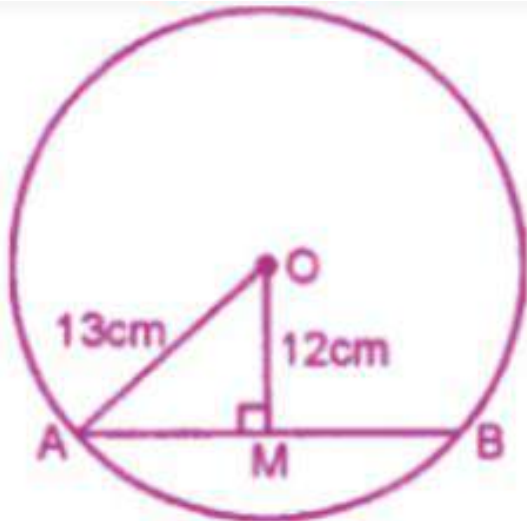


EXERCISE 15.1

1. Calculate the length of a chord which is at a distance of 12 cm from the centre of a circle of radius 13 cm.

Solution:

AB is chord of a circle with center O and OA is its radius $OM \perp AB$



Therefore, $OA = 13$ cm, $OM = 12$ cm

Now from right angled triangle OAM,

$OA^2 = OM^2 + AM^2$ by using Pythagoras theorem,

$$13^2 = 12^2 + AM^2$$

$$AM^2 = 13^2 - 12^2$$

$$AM^2 = 169 - 144$$

$$AM^2 = 25$$

$$AM = 5$$

We know that OM perpendicular to AB

Therefore, M is the midpoint of AB

$$AB = 2 AM$$

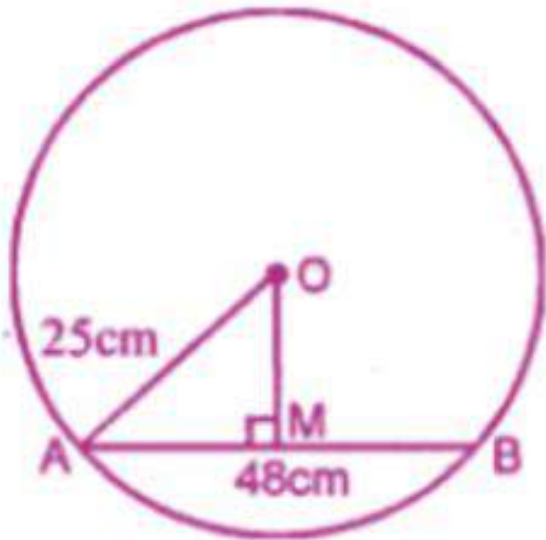
$$AB = 2 (5)$$

$$AB = 10 \text{ cm}$$

2. A chord of length 48 cm is drawn in a circle of radius 25 cm. Calculate its distance from the center of the circle.

Solution:

AB is the chord of the circle with centre O and radius OA
OM is perpendicular to AB



Therefore, $AB = 48$ cm

$OA = 25$ cm

$OM \perp AB$

M is the mid-point of AB

$AM = \frac{1}{2} AB = \frac{1}{2} \times 48 = 24$ cm

Now right ΔOAM ,

$$OA^2 = OM^2 + AM^2$$

(by Pythagoras Axiom)

$$(25)^2 = OM^2 + (24)^2$$

$$OM^2 = (25)^2 - (24)^2 = 625 - 576$$

$$= 49 = (7)^2$$

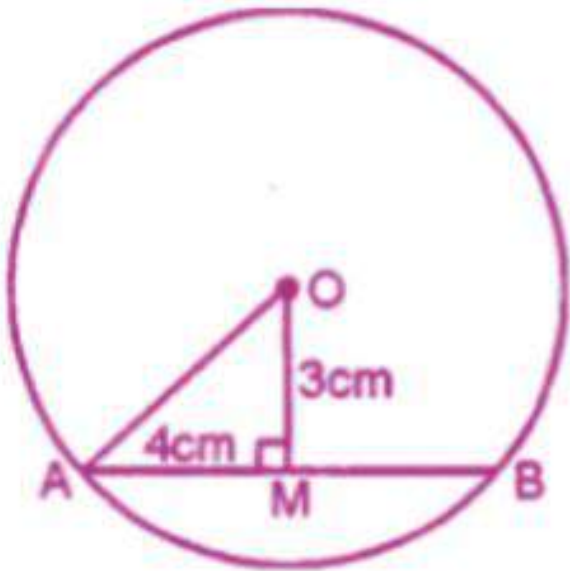
$$OM = 7$$
 cm

**3. A chord of length 8 cm is at a distance of 3 cm from the centre of the circle.
Calculate the radius of the circle.**

Solution:

