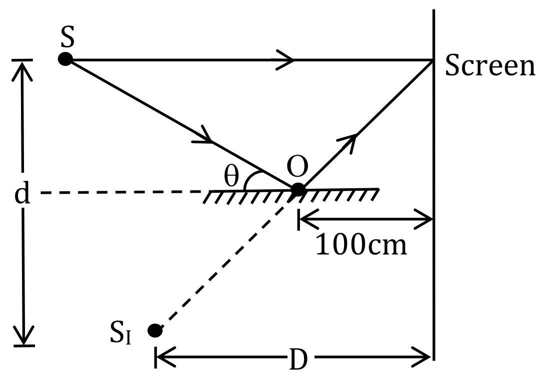
$d = 2 \times SO \sin\theta = 2 \times 20 \times 0.5 \times 10^{-3} = 2\times 10^{-2}$ cm

Fringe width

$\beta = \frac{\lambda D}{d}$

Substituting the values of λ, D & d

$\beta = \frac{440 \times 10^{-7} \times 120}{2 \times 10^{-2}} = 2.64$ mm



18. (b)

Given: Rod generates energy at a rate = 5×10^8 Watt/m^3

Shape is cylindrical, radius = 4mm

Length = 0.20 m

Specific heat of coolant(s) = $4 \times 10^3 \dfrac{J}{kg - K}$

Rate of coolant $\left(\dfrac{dm}{dt}\right) = 0.2$ kg/s

Temperature rise of coolant (ΔT) = ?

$\because$ We know that heat is given by

$Q = ms\,\Delta T$

So $\dfrac{dQ}{dt} = \dfrac{dm}{dt} S\Delta T$ .....(1)

$\dfrac{dQ}{dt} = 5 \times 10^8 \times$ volume of rod

$= 5 \times 10^8 \times \pi r^2 \ell$

$= 5 \times 10^8 \times \pi \times (4)^2 \times 10^{-6} \times .2$

$= 5 \times 10 \times \pi \times 16 \times 2 = 1600\,\pi$

Substituting the values in equation (1),

$0.2 \times 4 \times 10^3 \Delta T = 1600\pi$

$8 \times 10^2 \Delta T = 16 \times 10^2 \pi$

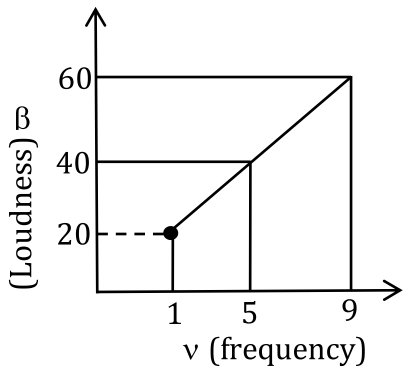
$\Delta T = 3.14 \times 2$

$\Delta T = 6.28°C$

So, the correct option is (b).

19. (b)

Loudness of sound is measured as $\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$ ..... (1)



At5 kHz, hearing capacity is calculated as

$$\frac{9-1}{60-20} = \frac{5-1}{\alpha-20}$$

$$\frac{8}{40} = \frac{4}{\alpha-20}$$

$\Rightarrow \alpha = 40$

Now from equation (1) intensity can be written as

$$I = I_0 10^{\left(\frac{\beta}{10}\right)}$$

$$(I)_{1kHz} = I_0 10^{\left(\frac{20}{10}\right)} = I_0(10)^2$$

$$(I)_{5kHz}\ I_0 10^{\left(\frac{40}{10}\right)} = I_0(10)^4$$

$$\frac{(I)_{1kHz}}{(I)_{5kHz}} = \frac{1}{100} \Rightarrow (I)_{5kHz} = 100(I)_{1kHz}$$

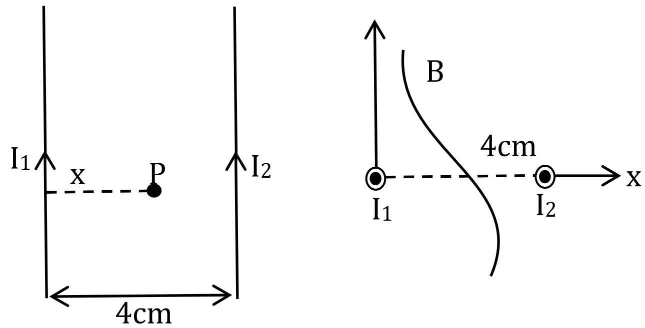
$\Rightarrow$The minimum intensity of sound that the person can hear at 5kHz is 100 times than that at 1kHz.

