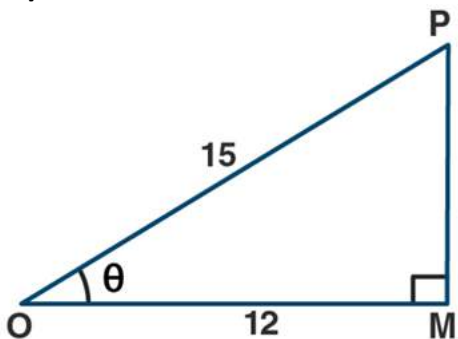


EXERCISE 17

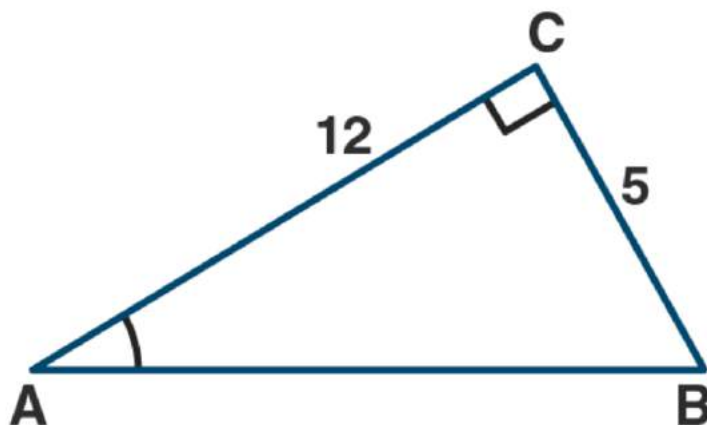
1. (a) From the figure (1) given below, find the values of:

- (i) $\sin \theta$
- (ii) $\cos \theta$
- (iii) $\tan \theta$
- (iv) $\cot \theta$
- (v) $\sec \theta$
- (vi) $\operatorname{cosec} \theta$



(b) From the figure (2) given below, find the values of:

- (i) $\sin A$
- (ii) $\cos A$
- (iii) $\sin^2 A + \cos^2 A$
- (iv) $\sec^2 A - \tan^2 A$.



Solution:

(a) From right angled triangle OMP,
By Pythagoras theorem, we get
 $OP^2 = OM^2 + MP^2$

$$\begin{aligned}MP^2 &= OP^2 + OM^2 \\MP^2 &= (15)^2 - (12)^2 \\MP^2 &= 225 - 144 \\MP^2 &= 81 \\MP^2 &= 9^2 \\MP &= 9\end{aligned}$$

$$\begin{aligned}\text{(i) } \sin \theta &= MP/OP \\&= 9/15 \\&= 3/5\end{aligned}$$

$$\begin{aligned}\text{(ii) } \cos \theta &= OM/OP \\&= 12/15 \\&= 4/5\end{aligned}$$

$$\begin{aligned}\text{(iii) } \tan \theta &= MP/OM \\&= 9/12 \\&= 3/4\end{aligned}$$

$$\begin{aligned}\text{(iv) } \cot \theta &= OM/MP \\&= 12/9 \\&= 4/3\end{aligned}$$

$$\begin{aligned}\text{(v) } \sec \theta &= OP/OM \\&= 15/12 \\&= 5/4\end{aligned}$$

$$\begin{aligned}\text{(vi) } \operatorname{cosec} \theta &= OP/MP \\&= 15/9 \\&= 5/3\end{aligned}$$

(b) From right angled triangle ABC,
By Pythagoras theorem, we get

$$\begin{aligned}AB^2 &= AC^2 + BC^2 \\AB^2 &= (12)^2 + (5)^2 \\AB^2 &= 144 + 25 \\AB^2 &= 169 \\AB^2 &= 13^2\end{aligned}$$

$$AB = 13$$

$$\begin{aligned} \text{(i) } \sin A &= BC/AB \\ &= 5/13 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \cos A &= AC/AB \\ &= 12/13 \end{aligned}$$

$$\begin{aligned} \text{(iii) } \sin^2 A + \cos^2 A &= (BC/AB)^2 + (AC/AB)^2 \\ &= (5/13)^2 + (12/13)^2 \\ &= (25/169) + (144/169) \\ &= (25 + 144)/169 \\ &= 169/169 \\ &= 1 \end{aligned}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\begin{aligned} \text{(iv) } \sec^2 A - \tan^2 A &= (AB/AC)^2 - (BC/AC)^2 \\ &= (13/12)^2 - (5/12)^2 \\ &= (169/144) - (25/144) \\ &= (169 - 25)/144 \\ &= 144/144 \\ &= 1 \end{aligned}$$

$$\sec^2 A - \tan^2 A = 1$$

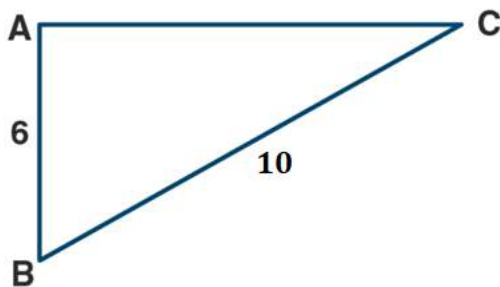
2. (a) From the figure (1) given below, find the values of:

(i) $\sin B$

(i) $\cos C$

(iii) $\sin B + \sin C$

(iv) $\sin B \cos C + \sin C \cos B$.



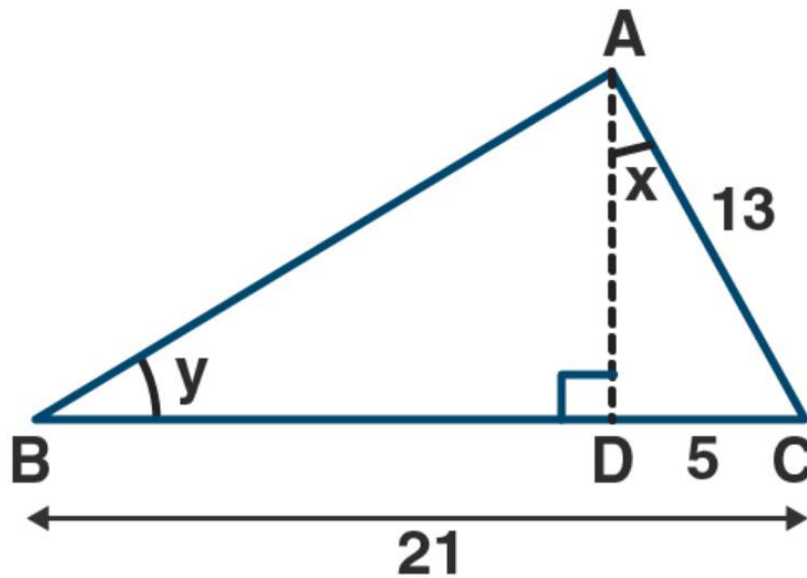
(b) From the figure (2) given below, find the values of:

(i) $\tan x$

(ii) $\cos y$

(iii) $\operatorname{cosec}^2 y - \cot^2 y$

(iv) $5/\sin x + 3/\sin y - 3 \cot y$.



Solution:

From right angled triangle ABC,
By Pythagoras theorem, we get

$$BC^2 = AC^2 + AB^2$$

$$AC^2 = BC^2 - AB^2$$

$$AC^2 = (10)^2 - (6)^2$$

$$AC^2 = 100 - 36$$

$$AC^2 = 64$$

$$AC^2 = 8^2$$

$$AC = 8$$

$$\begin{aligned} \text{(i) } \sin B &= \text{perpendicular/ hypotenuse} \\ &= AC/BC \\ &= 8/10 \\ &= 4/5 \end{aligned}$$

$$\text{(ii) } \cos C = \text{Base/hypotenuse}$$

$$\begin{aligned} &= AC/BC \\ &= 8/10 \\ &= 4/5 \end{aligned}$$

(iii) $\sin B = \text{Perpendicular/hypotenuse}$

$$\begin{aligned} &= AC/BC \\ &= 8/10 \\ &= 4/5 \end{aligned}$$

$\sin C = \text{perpendicular/hypotenuse}$

$$\begin{aligned} &= AB/BC \\ &= 6/10 \\ &= 3/5 \end{aligned}$$

Now,

$$\begin{aligned} \sin B + \sin C &= (4/5) + (3/5) \\ &= (4 + 3)/5 \\ &= 7/5 \end{aligned}$$

(iv) $\sin B = 4/5$

$$\cos C = 4/5$$

$\sin C = \text{perpendicular/hypotenuse}$

$$\begin{aligned} &= AB/BC \\ &= 6/10 \\ &= 3/5 \end{aligned}$$

$\cos B = \text{Base/Hypotenuse}$

$$\begin{aligned} &= AB/BC \\ &= 6/10 \\ &= 3/5 \end{aligned}$$

$\sin B \cos C + \sin C \cos B$

$$\begin{aligned} &= (4/5) \times (4/5) + (3/5) \times (3/5) \\ &= (16/25) + (9/25) \\ &= (16+9)/25 \\ &= 25/25 \\ &= 1 \end{aligned}$$