AB is the chord of a circle with center O

And radius OA and $OM \perp AB$



$$AB = 8 \text{ cm}$$

$$OM = 3 \text{ cm}$$

$$OM \perp AB$$

M is the mid-point of AB

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4 \text{ cm.}$$

Now in right $\triangle OAM$

$$OA^2 = OM^2 + AM^2$$

(By Pythagoras Axiom)

$$= (3)^2 + (4)^2 = 9 + 16 = 25$$

$$= (5)^2$$

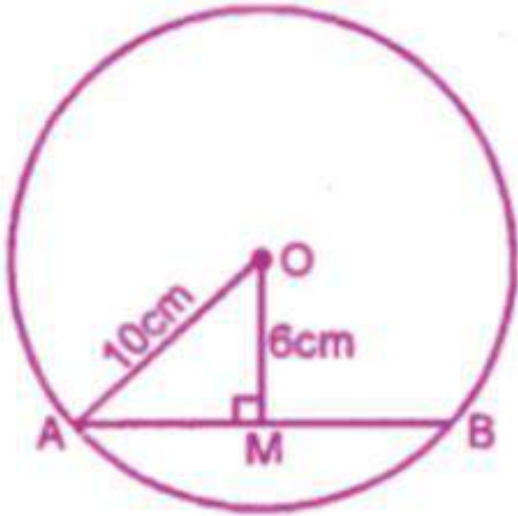
$$OA = 5 \text{ cm.}$$

4. Calculate the length of the chord which is at a distance of 6 cm from the centre of a circle of diameter 20 cm.

Solution:

AB is the chord of the circle with centre O

And radius OA and $OM \perp AB$



Diameter of the circle = 20 cm

Radius = $20/2 = 10$ cm

OA = 10 cm, OM = 6 cm

Now in right $\triangle OAM$,

$$OA^2 = AM^2 + OM^2$$

(By Pythagoras Axiom)

$$(10)^2 = AM^2 + (6)^2$$

$$AM^2 = 10^2 - 6^2$$

$$AM^2 = 100 - 36 = 64 = (8)^2$$

$$AM = 8 \text{ cm}$$

OM \perp AB

M is the mid-point of AB.

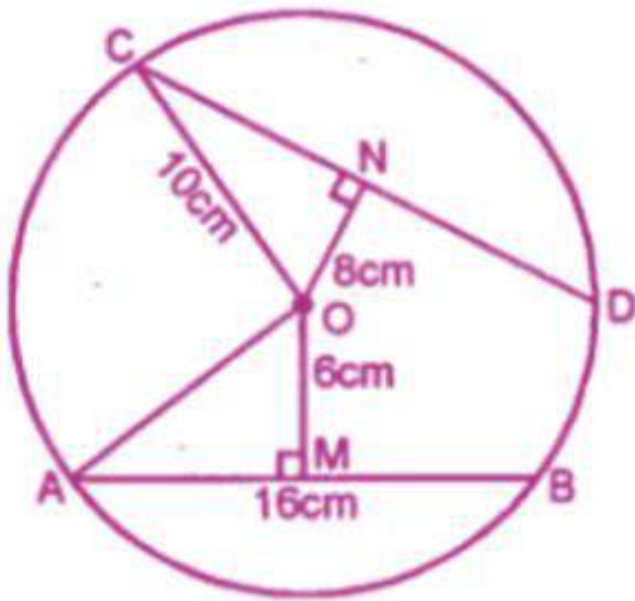
$$AB = 2 AM = 2 \times 8 = 16 \text{ cm.}$$

5. A chord of length 16 cm is at a distance of 6 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 8 cm from the centre.

Solution:

AB is a chord a circle with centre O and

OA is the radius of the circle and OM \perp AB



$AB = 16 \text{ cm}$, $OM = 6 \text{ cm}$

$OM \perp AB$

$AM = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8 \text{ cm}$

Now in right $\triangle OAM$

$$OA^2 = AM^2 + OM^2$$

(By Pythagoras Axiom)

$$= (8)^2 + (6)^2$$

$$64 + 36 = 100 = (10)^2$$

Now CD is another chord of the same circle

$ON \perp CD$ and OC is the radius.

In right $\triangle ONC$

$$OC^2 = ON^2 + NC^2$$

(By Pythagoras Axioms)

$$(10)^2 = (8)^2 + (NC)^2$$

$$100 = 64 + NC^2$$

$$NC^2 = 100 - 64 = 36 = (6)^2$$

$$NC = 6$$

But $ON \perp AB$

N is the mid-point of CD

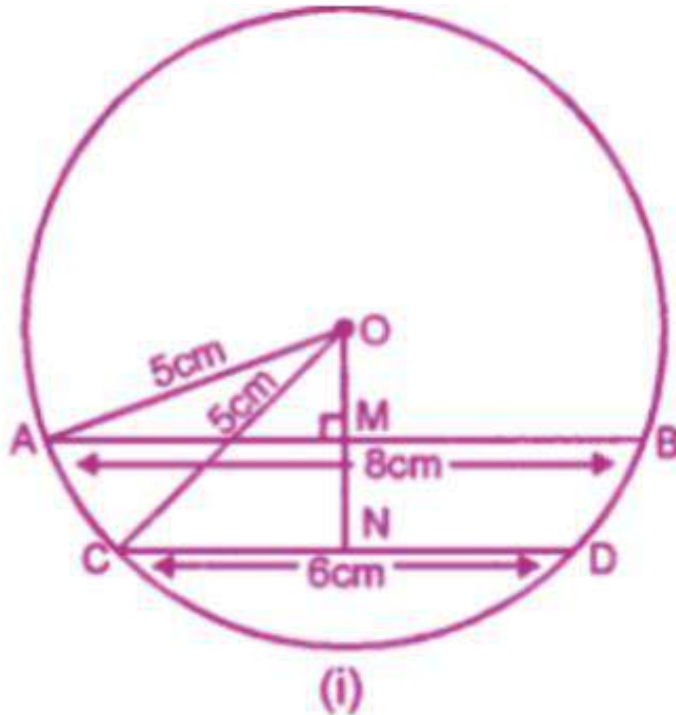
$$CD = 2 NC = 2 \times 6 = 12 \text{ cm}$$