So, the correct option is (b).

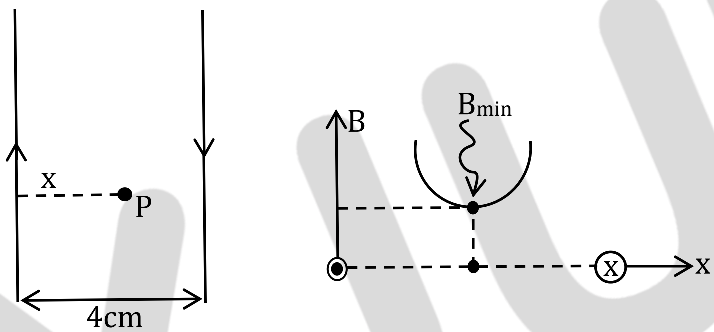
20. (a)

Given, two wires are infinity long.
Current in wires are I_1 and I_2 respectively.
Distance between wires = 4cm
Case-(I):When current in both the wires is in same direction.
Case-(II): When current in both the wires is in opposite direction.

Case-(I)



Case-(II)



In both the cases, when we draw graph we see only in case-II we get the minimum value of B.
So, in case II →
Net magnetic field at point P
B_p = magnetic field due to wire 1 at P + magnetic field due to wire 2 at P.

$B_p = \dfrac{\mu_0 I_1}{2\pi x} = \dfrac{\mu_0 I_2}{2\pi(4-x)}$ .....(1)

For minima of B_p, $\dfrac{dB_p}{dx} = 0$

Differentiating equation (1) with respect to x.

$\Rightarrow \dfrac{\mu_0 I}{2\pi}\left[-\dfrac{1}{x^2}\right] + \dfrac{\mu_0 I_2}{2\pi}\dfrac{1}{(4-x)^2} = 0$

$\dfrac{I_2}{x^2} = \dfrac{I_2}{(4-x)^2}$

$\dfrac{I_1}{I_2} = \left(\dfrac{x}{4-x}\right)^2 \Rightarrow \dfrac{I_1}{I_2} = \left(\dfrac{1}{4-x}\right)^2 \Rightarrow \dfrac{I_2}{I_1} = \dfrac{9}{1}$

So, the correct answer is (a).

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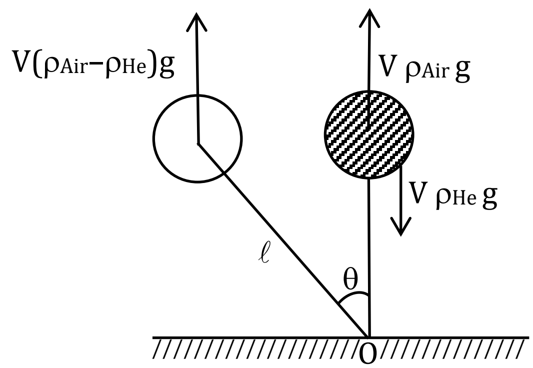
PART-II

21. (c)



density = $\frac{\text{mass}}{\text{volume}}$

$\rho = \frac{m}{v}$

Torque will be experienced by the balloon and i.e.