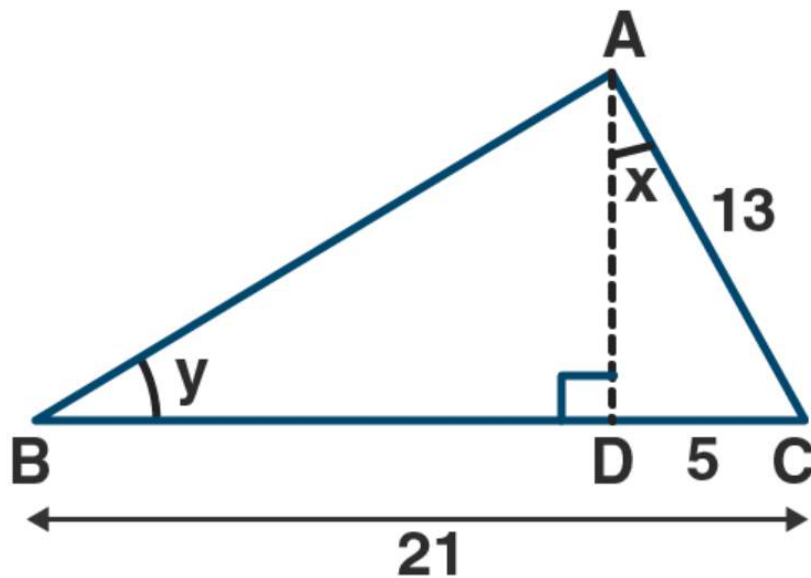
From Figure

$$AC = 13, CD = 5, BC = 21,$$

$$BD = BC - CD$$

$$= 21 - 5$$

= 16



From right angled ΔACD ,

By Pythagoras theorem we get

$$AC^2 = AD^2 + CD^2$$

$$AD^2 = AC^2 - CD^2$$

$$AD^2 = (13)^2 - (5)^2$$

$$AD^2 = 169 - 25$$

$$AD^2 = 144$$

$$AD^2 = (12)^2$$

$$AD = 12$$

From right angled ΔABD ,

By Pythagoras theorem we get

By Pythagoras theorem we get

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = 400$$

$$AB^2 = (20)^2$$

$$AB = 20$$

(i) $\tan x = \text{perpendicular/Base}$ (in right angled ΔACD)

$$= CD/AD$$

$$= 5/12$$

(ii) $\cos y = \text{Base}/\text{Hypotenuse}$ (in right angled $\triangle ABD$)

$$= BD/AB$$

$$= (20)/12 - (5/3)$$

$\cot y = \text{Base}/\text{Perpendicular}$ (in right $\triangle ABD$)

$$= BD/AB$$

$$= 16/20 = 4/5$$

(iii) $\cos y = \text{Hypotenuse}/\text{perpendicular}$ (in right angled $\triangle ABD$)

$$BD/AB$$

$$= 20/12$$

$$= 5/3$$

$\cot y = \text{Base}/\text{Perpendicular}$ (in right $\triangle ABD$)

$$AB/AD$$

$$= 16/12$$

$$= 4/3$$

$$\text{Cosec}^2 y - \cot^2 y = (5/3)^2 - (4/3)^2$$

$$= (25/9) - (16/9)$$

$$= (25-16)/9$$

$$= 9/9$$

$$= 1$$

$$\text{Hence, } \text{cosec}^2 y - \cot^2 y = 1$$

(iv) $\sin x = \text{Perpendicular}/\text{Hypotenuse}$ (in right angled $\triangle ACD$)

$$= AD/AB$$

$$= 12/20$$

$$= 3/5$$

$\cot y = \text{Base}/\text{Perpendicular}$ (in right angled $\triangle ABD$)

$$= BD/AD$$

$$= 16/12$$

$$= 4/3$$

$$(5/\sin x) + (3/\sin y) - 3\cot y$$

$$= 5/(5/13) + 3/(3/5) - 3 \times 4/3$$

$$= 5 \times 13/5 + 3 \times 5/3 - 3 \times 4/3$$

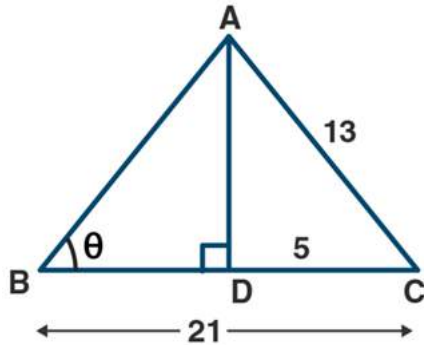
$$= 1 \times 13/1 + 1 \times 5/1 - 1 \times 4/1$$

$$= 13 + 5 - 4 = 18 - 4$$

$$= 14$$

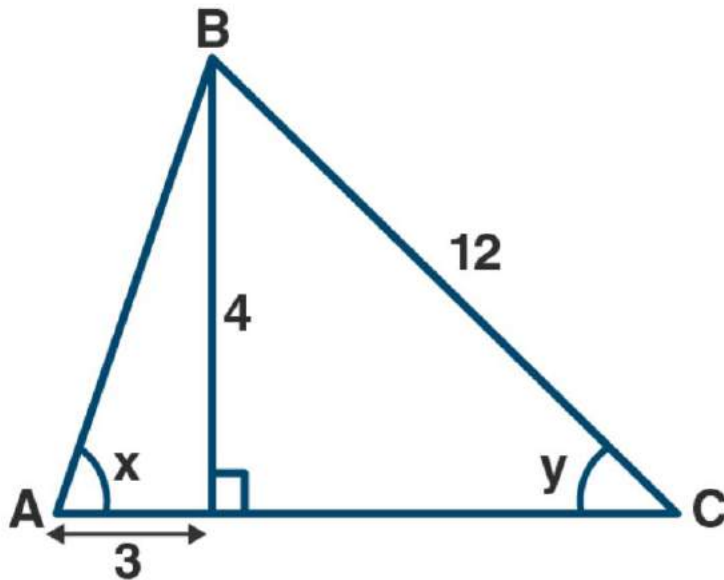
$$\text{Hence } 5/\sin x + 3/\sin y - 3\cot y = 14$$

3. (a) From the figure (1) given below, find the value of $\sec \theta$.



(b) From the figure (2) given below, find the values of:

- (i) $\sin x$
- (ii) $\cot x$
- (iii) $\cot^2 x - \operatorname{cosec}^2 x$
- (iv) $\sec y$
- (v) $\tan^2 y - 1/\cos^2 y$.



Solution:

(a) From the figure, $\sec \theta = AB / BD$

But in $\triangle ADC$, $\angle D = 90^\circ$

$$AC^2 = AD^2 + DC^2 \text{ (Pythagoras Theorem)}$$

$$(13)^2 = AD^2 + 25$$

$$AD^2 = 169 - 25$$

$$= 144$$

$$= (12)^2$$

$$AD = 12$$

(in right $\triangle ABD$)

$$AB^2 = AD^2 + BD^2$$

$$= (12)^2 + (16)^2$$

$$= 144 + 256$$

$$= 400$$

$$= (20)^2$$

$$AB = 20$$

$$\text{Now, } \sec \theta = AB / BD$$

$$= 20/16$$

$$= 5/4$$

(b) let given $\triangle ABC$

$$BD = 3, AC = 12, AD = 4$$

In right angled $\triangle ABD$

By Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (4)^2 + (3)^2$$

$$AB^2 = 16 + 9$$

$$AB^2 = 25$$

$$AB^2 = (5)^2$$

$$AB = 5$$

In right angled triangle ACD

By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$

$$CD^2 = AC^2 - AD^2$$

$$CD^2 = (12)^2 - (4)^2$$

$$CD^2 = 144 - 16$$

$$CD^2 = 128$$

$$CD = \sqrt{128}$$

$$CD = \sqrt{64 \times 2}$$

$$= 8\sqrt{2}$$

(i) $\sin x = \text{perpendicular/Hypotenuse}$

$$= AD/AB$$

$$= 4/5$$

$$\begin{aligned} \text{(ii) } \cot x &= \text{Base/Perpendicular} \\ &= \text{BD/AD} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{(iii) } \cot x &= \text{Base/ Perpendicular} \\ &= \text{BD/AD} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{(iv) } \operatorname{cosec} x &= \text{Hypotenuse / Perpendicular} \\ &= \text{AB/BD} \\ &= \frac{5}{4} \\ \cot^2 x - \operatorname{cosec}^2 x &= \left(\frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2 \\ &= \frac{9}{16} - \frac{25}{16} \\ &= \frac{9 - 25}{16} \\ &= \frac{-16}{16} \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{Perpendicular} &= \text{Hypotenuse/Base (in right angled } \triangle \text{ACD)} \\ &= \text{AD/CD} \\ &= \frac{12}{8\sqrt{2}} \\ &= \frac{3}{2\sqrt{2}} \\ \cot y &= \text{Base/ Hypotenuse} \\ &= \text{AD/CD} \\ &= \frac{4}{8\sqrt{2}} \\ &= \frac{1}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \cot y &= \text{Base / Hypotenuse (in right angled } \triangle \text{ACD)} \\ &= \text{CD/AC} \\ &= \frac{8\sqrt{2}}{12} \\ &= \frac{2\sqrt{2}}{3} \end{aligned}$$

$$\begin{aligned} \text{Now } \tan^2 y &= \frac{1}{\cos^2 y} \\ &= \frac{(1/2\sqrt{2})^2}{1/(2\sqrt{2}/3)^2} \\ &= \frac{1}{4} \times -\frac{1}{4} \times 2 \\ &= \frac{1}{8} - \frac{9}{8} \\ &= \frac{1-9}{8} \\ &= \frac{-8}{8} \\ &= -1 \\ \tan^2 y - \frac{1}{\cos^2 y} &= -1. \end{aligned}$$