6. In a circle of radius 5 cm, AB and CD are two parallel chords of length 8 cm and 6 cm respectively. Calculate the distance between the chords if they are on :

- (i) the same side of the centre.
- (ii) the opposite sides of the centre

Solution:

Two chords AB and CD of a Circle with centre O and radius OA or OC



$OA = OC = 5 \text{ cm}$

$AB = 8 \text{ cm}$

$CD = 6 \text{ cm}$

OM and ON are perpendiculars from O to AB and CD respectively.

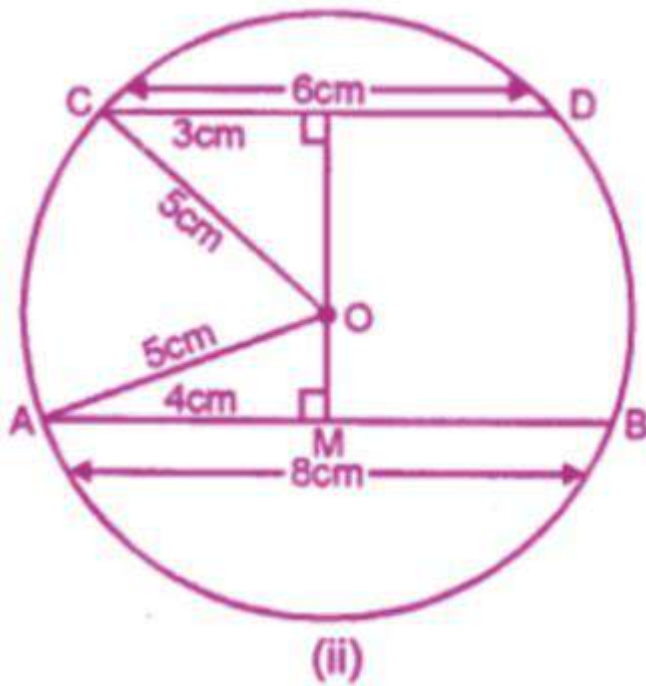
M and N are the Mid-points of AB and

CD respectively

In figure (i) chord are on the same side

And in figure (ii) chord are on the opposite

Sides of the centre



In right $\triangle OAM$

$$OA^2 = AM^2 + OM^2$$

(By Pythagoras Axiom)

$$(5)^2 = (4)^2 + OM^2$$

$$AM = \frac{1}{2} AB$$

$$25 = 16 + OM^2$$

$$OM^2 = 25 - 16 = 9 = (3)^2$$

$$OM = 3 \text{ cm}$$

Again in right $\triangle OCN$,

$$OC^2 = CN^2 + ON^2$$

$$(5)^2 = (3)^2 + ON^2$$

$$(CN = \frac{1}{2} CD)$$

$$25 = 9 + ON^2$$

$$ON^2 = 25 - 9 = 16 = (4)^2$$

$$ON = 4$$

In fig (i), distance $MN = ON - OM$

$$= 4 - 3 = 1 \text{ cm.}$$

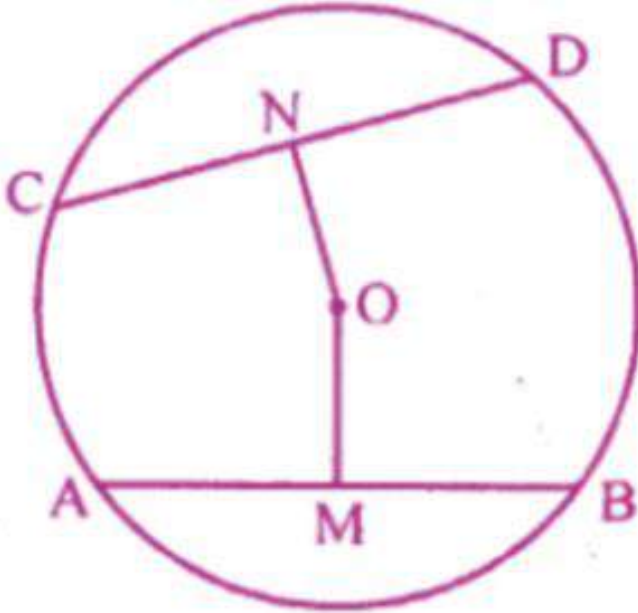
In fig (ii)