$\tau_0 = mg\,\ell\,\sin\theta$

$\tau_0 = V(\rho_{Air} - \rho_{He})g\,\ell\,\sin\theta$

For small angular displacement θ ; $\sin\theta \approx \theta$

$\therefore \tau_0 = V(\rho_{Air} - \rho_{He})g\,\ell\theta$

$\because\ \tau_0 = I\alpha$ I = moment of inertia

α = angular acceleration

$I = m\ell^2$ and $m = \rho V_{He}$

$\rho_{He}V\ell^2\alpha = V[\rho_{Air} - \rho_{He}]\,\ell\theta\,g$

$\alpha = \left[\frac{\rho_{Air} - \rho_{He}}{\rho_{He}}\right]\frac{g}{\ell}\theta$

(Angular frequency) $\omega = \sqrt{\left[\frac{\rho_{Air} - \rho_{He}}{\rho_{He}}\right]\frac{g}{\ell}}$

$\because\ \omega = \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi}{\omega}$

$\therefore\ T = 2\pi\sqrt{\frac{\ell}{g}\left[\frac{\rho_{He}}{\rho_{Air} - \rho_{He}}\right]}$

So, the correct option is (c).

22. (a)

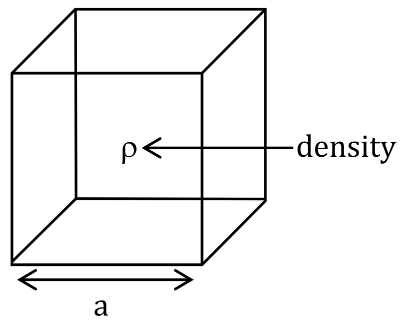
Given: Charge density of cube = ρ

Let, side of the cube = a

$\therefore$ Potential is given by (Let, at the centre of cube potential is V).

So, $V = \frac{kq}{r} \Rightarrow V \propto \frac{q}{r}$; q = charge, r = distance

Here, r = a



$V \propto \frac{q}{a}$ ....(1)

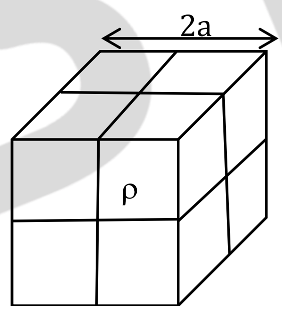
$\because$ Volume charge density $\rho = \frac{q}{\text{volume}} = \frac{q}{a^3}$

$\Rightarrow q = \rho a^3$ .....(2)

So, from equation (1) and (2),

$V \propto \frac{\rho a^3}{a}$

$\Rightarrow V \propto a^2$



Note:Big cube consist of 8 cube and side is 2a. At centre of big cube of side 2a, potential is $8V_0$. Potential at corner of big cube = $V_0 \times (2)^2 = 4V_0$

Required ratio = $\frac{8V_0}{4V_0}$ = 2 : 1

23. (d)

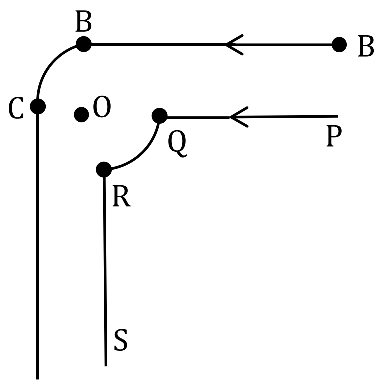


Fig. (A)

Here, we consider it like figure. (a). Now magnetic field at O,

$(\vec{B})_0 = (\vec{B})_{wireAB} + (\vec{B})_{BCArc} + (\vec{B})_{CDwire} + (\vec{B})_{PQwire} + (\vec{B})_{QR\,Arc} + (\vec{B})_{RS\,wire}$

As we know that when magnetic field and current are in same direction then their magnetic field will be zero.

So, $\vec{B}_{PQ} = \vec{B}_{RS} = 0$

$\vec{B}_{BC} = -\vec{B}_{QR}$

$(\vec{B})_{wireAB} = \vec{B}_{CD}$

$\vec{B}_{net} = (\vec{B})_{wire\,AB} + (\vec{B})_{wireCD}$

$\because$ Magnetic field through a conducting wire,

$$B = \frac{\mu_0 I}{4\pi r}$$

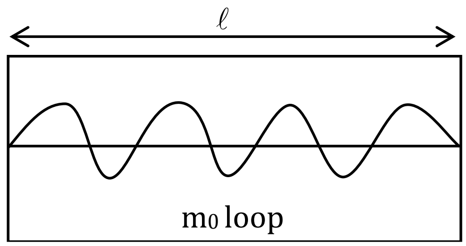
$$\therefore \vec{B}_{net} = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4\pi r}$$

$$\vec{B}_{net} = \frac{\mu_0 I}{2\pi r}$$

So, the correct option is (d).