4. (a) From the figure (1) given below, find the values of:

(i) $2 \sin y - \cos y$

(ii) $2 \sin x - \cos x$

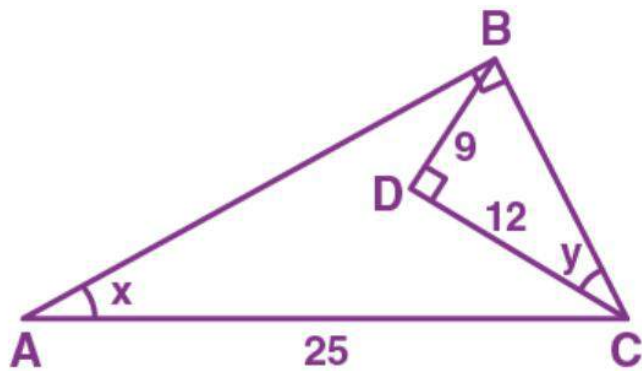
(iii) $1 - \sin x + \cos y$

(iv) $2 \cos x - 3 \sin y + 4 \tan x$

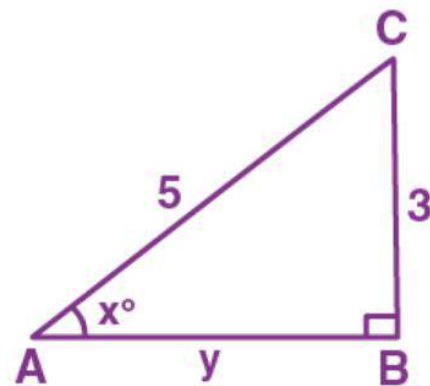
(b) In the figure (2) given below, ΔABC is right-angled at B. If $AB = y$ units, $BC = 3$ units and $CA = 5$ units, find

(i) $\sin x^\circ$

(ii) y .



(1)



(2)

Solution:

(a) In a right angled ΔBCD ,
Using Pythagoras theorem

$$BC^2 = BD^2 + CD^2$$

Substituting the values

$$BC^2 = 9^2 + 12^2$$

By further calculation

$$BC^2 = 81 + 144 = 225$$

$$BC^2 = 15^2$$

$$BC = 15$$

In a right angled ΔABC ,
Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

We can write it as

$$AB^2 = AC^2 - BC^2$$

Substituting the values

$$AB^2 = 25^2 - 15^2$$

By further calculation

$$AB^2 = 625 - 225 = 400$$

So we get

$$AB^2 = 20^2$$

$$AB = 20$$

(i) We know that

In right angled $\triangle BCD$

$\sin y = \text{perpendicular/hypotenuse}$

$$\sin y = BD/BC$$

Substituting the values

$$\sin y = 9/15 = 3/5$$

In right angled $\triangle BCD$

$\cos y = \text{base/hypotenuse}$

$$\cos y = CD/BC$$

Substituting the values

$$\cos y = 12/15 = 4/5$$

Here

$$2\sin y - \cos y = 2 \times 3/5 - 4/5$$

We can write it as

$$= 6/5 - 4/5$$

$$= 2/5$$

Therefore, $2 \sin y - \cos y = 2/5$

(ii) In right angled $\triangle ABC$

$\sin x = \text{perpendicular/hypotenuse}$

$$\sin x = BC/AC$$

Substituting the values

$$\sin x = 15/25 = 3/5$$

In right angled $\triangle ABC$

$\cos x = \text{base/hypotenuse}$

$\cos x = AB/AC$

Substituting the values

$\cos x = 20/25 = 4/5$

Here

$2 \sin x - \cos x = 2 \times 3/5 - 4/5$

We can write it as

$= 6/5 - 4/5$

$= 2/5$

Therefore, $2 \sin x - \cos x = 2/5$.

(iii) In right angled $\triangle ABC$

$\sin x = \text{perpendicular/hypotenuse}$

$\sin x = BC/AC$

Substituting the values

$\sin x = 12/25 = 3/5$

In right angled $\triangle BCD$

$\cos y = \text{base/hypotenuse}$

$\cos y = CD/BC$

Substituting the values

$\cos y = 12/15 = 4/5$

Here

$1 - \sin x + \cos y = 1 - 3/5 + 4/5$

By further calculation

$= (5 - 3 + 4)/5$

So we get

$= (9 - 3)/5$

$= 6/5$

Therefore, $1 - \sin x + \cos y = 6/5$.

(iv) In right angled $\triangle BCD$

$\cos x = \text{base/hypotenuse}$

$$\cos x = AB/AC$$

Substituting the values

$$\cos x = 20/25 = 4/5$$

In right angled $\triangle BCD$

$\sin y = \text{perpendicular/hypotenuse}$

$$\sin y = BD/BC$$

Substituting the values

$$\sin y = 9/15 = 3/5$$

In right angled $\triangle ABC$

$\tan x = \text{perpendicular/base}$

$$\tan x = BC/AB$$

Substituting the values

$$\tan x = 15/20 = 3/4$$

Here

$$2 \cos x - 3 \sin y + 4 \tan x = 2 \times 4/5 - 3 \times 3/5 + 4 \times 3/4$$

By further calculation

$$= 8/5 - 9/5 + 3$$

Taking LCM

$$= (8 - 9 + 15)/5$$

So we get

$$= (23 - 9)/5$$

$$= 14/5$$

(b) It is given that

$AB = y$ units, $BC = 3$ units, $CA = 5$ units

(i) In right angled $\triangle ABC$

$\sin x = \text{perpendicular/hypotenuse}$

$$\sin x = BC/AC$$

Substituting the values

$$\sin x = 3/5$$

(ii) In right angled $\triangle ABC$

Using Pythagoras theorem

$$AC^2 = BC^2 + AB^2$$

We can write it as

$$AB^2 = AC^2 - BC^2$$

Substituting the values

$$AB^2 = 5^2 - 3^2$$

By further calculation

$$AB^2 = 25 - 9 = 16$$

So we get

$$AB^2 = 4^2$$

$$AB = 4$$

$$y = 4 \text{ units}$$

Therefore, $y = 4$ units.

5. In a right-angled triangle, it is given that angle A is an acute angle and that $\tan A = 5/12$. Find the values of:

(i) $\cos A$

(ii) $\operatorname{cosec} A - \cot A$.

Solution:

Here, ABC is right angled triangle

$\angle A$ is an acute angle and $\angle C = 90^\circ$

$$\tan A = 5/12$$

$$BC/AC = 5/12$$

$$\text{Let } BC = 5x \text{ and } AC = 12x$$

From right angled $\triangle ABC$

By Pythagoras theorem, we get

$$AB^2 = (5x)^2 + (12x)^2$$

$$AB^2 = 25x^2 + 144x^2$$

$$AB^2 = 169x^2$$

(i) $\cos A = \text{Base} / \text{Hypotenuse}$

$$= AC / AB$$

$$= 12x / 13x$$

$$= 12/13$$

(ii) $\operatorname{cosec} A = \text{Hypotenuse} / \text{perpendicular}$

$$= AC / BC$$

$$= 13x / 5x$$

$$= 13/5$$

$$\operatorname{cosec} A - \cot A = 13/5 - 12/5$$

$$= (13-12)/5$$

$$= 1/5$$

6. (a) In ΔABC , $\angle A = 90^\circ$. If $AB = 7$ cm and $BC - AC = 1$ cm, find:

(i) $\sin C$

(ii) $\tan B$

(b) In ΔPQR , $\angle Q = 90^\circ$. If $PQ = 40$ cm and $PR + QR = 50$ cm, find:

(i) $\sin P$

(ii) $\cos P$

(iii) $\tan R$.