$$MN = OM + ON = 3 + 4 = 7 \text{ Cm}$$

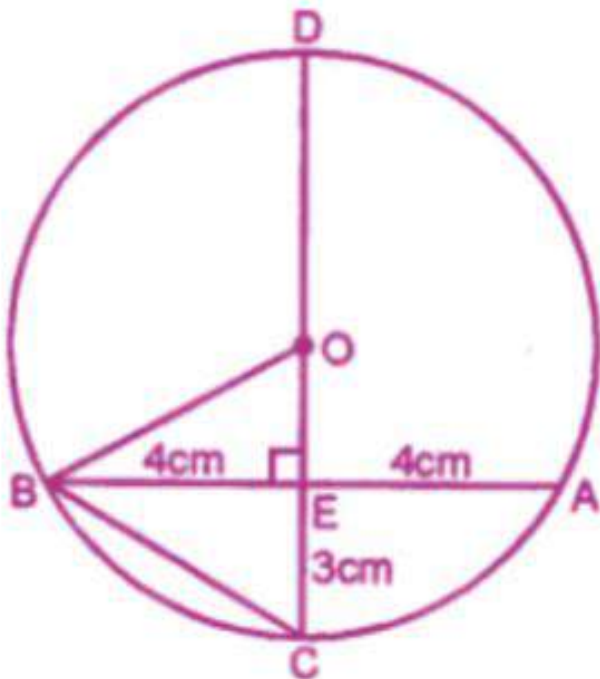
7. (a) In the figure given below, O is the centre of the circle. AB and CD are two chords

of the circle, OM is perpendicular to AB and ON is perpendicular to CD . $AB = 24$ cm, $OM = 5$ cm, $ON = 12$ cm. Find the:

- (i) radius of the circle.
- (ii) length of chord CD .



(b) In the figure (ii) given below, CD is the diameter which meets the chord AB in E such that $AE = BE = 4$ cm. If $CE = 3$ cm, find the radius of the circle.



Solution:

(a) Given : $AB = 24$ cm, $OM = 5$ cm, $ON = 12$ cm

$OM \perp AB$

M is midpoint of AB

$AM = 12$ cm

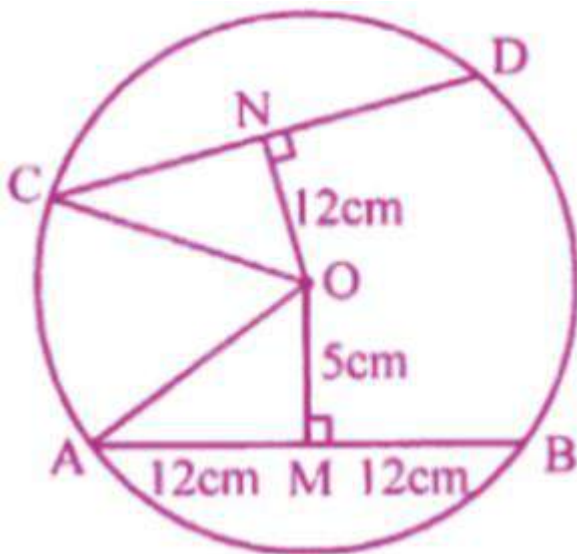
(i) Radius of circle $OA = \sqrt{OM^2 + AM^2}$

(ii) Again $OC^2 = ON^2 + CN^2$

$$13^2 = 12^2 + CN^2$$

$$CN = \sqrt{(13^2 - 12^2)} = \sqrt{(169 - 144)} = \sqrt{25}$$

$$CN = 5 \text{ cm}$$



As $ON \perp CD$, N is mid-Point of CD

$$CD = 2CN = 2 \times 5 = 10 \text{ cm}$$

(b) $AB = 8$ cm, $EC = 3$ cm

Let radius $OB = OC = r$

$$OE = (r-3) \text{ cm.}$$

Now in right $\triangle OBE$

$$OB^2 = BE^2 + OE^2$$

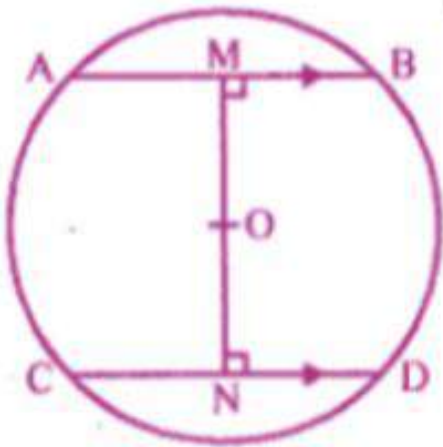
$$r^2 = (4)^2 + (r-3)^2$$

$$r^2 = 16 + r^2 - 6r + 9$$

$$6r = 25 \quad (r = 25/6 = 4 \frac{1}{6} \text{ cm})$$

8. In the adjoining figure, AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length

24 cm and 18 cm respectively.



Solution:

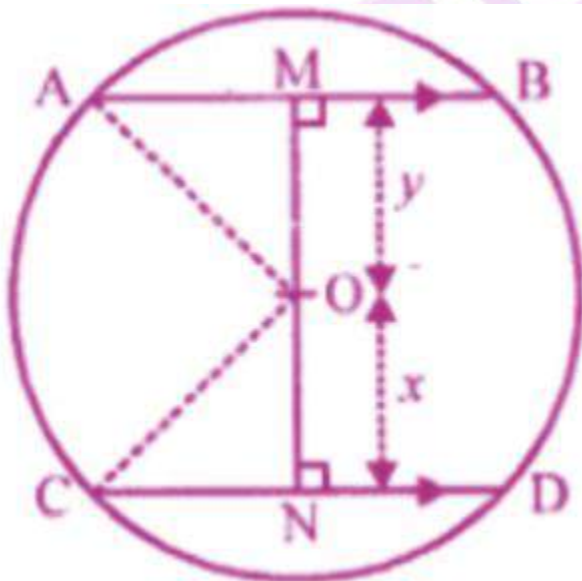
In the figure, chords $AB \parallel CD$

O is the centre of the circle

Radius of the Circle = 15 cm

Length of $AB = 24$ cm and $CD = 18$ cm

Join OA and OC



$AB = 24$ cm and $OM \perp AB$

$AM = MB = 24/2 = 12$ cm

Similarly $ON \perp CD$

$CN = ND = 18/2 = 9$ cm

Similarly In right ΔCNO

$$OC^2 = CN^2 + ON^2 \quad (15)^2 = (9)^2 + ON^2$$

$$225 = 81 + ON^2$$

$$ON^2 = 225 - 81 = 144 = (12)^2$$

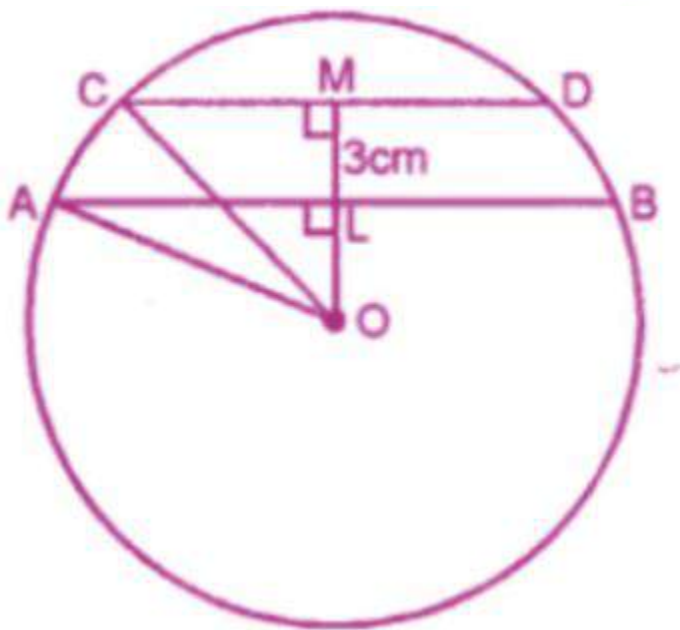
$$ON = 12 \text{ cm}$$

$$\text{Now } MN = OM + ON = 9 + 12 = 21 \text{ cm}$$

9. AB and CD are two parallel chords of a circle of lengths 10 cm and 4 cm respectively. If the chords lie on the same side of the centre and the distance between them is 3 cm, find the diameter of the circle.

Solution :

AB and CD are two parallel chords and $AB = 10 \text{ cm}$, $CD = 4 \text{ cm}$ and distance between AB and CD = 3 cm



Let radius of circle $OA = OC = r$

$OM \perp CD$ which intersects AB in L.

Let $OL = x$, then $OM = x + 3$

Now right ΔOLA

$$OA^2 = AL^2 + OL^2$$

$$r^2 = (5)^2 + x^2 = 25 + x^2$$

(l is mid- point of AB)

Again in right $\triangle OCM$

$$OC^2 = CM^2 + OM^2$$

$$r^2 = (2)^2 + (x + 3)^2$$

(M is mid-point of CD)

$$r^2 = 4 + (x + 3)^2$$

(M is mid-Point of CD)

$$r^2 = 4 + (x + 3)^2$$

from (i) and (ii)

$$25 + x^2 = 4 + (x + 3)^2$$

$$25 + x^2 = 4 + x^2 + 9 + 6x$$

$$6x = 25 - 13 = 12$$

$$x = 12/6 = 2 \text{ cm}$$

Substituting the value of x in (i)

$$r^2 = 25 + x^2 = 25 + (2)^2 = 25 + 4$$

$$r^2 = 29$$

$$r = \sqrt{29} \text{ cm}$$

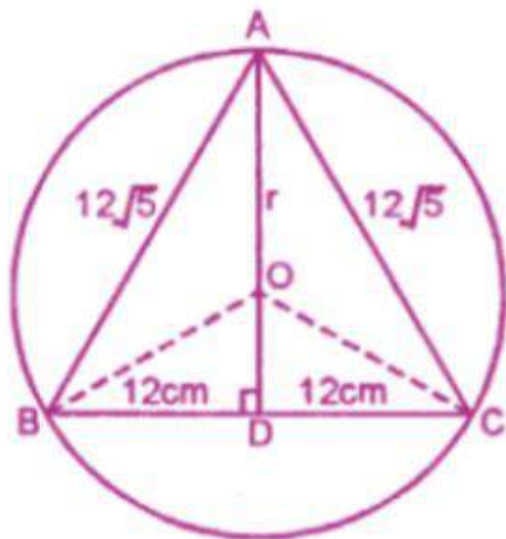
Diameter of the circle = $2r$

$$= 2 \times \sqrt{29} \text{ cm} = 2\sqrt{29} \text{ cm}$$

10. ABC is an isosceles triangle inscribed in a circle. If $AB = AC = 12\sqrt{5}$ cm and $BC = 24$ cm, find the radius of the circle.