24. (c)

Given electron makes a transition from state k = 1 to the state n in a confined box length ℓ.



$\because$ kinetic energy of electron = $\dfrac{hc}{\lambda}$ ....(1)

$\because$ From de-Broglie's wavelength formula

$$\lambda_{\text{de-Broglie}} = \frac{h}{\sqrt{2mKE}} \qquad .....(2)$$

From equation (1) and (2),

$$\lambda_{\text{de-Broglie}} = \frac{h}{\sqrt{2m\frac{hc}{\lambda}}} = \frac{\sqrt{h\lambda}}{\sqrt{2mc}}$$

$$\because \frac{m_0\lambda_{\text{de-Broglie}}}{2} = \ell$$

$$\therefore \frac{m_0}{2} \times \frac{\sqrt{h\lambda}}{2mc} = \sqrt{\frac{35h\lambda}{8mc}}$$

$$\Rightarrow \frac{m_0}{2\sqrt{2}} = \frac{\sqrt{35}}{2\sqrt{2}}$$

$m_0 = \sqrt{35} = 5.8 \approx 6$

i.e. electron get excited to state 6,

$\therefore$ n = 6

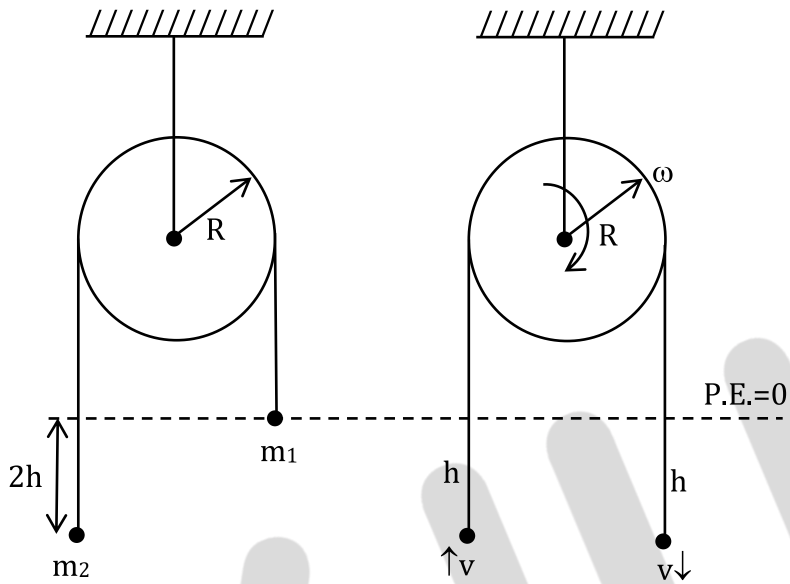
So, the correct answer is (c).

25. (a)

System will be = Pulley + Earth

Total energy is conserved so it implies that

Gain in kinetic energy + loss in potential energy.



Kinetic energy initially is zero because system is at rest. So, kinetic energy final will be equal to

$\frac{1}{2}m_1v^2$(i.e. K.E. of mass m_1) + $\frac{1}{2}m_2v^2$(i.e. K.E. of mass m_2) + rotational energy of disc (pulley)

$$\frac{1}{2}V^2\left[m_1m_2+\frac{I}{R^2}\right] \qquad \left\{\because \omega=\frac{V}{R}\right.$$