Solution:

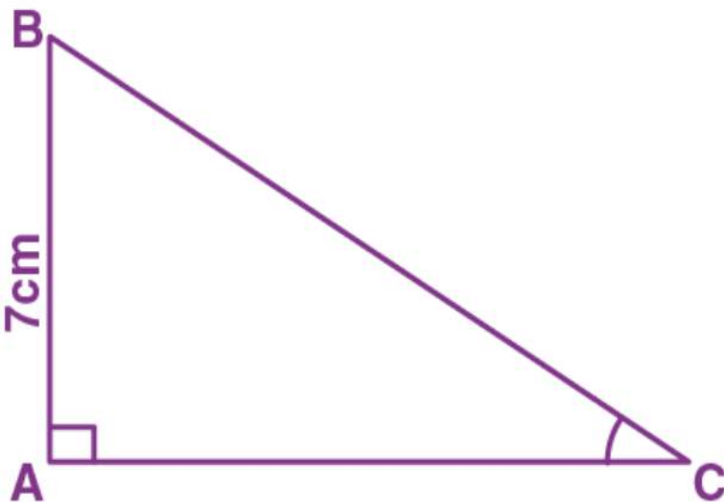
(a) In right ΔABC

$$\angle A = 90^\circ$$

$$AB = 7 \text{ cm}$$

$$BC - AC = 1 \text{ cm}$$

$$BC = 1 + AC$$



We know that

$$BC^2 = AB^2 + AC^2$$

Substituting the value of BC

$$(1 + AC)^2 = AB^2 + AC^2$$

$$1 + AC^2 + 2AC = 7^2 + AC^2$$

By further calculation

$$1 + AC^2 + 2AC = 49 + AC^2$$

$$2AC = 49 - 1 - 48$$

So we get

$$AC = 48/2 = 24 \text{ cm}$$

Here

$$BC = 1 + AC$$

Substituting the value

$$BC = 1 + 24 = 25 \text{ cm}$$

$$(i) \sin C = AB/BC = 7/25$$

$$(ii) \tan B = AC/AB = 24/7$$

(b) In right ΔPQR

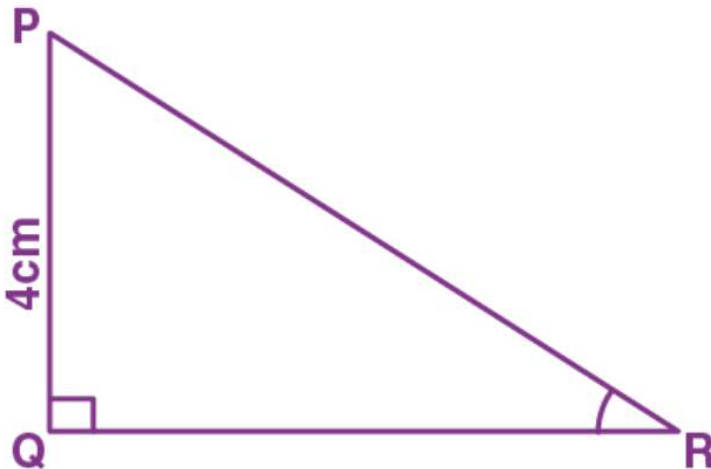
$$\angle Q = 90^\circ$$

$$PQ = 40 \text{ cm}$$

$$PQ + QR = 50 \text{ cm}$$

We can write it as

$$PQ = 50 - QR$$



Using Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$

$$(50 - QR)^2 = (40)^2 + QR^2$$

By further calculation

$$2500 + QR^2 - 100QR = 1600 + QR^2$$

So we get

$$2500 - 1600 = 100QR$$

$$100QR = 900$$

By division

$$QR = 900/100 = 9$$

We get

$$PR = 50 - 9 = 41$$

$$(i) \sin P = QR/PR = 9/41$$

$$(ii) \cos P = PQ/PR = 40/41$$

$$(iii) \tan R = PQ/QR = 40/9$$

7. In triangle ABC, AB = 15 cm, AC = 15 cm and BC = 18 cm. Find

(i) $\cos \angle ABC$

(ii) $\sin \angle ACB$.

Solution:

Here ABC is a triangle in which

AB = 15 cm, AC = 15 cm and BC = 18 cm

Draw AD perpendicular to BC, D is mid-point of BC.

Then, BD = DC = 9 cm

in right angled triangle ABD,

By Pythagoras theorem, we get

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2$$

$$AD^2 = (15)^2 - (9)^2$$

$$AD^2 = 225 - 81$$

$$AD^2 = 144$$

$$AD = 12 \text{ cm}$$

(i) $\cos \angle ABC = \text{Base} / \text{Hypotenuse}$

(In right angled $\triangle ABD$, $\angle ABC = \angle ABD$)

$$= BD / AB$$

$$= 9/15$$

$$= 3/5$$

$$\begin{aligned} \text{(ii) } \sin \angle ACB &= \sin \angle ACD \\ &= \text{perpendicular/ Hypotenuse} \\ &= AD/AC \\ &= 12/15 \\ &= 4/5 \end{aligned}$$

8. (a) In the figure (1) given below, ΔABC is isosceles with $AB = AC = 5$ cm and $BC = 6$ cm. Find

(i) $\sin C$

(ii) $\tan B$

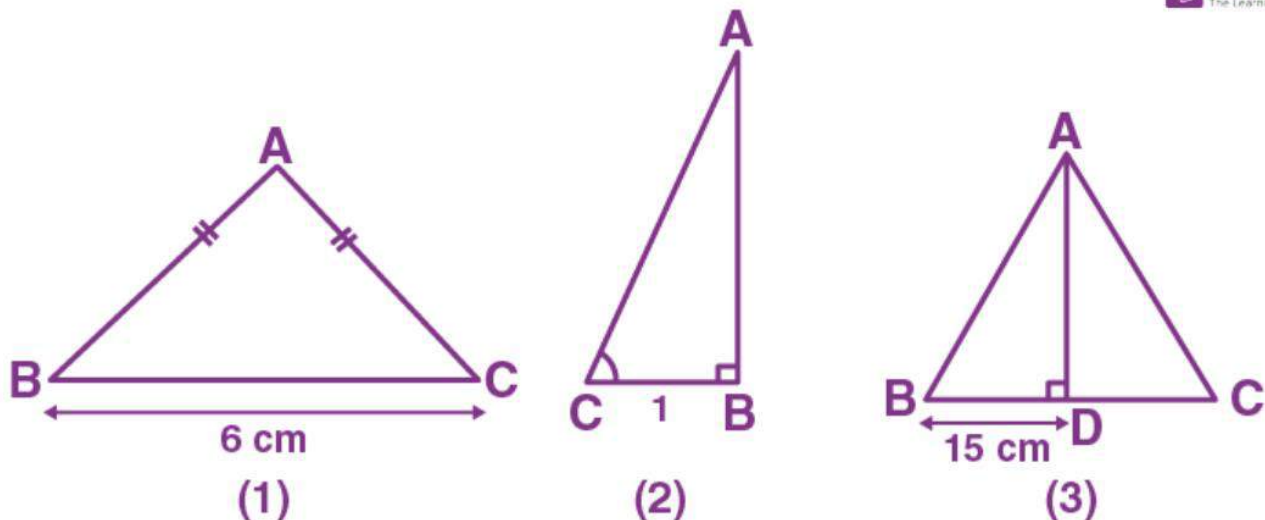
(iii) $\tan C - \cot B$.

(b) In the figure (2) given below, ΔABC is right-angled at B. Given that $\angle ACB = \theta$, side $AB = 2$ units and side $BC = 1$ unit, find the value of $\sin^2 \theta + \tan^2 \theta$.

(c) In the figure (3) given below, AD is perpendicular to BC, $BD = 15$ cm, $\sin B = 4/5$ and $\tan C = 1$.

(i) Calculate the lengths of AD, AB, DC and AC.

(ii) Show that $\tan^2 B - 1/\cos^2 B = -1$.