Solution :

$AB = AC = 12\sqrt{5}$ and $BC = 24$ cm.



Join OB and OC and OA

Draw $AD \perp BC$ which will pass through
Centre O

OD bisect BC in D

$$BD = DC = 12 \text{ cm}$$

In right $\triangle ABD$

$$AB^2 = AD^2 + BD^2$$

$$(12\sqrt{5})^2 = AD^2 + BD^2$$

$$(12\sqrt{5})^2 = AD^2 + (12)^2$$

$$144 \times 5 = AD^2 + 144$$

$$720 - 144 = AD^2$$

$$AD^2 = 576 \quad (AD = \sqrt{576} = 24)$$

Let radius of the circle = $OA = OB = OC = r$

$$OD = AD - AO = 24 - r$$

Now in right $\triangle OBD$,

$$OB^2 = BD^2 + OD^2$$

$$r^2 = (12)^2 + (24 - r)^2$$

$$r^2 = 144 + 576 + r^2 - 48r$$

$$48r = 720$$

$$48r = 720$$

$$r = 720/48 = 15$$

$$48r = 720$$

$$r = 720/48 = 15 \text{ cm.}$$

Radius = 15 cm.

11. An equilateral triangle of side 6 cm is inscribed in a circle. Find the radius of the circle.

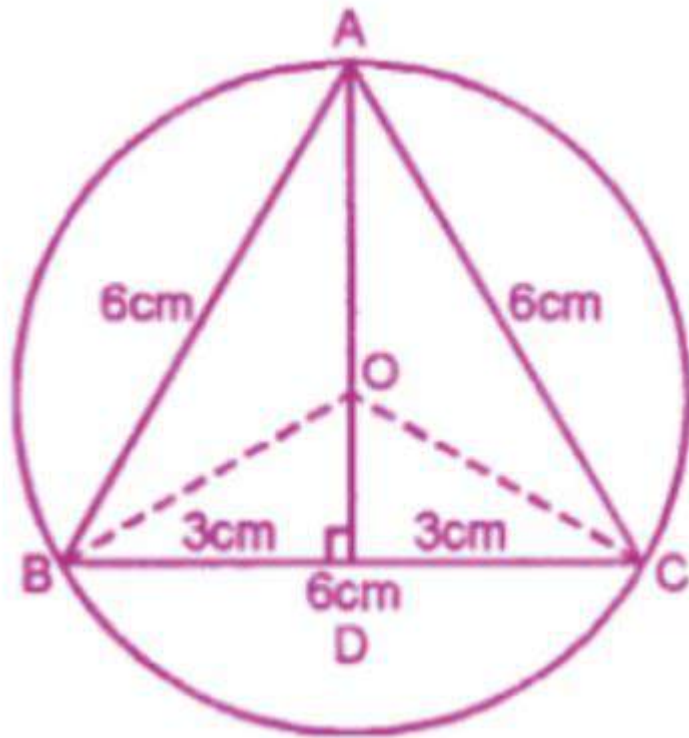
Solution :

ABC is an equilateral triangle inscribed in a

Circle with centre O. Join OB and OC,

From A, Draw $AD \perp BC$ which will pass

Through the centre O of the circle.



Each side of $\triangle ABC = 6$ cm.

$AD = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}$ cm.

$OD = AD - AO = 3\sqrt{3} - r$

Now in right $\triangle OBD$

$OB^2 = BD^2 + OD^2$

$r^2 = (3)^2 + (3\sqrt{3} - r)^2$

$r^2 = 9 + 27 + r^2 - 6\sqrt{3}r$

(D is mid-point of BC)

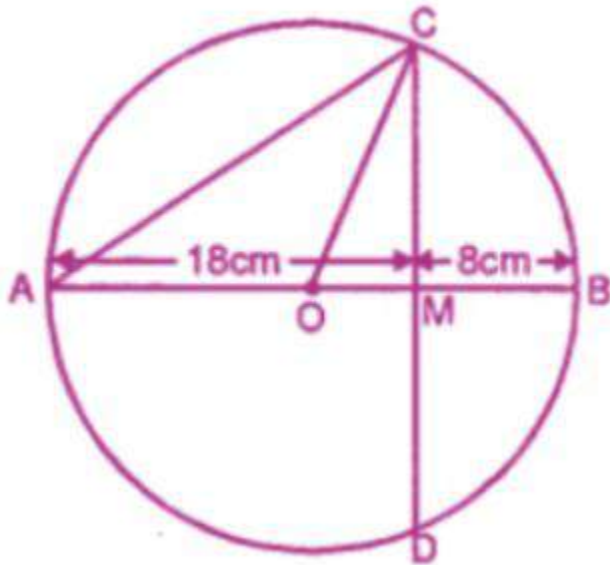
$6\sqrt{3}r = 36$

$R = \frac{36}{6\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ cm

Radius = $2\sqrt{3}$ cm

12. AB is a diameter of a circle. M is a point in AB such that AM = 18 cm and MB = 8 cm. Find the length of the shortest chord through M.

Solution:



$AM = 18$ cm and $MB = 8$ cm

$AB = AM + MB = 18 + 8 = 26$ cm

Radius of the circle $= 26/2 = 13$ cm

Let CD is the shortest chord drawn through M .