Now, applying conservation of energy,

$(K.E.)_f$ + P.E. = 0

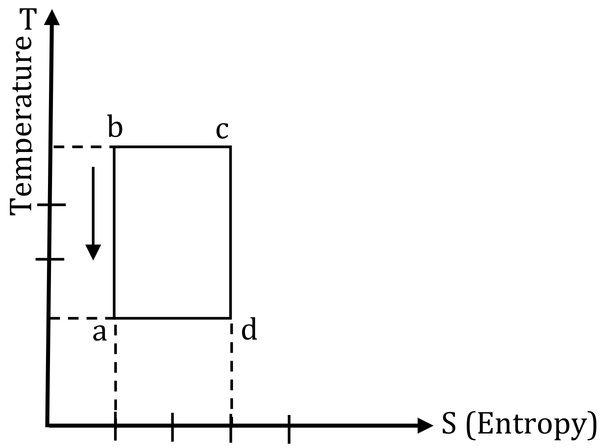
$$\therefore \frac{1}{2}v^2[m_1+m_2+\frac{I}{R_2}] + m_2gh - m_1gh = 0$$

$$v^2 = \frac{2gh(m_1-m_2)}{\left(m_1+m_2+\frac{I}{R^2}\right)} \Rightarrow v = \sqrt{\frac{2gh(m_1-m_2)}{\left(m_1+m_2+\frac{I}{R^2}\right)}}$$

$$\text{So, } v \propto \sqrt{\frac{(m_1-m_2)}{\left(m_1+m_2+\frac{I}{R^2}\right)}}$$

26. (a)

Given, curve



As we can see from diagram that bc process is an isothermal process i.e. temperature remains constant at bc.

$\because$ Change in internal energy also remains constant.

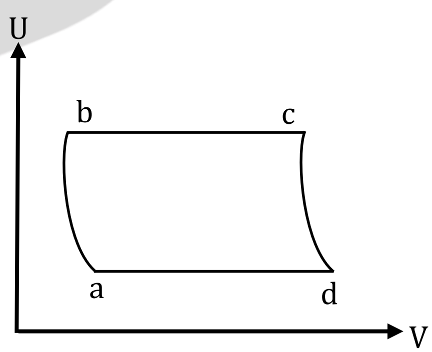
cd process is an isentropic process so S(entropy) remains constant.

An isentropic process is a thermodynamic process, in which the entropy of the fluid or gas remains constant. It means the isentropic process is a special case of an adiabatic process in which there is no transfer of heat or matter. It is a reversible adiabatic process.

For a $\rightarrow$ b, temperature will be increased

For c $\rightarrow$ d, temperature will be decreased

So, the graph will be



Therefore,the correct option is (a).

27. (Bonus)

Given, heat capacity of one mole of an ideal gas (C_v) = $\frac{3R(1+aRT)}{2}$

In reversible adiabatic process, the equation is given by

$TV^{\gamma-1}$ = C (constant) ……(1)

Where T = temperature, V = volume

$\gamma =$ ration of specific heats i.e. $\gamma = \frac{C_p}{C_v}$

The relation between $\gamma$and degree of freedom (f) is given by $\gamma = 1 + \frac{2}{f}$ …..(2)

Now from equation (1) and (2),

$$TV^{1+\frac{2}{f}-1} = C$$

$$\Rightarrow TV^{\frac{2}{f}} = C$$

Specific heat at constant volume $C_v = \frac{fR}{2}$ which will be equal to $\frac{3R(1+aRT)}{2}$ i.e.

$$\frac{fR}{2} = \frac{3R(1+aRT)}{2}$$

$$\frac{f}{2} = \frac{3}{2}(1+a\,RT)$$

$$\frac{2}{f} = \frac{2}{3}(1+a\,RT)^{-1}$$

If aRT is very small then $(1+aRT)^{-1}$ will be nearby equal to $\approx e^{aRT}$ {from Binomial expansion}

$$\Rightarrow \frac{2}{f} = \frac{2}{3}e^{aRT}$$

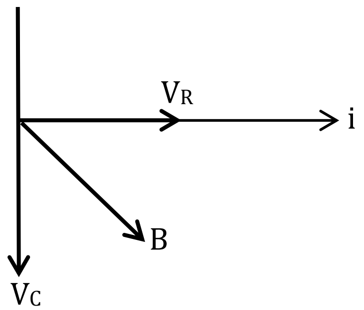
$$\therefore\ TV^{\gamma-1} = TV^{2/f}$$

$$\Rightarrow TV^{\frac{2}{3}e^{aRT}} = \text{constant (c)}$$

28. (c)