Solution:

(a) It is given that

ΔABC is isosceles with $AB = AC = 5$ cm and $BC = 6$ cm

Construct AD perpendicular to BC

D is the mid point of BC

So $BD = CD$

Here

$$BD = CD = 6/2 = 3 \text{ cm}$$

In right angled $\triangle ABD$

Using Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

We can write it as

$$AD^2 = AB^2 - BD^2$$

Substituting the values

$$AD^2 = 5^2 - 3^2$$

By further calculation

$$AD^2 = 25 - 9 = 16$$

So we get

$$AD^2 = 4^2$$

$$AD = 4 \text{ cm}$$

(i) In right angled $\triangle ACD$

$\sin C = \text{perpendicular/hypotenuse}$

$$\sin C = AD/AC = 4/5$$

(ii) In right angled $\triangle ABD$

$\tan B = \text{perpendicular/base}$

$$\tan B = AD/BD = 4/3$$

(iii) In right angled $\triangle ACD$

$\tan C = \text{perpendicular/base}$

$$\tan C = AD/CD = 4/3$$

In right angled $\triangle ABD$

$\cot B = \text{base/perpendicular}$

$$\cot B = BD/AD = \frac{3}{4}$$

Here

$$\tan C - \cot B = 4/3 - \frac{3}{4}$$

Taking LCM

$$\tan C - \cot B = (16 - 9)/12 = 7/12$$

(b) It is given that

ΔABC is right-angled at B

AB = 2 units and BC = 1 unit

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 2^2 + 1^2$$

$$AC^2 = 4 + 1 = 5$$

So we get

$$AC^2 = 5$$

$$AC = \sqrt{5} \text{ units}$$

In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

$$\sin \theta = AB/AC = 2/\sqrt{5}$$

In right angled ΔABC

$\tan \theta = \text{perpendicular/base}$

$$\tan \theta = AB/BC = 2/1$$

We know that

$$\sin^2 \theta + \tan^2 \theta = (2/\sqrt{5})^2 + (2/1)^2$$

By further calculation

$$= 4/5 + 4/1$$

Taking LCM

$$= (4 + 20)/5$$

$$= 24/5$$

$$= 4 \frac{4}{5}$$

(c) (i) In ΔABC

AD is perpendicular to BC

$$BD = 15 \text{ cm}$$

$$\sin B = 4/5$$

$$\tan C = 1$$

In $\triangle ABD$

$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$$\sin B = \frac{AD}{AB} = \frac{4}{5}$$

Consider $AD = 4x$ and $AB = 5x$

Using Pythagoras theorem

In right angled $\triangle ABD$

$$AB^2 = AD^2 + BD^2$$

We can write it as

$$BD^2 = AB^2 - AD^2$$

Substituting the values

$$(15)^2 = (5x)^2 - (4x)^2$$

$$225 = 25x^2 - 16x^2$$

By further calculation

$$225 = 9x^2$$

$$x^2 = \frac{225}{9} = 25$$

So we get

$$x = \sqrt{25} = 5$$

Here

$$AD = 4 \times 5 = 20$$

$$AB = 5 \times 5 = 25$$

In right angled $\triangle ACD$

$\tan C = \frac{\text{perpendicular}}{\text{base}}$

So we get

$$\tan C = \frac{AD}{CD} = \frac{1}{1}$$

Consider $AD = X$ then $CD = x$

In right angled $\triangle ADC$

Using Pythagoras theorem

$$AC^2 = AD^2 + CD^2$$

Substituting the values

$$AC^2 = x^2 + x^2 \dots\dots(1)$$

So the equation becomes

$$AC^2 = 20^2 + 20^2$$

$$AC^2 = 400 + 400 = 800$$

So we get

$$AC = \sqrt{800} = 20\sqrt{2}$$

Length of AD = 20 cm

Length of AB = 25 cm

Length of DC = 20 cm

Length of AC = $20\sqrt{2}$ cm

(ii) In right angled $\triangle ABD$

$\tan B = \text{perpendicular/base}$

So we get

$$\tan B = AD/BD$$

Substituting the values

$$\tan B = 20/15 = 4/3$$

In right angled $\triangle ABD$

$\cos B = \text{base/hypotenuse}$

So we get

$$\cos B = BD/AB$$

Substituting the values

$$\cos B = 15/25 = 3/5$$

Here

$$\text{LHS} = \tan^2 B - 1/\cos^2 B$$

Substituting the values

$$= (4/3)^2 - 1/(3/5)^2$$

By further calculation

$$= (4)^2/(3)^2 - (5)^2/(3)^2$$

$$= 16/9 - 25/9$$

So we get

$$= (16 - 25)/9$$

$$= -9/9$$

$$= -1$$

$$= \text{RHS}$$

Hence, proved.

9. If $\sin \theta = 3/5$ and θ is acute angle, find

(i) $\cos \theta$

(ii) $\tan \theta$.

Solution:

Let ΔABC be a right angled at B

Let $\angle ACB = \theta$

Given that, $\sin \theta = 3/5$

$AB/AC = 3/5$

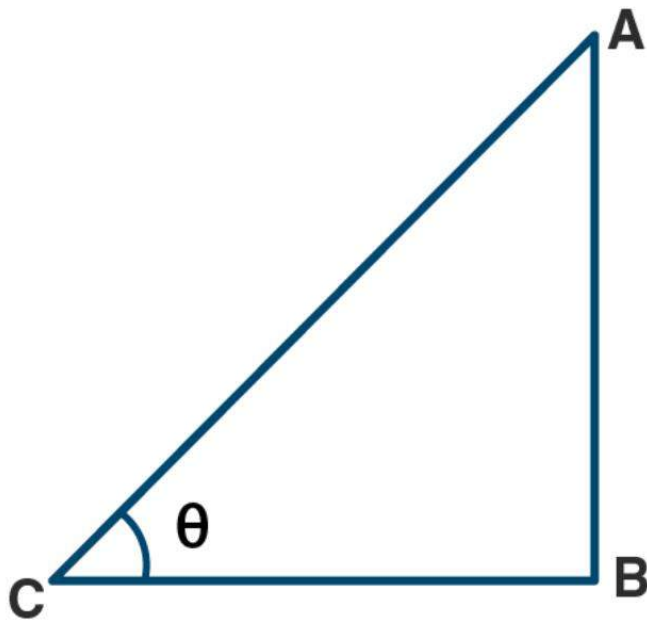
Let $AB = 3x$

then $AC = 5x$

In right angled ΔABC ,

By Pythagoras theorem,

We get



$$(5x)^2 = (3x)^2 + BC^2$$

$$BC^2 = (5x)^2 - (3x)^2$$

$$BC^2 = (2x)^2$$

$$BC = 4x$$

$$\begin{aligned} \text{(i) } \cos \theta &= \text{Base/ Hypotenuse} \\ &= BC / AC \\ &= 4x / 5x \\ &= 4/5 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \tan \theta &= \text{perpendicular/Base} \\ &= AB/BC \\ &= 3x/4x \\ &= \frac{3}{4} \end{aligned}$$

10. Given that $\tan \theta = 5/12$ and θ is an acute angle, find $\sin \theta$ and $\cos \theta$.

Solution:

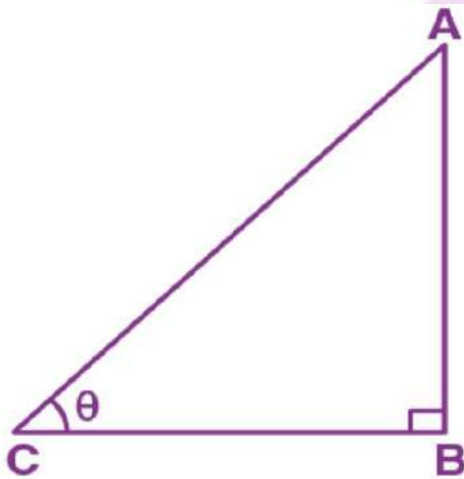
Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\tan \theta = 5/12$$

$$AB/BC = 5/12$$

Consider $AB = 5x$ and $BC = 12x$



In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (5x)^2 + (12x)^2$$

By further calculation

$$AC^2 = 25x^2 + 144x^2 = 169x^2$$

So we get

$$AC^2 = (13x)^2$$

$$AC = 13x$$

In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

So we get

$$\sin \theta = AB/AC = 5x/13x = 5/13$$

In right angled ΔABC

$\cos \theta = \text{base/hypotenuse}$

So we get

$$\cos \theta = BC/AC$$

Substituting the values

$$\cos \theta = 12x/13x = 12/13$$

11. If $\sin \theta = 6/10$, find the value of $\cos \theta + \tan \theta$.

Solution:

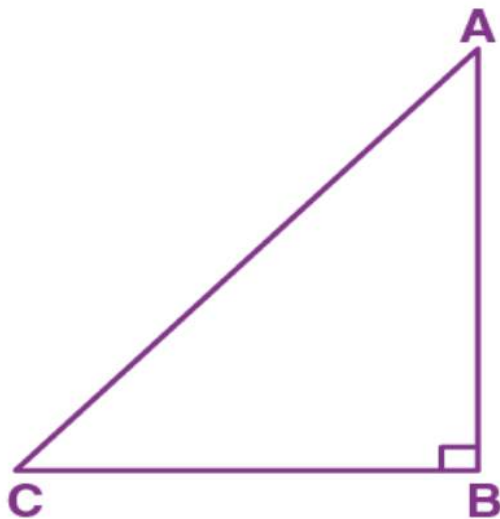
Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\sin \theta = AB/AC$$

$$\sin \theta = 6/10$$

Take $AB = 6x$ then $AC = 10x$



In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$(10x)^2 = (6x)^2 + BC^2$$

By further calculation

$$BC^2 = 100x^2 - 36x^2 = 64x^2$$

So we get

$$BC^2 = (8x)^2$$

$$BC = 8x$$

In right angled ΔABC

$\cos \theta = \text{base/hypotenuse}$

$$\cos \theta = BC/AC$$

Substituting the values

$$\cos \theta = 8x/10x = 4/5$$

In right angled ΔABC

$\tan \theta = \text{perpendicular/base}$

$$\tan \theta = AB/BC$$

Substituting the values

$$\tan \theta = 6x/8x = \frac{3}{4}$$

Here

$$\cos \theta + \tan \theta = 4/5 + \frac{3}{4}$$

Taking LCM

$$= (16 + 15)/20$$

$$= 31/20$$

$$= 1 \frac{11}{20}$$

12. If $\tan \theta = 4/3$, find the value of $\sin \theta + \cos \theta$ (both $\sin \theta$ and $\cos \theta$ are positive).