$CD \perp AB$

Join OC

$OM = AM - AO = 18 - 13 = 5$ cm

$OC = OA = 13$ cm

Now in right $\triangle OMC$

$OC^2 = OM^2 + MC^2$

$(13)^2 = (5)^2 + MC^2$ ($MC^2 = 13^2 - 5^2$)

$MC^2 = 169 - 25 = 144 = (12)^2$

$MC = 12$

M is Mid-Point of CD

$CD = 2 \times MC = 2 \times 12 = 24$ cm

EXERCISE 15.2

1. If arcs APB and CQD of a circle are congruent, then find the ratio of AB: CD.

Solution:

arc APB = arc CQD (given)

AB = CD

(if two arcs are congruent, then their

Corresponding chords are equal)

Ratio of AB and CD = $AB / CD = AB / AB = 1/1$

AB : CD = 1 : 1

2. A and B are points on a circle with centre O. C is a point on the circle such that OC bisects $\angle AOB$, prove that OC bisects the arc AB.

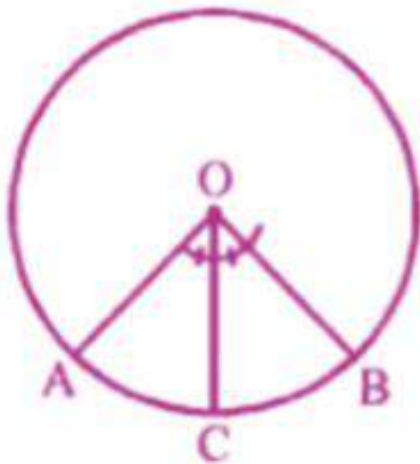
Solution:

Given : in a given circle with centre O, A

And B are Two points on the circle. C i

another point on the circle such that

$\angle AOC = \angle BOC$



To prove : arc AC = arc BC

Proof : OC is the bisector of $\angle AOB$

Or $\angle AOC = \angle BOC$

But these are the angle subtended by the
arc AC and BC

arc AC = arc BC.

3. Prove that the angle subtended at the centre of a circle is bisected by the radius passing through the mid-point of the arc.

Solution :

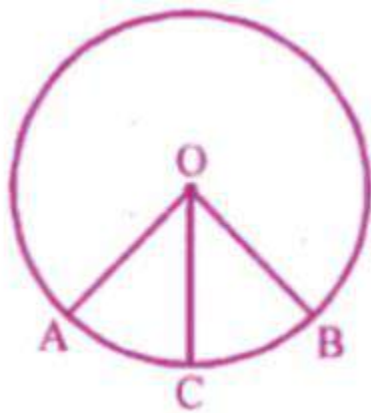
Given : AB is the arc of the circle with Centre O and C is the mid-Point of arc AB.

To prove : OC bisects the $\angle AOB$

I.e $\angle AOC = \angle BOC$

Proof : C is the mid-point of arc AB.

arc AC = arc BC



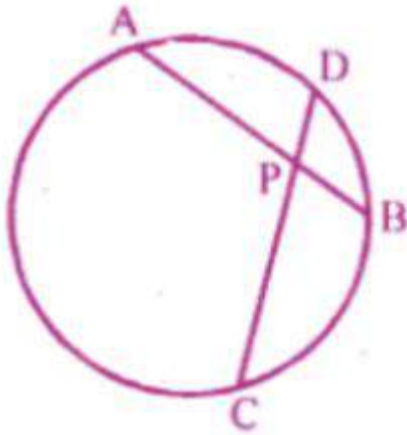
But arc AC and arc BC subtend $\angle AOC$ and $\angle BOC$ at the centre

$\angle AOC = \angle BOC$

Hence OC Bisects the $\angle AOB$.

4. In the given figure, two chords AB and CD of a circle intersect at P. If AB = CD, prove that arc AD = arc CB.

Solution :



Given: two chord AB and CD of a Circle

Intersect at P and $AB = CD$

To prove : arc AD = arc CB

Proof : $AB = CD$ (given)

minor arc AB = minor arc CD

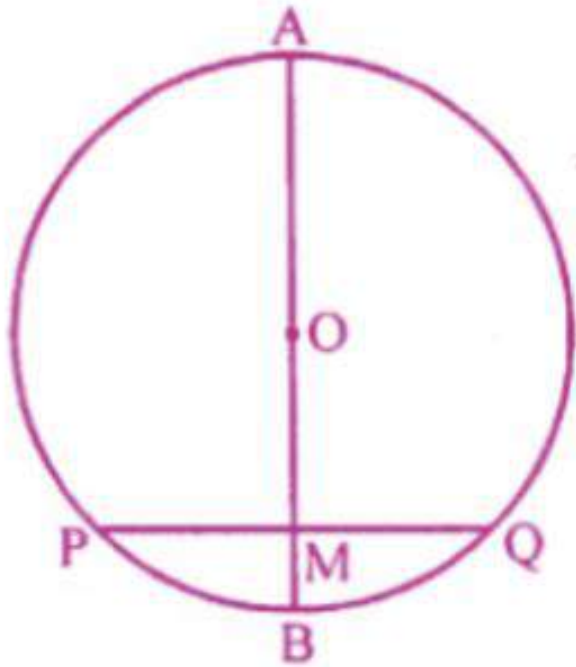
subtracting arc BD from both sides

arc AB = arc BD = arc CD – arc BD

arc AD = arc CD

CHAPTER TEST

1. In the given figure, a chord PQ of a circle with centre O and radius 15 cm is bisected at M by a diameter AB. If $OM = 9$ cm, find the lengths of : (i) PQ (ii) AP (iii) BP