Given: Input voltage (V_i) is $V_i(t) = A\cos(2\pi ft)$,

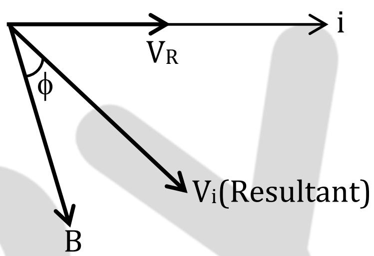
Output voltage (V_0) is $V_0(t) = B\cos(2\pi ft + \phi)$



Resultant of V_C, V_R and B give V_i(input voltage) and clearly we can see angle between V_i and B is ϕ.

When frequency f is very high, reactance X_C tends to zero then $V_C = 0$.

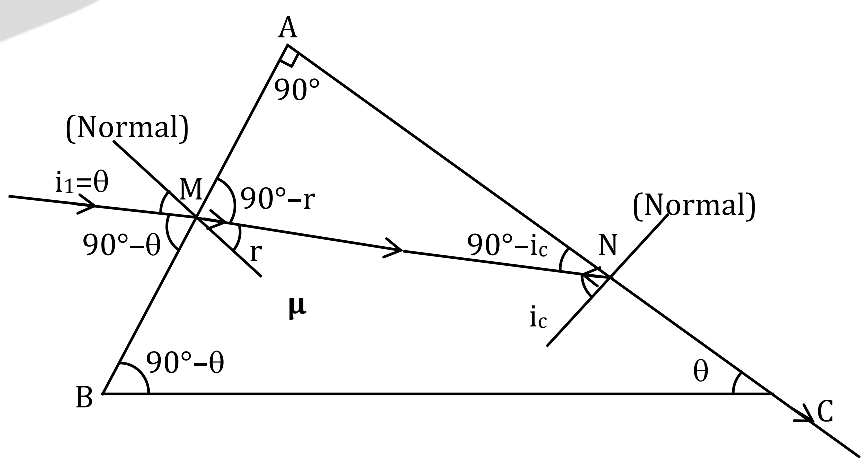
$\therefore$ Resultant of V_C, V_R and B lie between B and V_R.



B log behind ϕ. So, at higher frequency ϕ become negative.

So, the correct option is (c).

29. (a)



i_c = critical angle
r = angle of refraction
i_1 = angle of incidence

At M, $\frac{\sin i_1}{\sin r} = \mu$ (from Snell's law)

In this case, from triangle AMN

$90° + 90° - r + 90 - i_c = 180°$

$r + i_c = 90°$

$r = 90° - i_c$

From Snell's law, $\frac{\sin i_1}{\sin r} = \mu$

$$\frac{\sin\theta}{\sin(90 - i_c)} = \mu$$

$$\frac{\sin\theta}{\cos i_c} = \mu$$

$$\frac{\sin\theta}{\sqrt{1-\sin^2 i_c}} = \mu$$

(Squaring and rearrange) $\frac{\sin^2\theta}{\mu^2} = 1 - \sin^2 i_c$

$$\frac{\sin^2\theta}{\mu^2} = 1 - \frac{1}{\mu^2} \qquad \left\{\because \mu = \frac{1}{\sin i_c}\right\}$$

$$\frac{\sin^2\theta}{\mu^2} = \frac{\mu^2 - 1}{\mu^2}$$

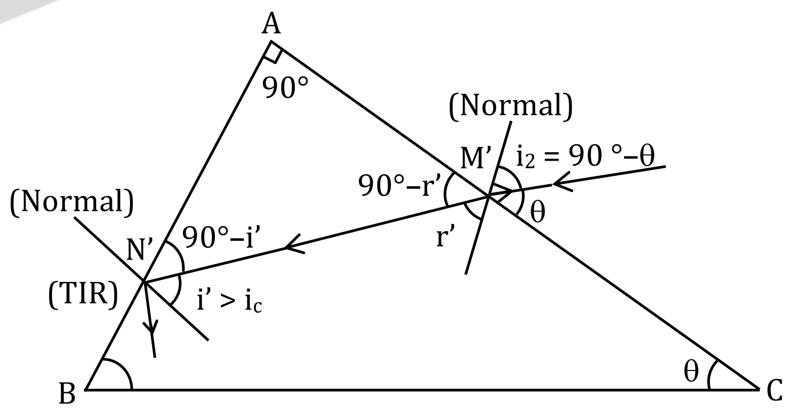
$$\sin^2\theta = \mu^2 - 1$$

$\Rightarrow$ $\mu = \sqrt{\sin^2\theta + 1}$ or $\sin^2\theta = \mu^2 - 1$ .....(1)