Solution:

Let ΔABC be a right angled

$$\angle ACB = \theta$$

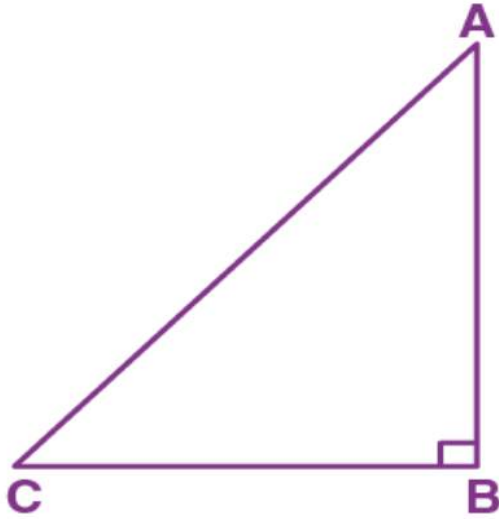
Given that, $\tan \theta = 4/3$

$$(AB/BC = 4/3)$$

Given that, $\tan \theta = 4/3$

$$(AB/BC = 4/3)$$

Let $AB = 4x$,
then $BC = 3x$



In right angled $\triangle ABC$

By Pythagoras theorem, we get

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$

$$(AC^2 = (4x)^2 + (3x)^2$$

$$AC^2 = 16x^2 + 9x^2$$

$$AC^2 = 25x^2$$

$$AC^2 = (5x)^2$$

$$AC = 5x$$

$$\sin \theta = \text{perpendicular/Hypotenuse}$$

$$= AB/AC$$

$$= 4x/5x$$

$$= 4/5$$

$$\cos \theta = \text{Base/Hypotenuse}$$

$$= BC/AC$$

$$= 3x/5x$$

$$= 3/5$$

$$\sin \theta + \cos \theta$$

$$= 4/5 + 3/5$$

$$= (4 + 3)/5$$

$$= 7/5$$

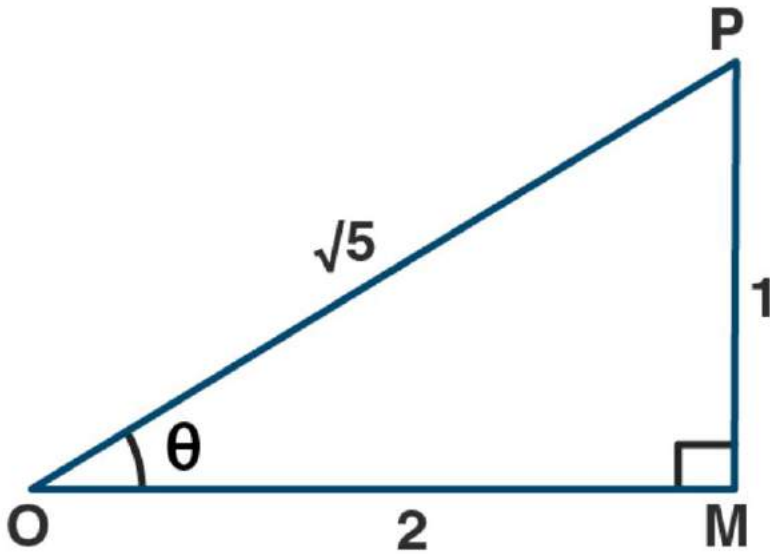
Hence, $\sin \theta + \cos \theta = 7/5 = 1 \frac{2}{5}$

13. If cosec $\theta = \sqrt{5}$ and θ is less than 90° , find the value of $\cot \theta - \cos \theta$.

Solution:

Given cosec $\theta = \sqrt{5}/1 = OP/PM$

$OP = \sqrt{5}$ and $PM = 1$



Now $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$$(\sqrt{5})^2 = OM^2 + 1^2$$

$$5 = OM^2 + 1$$

$$OM^2 = 5 - 1$$

$$OM^2 = 4$$

$$OM = 2$$

Now $\cot \theta = OM/PM$

$$= 2/1$$

$$= 2$$

$\cos \theta = OM/OP$

$$= 2/\sqrt{5}$$

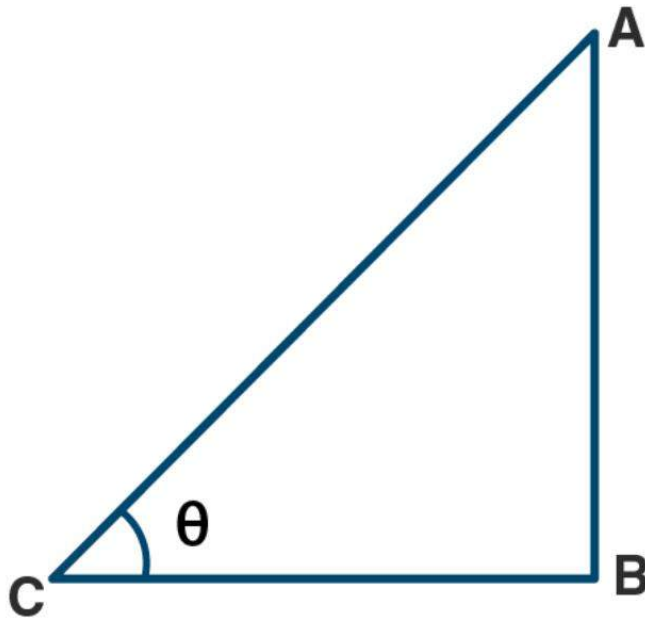
Now $\cot \theta - \cos \theta = 2 - (2/\sqrt{5})$

$$= 2(\sqrt{5} - 1)/\sqrt{5}$$

14. Given $\sin \theta = p/q$, find $\cos \theta + \sin \theta$ in terms of p and q .

Solution:

Given that $\sin \theta = p/q$



Which implies,

$$AB/AC = p/q$$

Let $AB = px$

And then $AC = qx$

In right angled triangle ABC

By Pythagoras theorem,

We get

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = q^2x^2 - p^2x^2$$

$$BC^2 = (q^2 - p^2)x^2$$

$$BC = \sqrt{(q^2 - p^2)}x$$

In right angled triangle ABC,

$\cos \theta = \text{base/ hypotenuse}$

$$= BC/AC$$

$$= \sqrt{(q^2 - p^2)}x/qx$$

$$= \sqrt{(q^2 - p^2)}/ q$$

Now,

$$\sin \theta + \cos \theta = p/q + \sqrt{(q^2 - p^2)}/ q$$

$$= [p + \sqrt{(q^2 - p^2)}]/ q$$

15. If θ is an acute angle and $\tan \theta = 8/15$, find the value of $\sec \theta + \operatorname{cosec} \theta$.

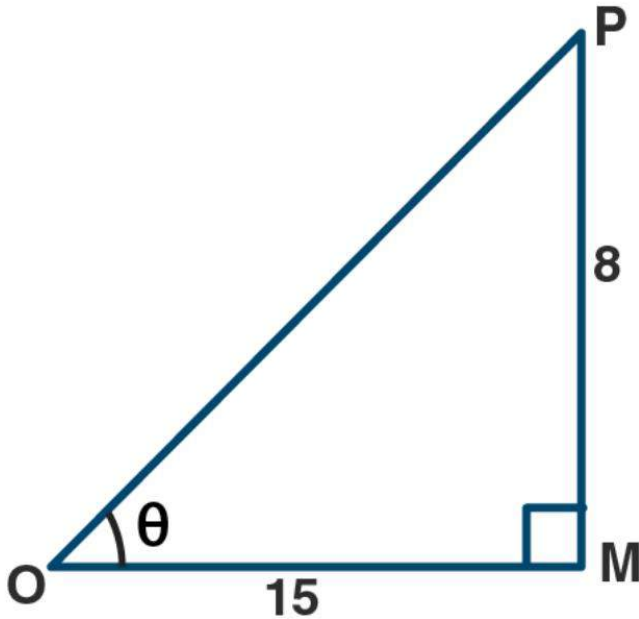
Solution:

Given $\tan \theta = 8/15$

θ is an acute angle

in the figure triangle OMP is a right angled triangle,

$\angle M = 90^\circ$ and $\angle Q = \theta$



$$\tan \theta = \frac{PM}{OM} = \frac{8}{15}$$

Therefore, $PM = 8$, $OM = 15$

But $OP^2 = OM^2 + PM^2$ using Pythagoras theorem,

$$= 15^2 + 8^2$$

$$= 225 + 64$$

$$= 289$$

$$= 17^2$$

Therefore, $OP = 17$

$$\sec \theta = \frac{OP}{OM}$$

$$= \frac{17}{15}$$

$$\operatorname{Cosec} \theta = \frac{OP}{PM}$$

$$= \frac{17}{8}$$

Now,

$$\sec \theta + \operatorname{cosec} \theta = \left(\frac{17}{15}\right) + \left(\frac{17}{8}\right)$$

$$= \frac{(136 + 255)}{120}$$

$$= 391/120$$
$$= 3 \frac{31}{120}$$

16. Given A is an acute angle and $13 \sin A = 5$, evaluate:

$(5 \sin A - 2 \cos A) / \tan A$.