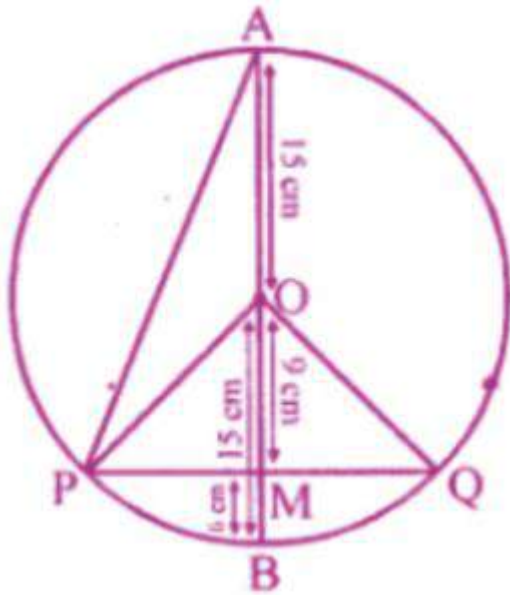


Solution:

Given, radius = 15 cm

$OA = OB = OP = OQ = 15$ cm

Also, $OM = 9$ cm



$$MB = OB - OM = 15 - 9 = 6 \text{ cm}$$

$$AM = OA + OM = 15 + 9 \text{ cm} = 24 \text{ cm}$$

In $\triangle OMP$, By using Pythagoras Theorem,

$$OP^2 = OM^2 + PM^2$$

$$15^2 = 9^2 + PM^2$$

$$PM^2 = 225 - 81$$

$$PM = \sqrt{144} = 12 \text{ cm}$$

Also, In $\triangle OMQ$

By using Pythagoras Theorem

$$OQ^2 = OM^2 + QM^2$$

$$15^2 = OM^2 + QM^2$$

$$15^2 = 9^2 + QM^2 \quad (QM^2 = 225 - 81)$$

$$QM = \sqrt{144} = 12 \text{ cm}$$

$$PQ = PM + QM$$

(As radius is bisected at M)

$$PQ = 12 + 12 \text{ cm} = 24 \text{ cm}$$

(ii) Now in $\triangle APM$

$$AP^2 = AM^2 + PM^2$$

$$AP^2 = 24^2 + 12^2$$

$$AP^2 = 576 + 144$$

$$AP = \sqrt{720} = 12\sqrt{5} \text{ cm}$$

(iii) Now in $\triangle BMP$

$$BP^2 = BM^2 + PM^2$$

$$BP^2 = 6^2 + 12^2$$

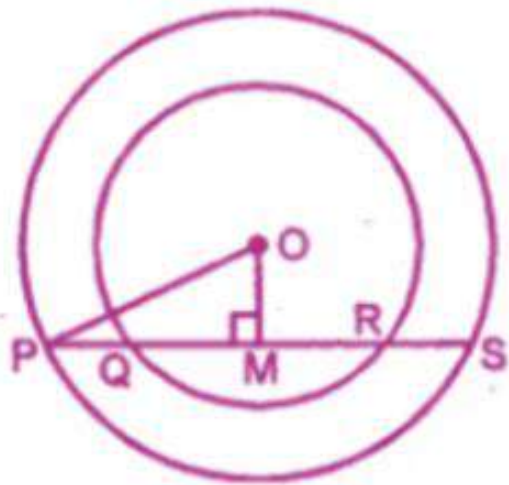
$$BP^2 = 36 + 144$$

$$BP = \sqrt{180} = 6\sqrt{5} \text{ cm}$$

2. The radii of two concentric circles are 17 cm and 10 cm ; a line PQRS cuts the larger circle at P and S and the smaller circle at Q and R. If QR = 12 cm, calculate PQ.

Solution :

A line PQRS intersects the outer circle at P
And S and inner circle at Q and R radius of
Outer circle $OP = 17 \text{ cm}$ and radius of inner
Circle $OQ = 10 \text{ cm}$



$$QR = 12 \text{ cm}$$

From O, draw $OM \perp PS$

$$QM = \frac{1}{2} QR = \frac{1}{2} \times 12 = 6 \text{ cm}$$

In right ΔOQM

$$OQ^2 = OM^2 + QM^2$$

$$(10)^2 = OM^2 + (6)^2$$

$$OM^2 = 10^2 - 6^2$$

$$= 100 - 36 = 64 = (8)^2$$

$$OM = 8 \text{ cm}$$

Now in right ΔOPM

$$OP^2 = OM^2 + PM^2$$

$$(17)^2 = OM^2 + PM^2$$

$$PM^2 = (17)^2 - (8)^2$$

$$= 289 - 64 = 225 = (15)^2$$

$$PM = 15 \text{ cm}$$

$$PQ = PM - QM = 15 - 6 = 9 \text{ cm}$$