$0 < \theta\ 90°$

$\Rightarrow$ $1 < \mu < \sqrt{2}$ ......(2)

Case-II :



In case-II from triangle AM'N' = 90° + 90–r' + 90–i' = 180°

$\Rightarrow$ i' + r' = 90°

Applying Snell's law, $\frac{\sin i_2}{\sin r'} = \mu$

$\frac{\sin(90-\theta)}{\sin(90-i')} = \mu$

$\frac{\cos\theta}{\mu} = \cos i'$ .....(a)

Now according to condition of total internal reflection,

$i' > i_c$

$\sin i' > \sin i_c$

$\sin^2 i' > \sin^2 i_c$

$1-\cos^2 i' > \frac{1}{\mu^2}$ $\left\{\because \mu = \frac{1}{\sin i_c}\right\}$

$1-\frac{1}{\mu^2} > \cos^2 i'$ .....(b)

From equation (a) and (b),

$\frac{\mu^2-1}{\mu^2} > \frac{\cos^2\theta}{\mu^2}$

$\mu^2-1 > \cos^2\theta$

$\mu^2 > \cos^2\theta + 1$

$\mu^2 > 1-\sin^2\theta + 1$

$\mu^2 > 2-\sin^2\theta$

Substituting the value from equation (1),

$\mu^2 > 2-(\mu-1)$

$\mu^2 > 2-\mu^2 + 1$

$2\mu^2 > 3$

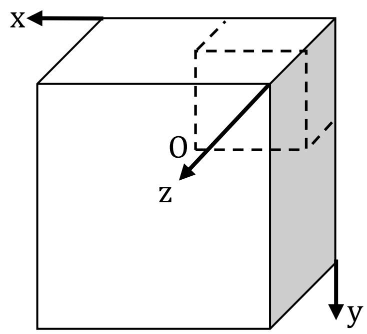
$\Rightarrow \mu > \sqrt{\frac{3}{2}}$ .....(3)

So, from equation (2) and (3),

$\sqrt{\frac{3}{2}} < \mu \ \sqrt{2}$

So, the correct option is (a).

30. (a)



$\therefore$ Centre of mass at point O is given by $\dfrac{m_1x_1+m_2x_2}{m_1+m_2}$

Here, we have to find the equation in terms of X.

$\because$ Density $(\rho) = \dfrac{\text{mass(m)}}{\text{volume(V)}}$

Centre of mass of remaining cube x coordinate = b

$$X_{CM} = \frac{\rho a^3 \times \frac{a}{2} - \rho b^3 \times \frac{b}{2}}{\rho a^3 - \rho b^3}$$

We will consider removed mass as a negative mass.

$$b = \frac{\frac{\rho a^4}{2} - \frac{\rho b^4}{2}}{\rho a^3 - \rho b^3}$$

$a^3b - b^4 = \dfrac{a^4}{2} - \dfrac{b^4}{2}$

$2a^3b - 2b^4 = a^4 - b^4$

Put $a = bx \Rightarrow 2b^4x^3\,b - 2b^4 = b^4x^4 - b^4$

$2x^3 - 1 = x^4$

$2x^3 - 2 + 1 = x^4$

$2(x^3-1) = (x^2-1)\,(x^2+1)$

$2(x-1)(x^2+1+x) = (x-1)\,(x+1)\,(x^2+1)$

$2x^2 + 2 + 2x = x^3 + x + x^2 + 1$

$x^3 - x^2 - x + 1 = 0$

So, the correct option is (a).