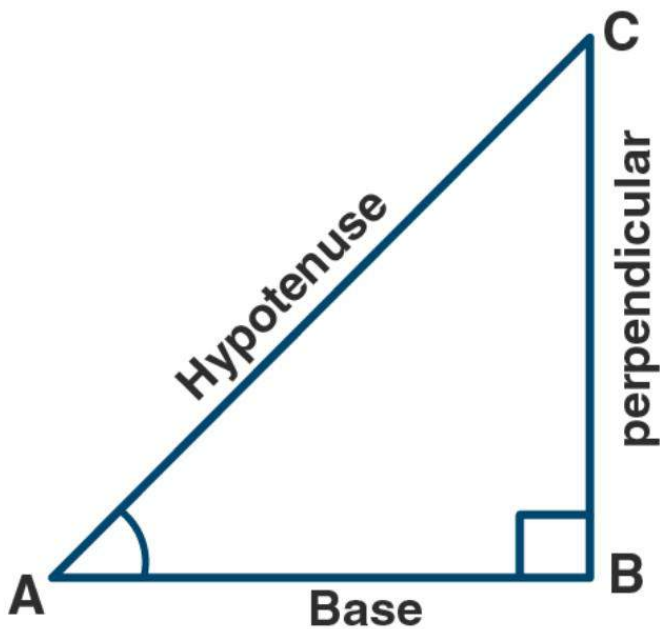
Solution:

Let triangle ABC be a right angled triangle at B and A is an acute angle

Given that $13 \sin A = 5$

$$\sin A = 5/13$$

$$AB/AC = 5/13$$



$$\text{Let } AB = 5x$$

$$AC = 13x$$

In right angled triangle ABC,

Using Pythagoras theorem,

We get

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = (13x)^2 - (5x)^2$$

$$BC^2 = 169x^2 - 25x^2$$

$$BC^2 = 144x^2$$

$$BC = 12x$$

$$\sin A = 5/13$$

$$\begin{aligned}\cos A &= \text{base/ hypotenuse} \\ &= BC/AC \\ &= 12x/ 13x \\ &= 12/13\end{aligned}$$

$$\begin{aligned}\tan A &= \text{perpendicular/ base} \\ &= AB/BC \\ &= 5x/ 12x \\ &= 5/ 12\end{aligned}$$

Now,

$$\begin{aligned}(5 \sin A - 2 \cos A) / \tan A &= [(5) (5/13) - (2) (12/13)] / (5/12) \\ &= (1/13) / (5/12) \\ &= 12/65\end{aligned}$$

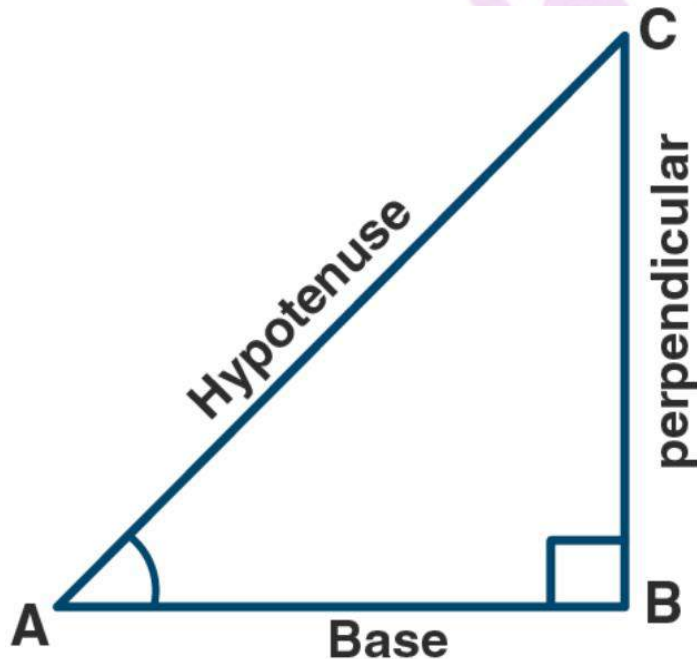
$$\text{Hence } (5 \sin A - 2 \cos A) / \tan A = 12/65$$

17. Given A is an acute angle and cosec A = $\sqrt{2}$, find the value of $(2 \sin^2 A + 3 \cot^2 A) / (\tan^2 A - \cos^2 A)$.

Solution:

Let triangle ABC be a right angled at B and A is a acute angle.

Given that cosec A = $\sqrt{2}$



Which implies,
 $AC/BC = \sqrt{2}/1$

Let $AC = \sqrt{2}x$

Then $BC = x$

In right angled triangle ABC

By using Pythagoras theorem,

We get

$$AC^2 = AB^2 + BC^2$$

$$(\sqrt{2}x)^2 = AB^2 + x^2$$

$$AB^2 = 2x^2 - x^2$$

$$AB = x$$

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$$= \frac{BC}{AC}$$

$$= \frac{1}{\sqrt{2}}$$

$\cot A = \frac{\text{base}}{\text{perpendicular}}$

$$= \frac{x}{x}$$

$$= 1$$

$\tan A = \frac{\text{perpendicular}}{\text{base}}$

$$= \frac{BC}{AB}$$

$$= \frac{x}{x}$$

$$= 1$$

$\cos A = \frac{\text{base}}{\text{hypotenuse}}$

$$= \frac{AB}{AC}$$

$$= \frac{x}{\sqrt{2}x}$$

$$= \frac{1}{\sqrt{2}}$$

Substituting these values we get

$$2 \sin^2 A + 3 \cot^2 A / (\tan^2 A - \cos^2 A) = 8$$

18. The diagonals AC and BD of a rhombus ABCD meet at O. If AC = 8 cm and BD = 6 cm, find $\sin \angle OCD$.

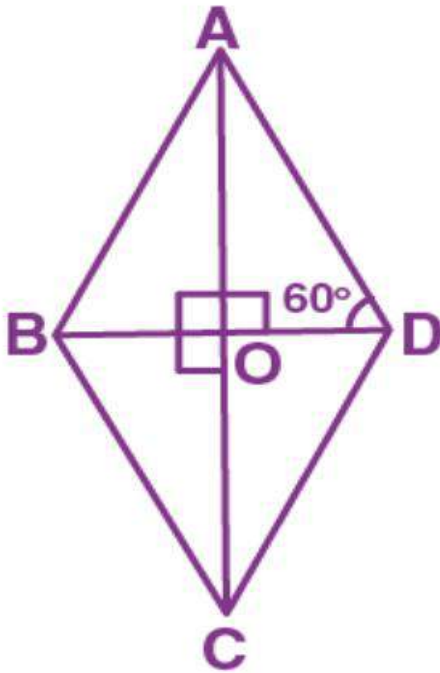
Solution:

It is given that

Diagonals AC and BD of rhombus ABCD meet at O

AC = 8 cm and BD = 6 cm

O is the mid point of AC



We know that

$$AO = OC = AC/2 = 8/2 = 4 \text{ cm}$$

O is the mid point of BD

$$BO = OD = BD/2 = 6/2 = 3 \text{ cm}$$

In right angled $\triangle COD$

$$CD^2 = OC^2 + OD^2$$

Substituting the values

$$CD^2 = 4^2 + 3^2$$

So we get

$$CD^2 = 16 + 9 = 25$$

$$CD^2 = 5^2$$

$$CD = 5 \text{ cm}$$

In right angled $\triangle COD$

$\sin \angle OCD = \text{perpendicular} / \text{hypotenuse}$

So we get

$$\sin \angle OCD = OD/CD = 3/5$$

19. If $\tan \theta = 5/12$, find the value of $(\cos \theta + \sin \theta) / (\cos \theta - \sin \theta)$.

Solution:

Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\tan \theta = AB/BC = 5/12$$

Take $AB = 5x$ then $BC = 12x$

In right angled ΔABC ,

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (5x)^2 + (12x)^2$$

By further calculation

$$AC^2 = 25x^2 + 144x^2 = 169x^2$$

So we get

$$AC^2 = (13x)^2$$

$$AC = 13x$$

In right angled ΔABC

$\cos \theta = \text{base/hypotenuse}$

$$\cos \theta = BC/AC$$

Substituting the values

$$\cos \theta = 12x/13x = 12/13$$

In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

$$\sin \theta = AB/AC$$

Substituting the values

$$\sin \theta = 5x/13x = 5/13$$

Here

$$(\cos \theta + \sin \theta) / (\cos \theta - \sin \theta) = [12/13 + 5/13] / [12/13 - 5/13]$$

Taking LCM

$$= [(12 + 5) / 13] / [(12 - 5) / 13]$$

So we get

$$= 17/13 / 7/13$$

$$= 17/13 \times 13/7$$

$$= 17/7$$

Therefore, $(\cos \theta + \sin \theta) / (\cos \theta - \sin \theta) = 17/7 = 2 \frac{3}{7}$.

20. Given $5 \cos A - 12 \sin A = 0$, find the value of $(\sin A + \cos A) / (2 \cos A - \sin A)$.

Solution:

It is given that

$$5 \cos A - 12 \sin A = 0$$

We can write it as

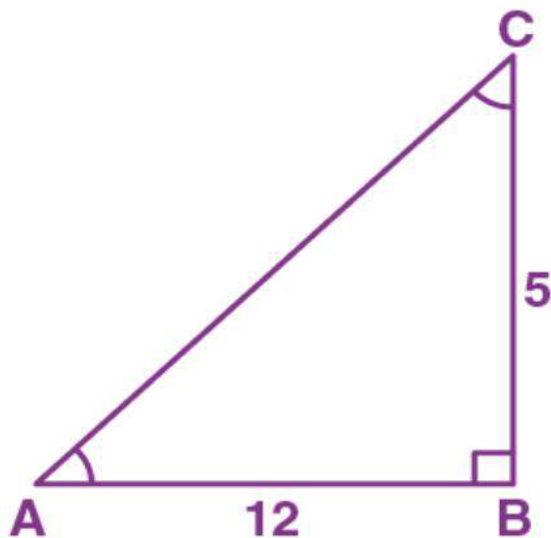
$$5 \cos A = 12 \sin A$$

So we get

$$\sin A / \cos A = 5/12$$

We know that $\sin A / \cos A = \tan A$

$$\tan A = 5/12$$



Consider ΔABC right angled at B and $\angle A$ is acute angle

Here

$$\tan A = BC/AB = 5/12$$

Take $BC = 5x$ then $AB = 12x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = BC^2 + AB^2$$

Substituting the values

$$AC^2 = (5x)^2 + (12x)^2$$

$$AC^2 = 25x^2 + 144x^2 = 169x^2$$

So we get

$$AC^2 = (13x)^2$$

$$AC = 13x$$

In right angled ΔABC

$\sin A = \text{perpendicular/hypotenuse}$

So we get

$$\sin A = BC/AC = 5x/13x = 5/13$$

In right angled ΔABC

$\cos A = \text{base/hypotenuse}$

So we get

$$\cos A = AB/AC = 12x/13x = 12/13$$

Here

$$(\sin A + \cos A) / (2 \cos A - \sin A) = [5/13 + 12/13] / [2 \times 12/13 - 5/13]$$

By further calculation

$$= [(5 + 12)/13] / [24/13 - 5/13]$$

So we get

$$= [(5 + 12)/13] / [(24 - 5)/13]$$

$$= 17/13 / 19/13$$

$$= 17/13 \times 13/19$$

$$= 17/19$$

Therefore, $(\sin A + \cos A) / (2 \cos A - \sin A) = 17/19$

21. If $\tan \theta = p/q$, find the value of $(p \sin \theta - q \cos \theta) / (p \sin \theta + q \cos \theta)$.

Solution:

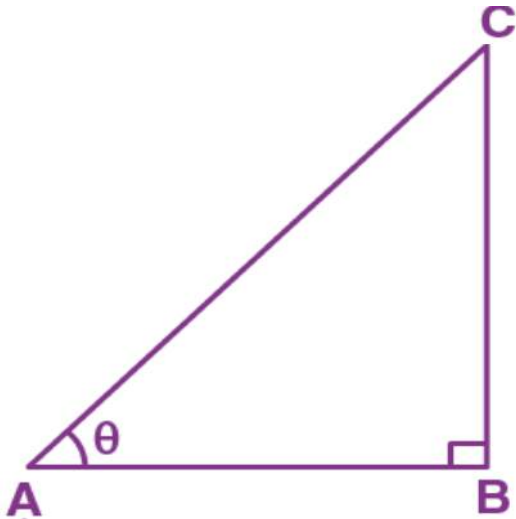
It is given that

$$\tan \theta = p/q$$

Consider ΔABC be right angled at B and $\angle BCA = \theta$

$$\tan \theta = BC/AB = p/q$$

$$BC = px \text{ then } AB = qx$$



In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = BC^2 + AB^2$$

Substituting the values

$$AC^2 = (px)^2 + (qx)^2$$

$$AC^2 = p^2x^2 + q^2x^2$$

$$AC^2 = x^2 (p^2 + q^2)$$

So we get

$$AC = \sqrt{x^2 (p^2 + q^2)}$$

$$AC = x(\sqrt{p^2 + q^2})$$

In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

$$\sin \theta = BC/AC$$

Substituting the values

$$\sin \theta = px / x(\sqrt{p^2 + q^2})$$

So we get

$$\sin \theta = p / (\sqrt{p^2 + q^2})$$

In right angled ΔABC

$\cos \theta = \text{base/hypotenuse}$

$$\cos \theta = AB/AC$$

Substituting the values

$$\cos \theta = qx / x(\sqrt{p^2 + q^2})$$

So we get

$$\cos \theta = q / (\sqrt{p^2 + q^2})$$

Here

$$\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} = \frac{p\left(\frac{p}{\sqrt{p^2+q^2}}\right) - q\left(\frac{q}{\sqrt{p^2+q^2}}\right)}{p\left(\frac{p}{\sqrt{p^2+q^2}}\right) + q\left(\frac{q}{\sqrt{p^2+q^2}}\right)}$$

By further calculation

$$\begin{aligned} &= \frac{\frac{p^2}{\sqrt{p^2+q^2}} - \frac{q^2}{\sqrt{p^2+q^2}}}{\frac{p^2}{\sqrt{p^2+q^2}} + \frac{q^2}{\sqrt{p^2+q^2}}} \\ &= \frac{\frac{p^2 - q^2}{\sqrt{p^2+q^2}}}{\frac{p^2 + q^2}{\sqrt{p^2+q^2}}} \end{aligned}$$

So we get

$$\begin{aligned} &= \frac{p^2 - q^2}{\sqrt{p^2 + q^2}} \times \frac{\sqrt{p^2 + q^2}}{p^2 + q^2} \\ &= \frac{p^2 - q^2}{p^2 + q^2} \end{aligned}$$

Therefore, $\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} = \frac{p^2 - q^2}{p^2 + q^2}$

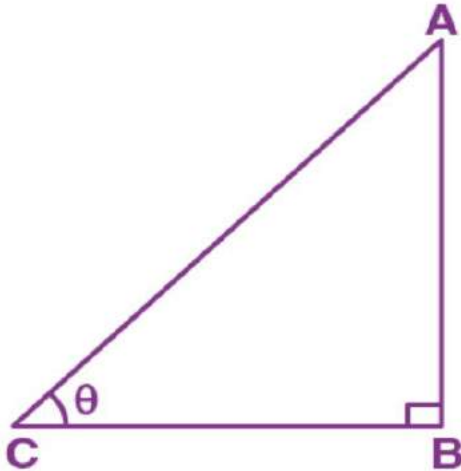
22. If $3 \cot \theta = 4$, find the value of $(5 \sin \theta - 3 \cos \theta) / (5 \sin \theta + 3 \cos \theta)$.

Solution:

It is given that

$$3 \cot \theta = 4$$

$$\cot \theta = 4/3$$



Consider ΔABC be right angled at B and $\angle ACB = \theta$

$$\cot \theta = BC/AB = 4/3$$

Take $BC = 4x$ then $AB = 3x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (3x)^2 + (4x)^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

So we get

$$AC^2 = (5x)^2$$

$$AC = 5x$$

In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

$$\sin \theta = AB/AC$$

Substituting the values

$$\sin \theta = 3x/5x = 3/5$$

In right angled ΔABC

$\cos \theta = \text{base/hypotenuse}$

$$\cos \theta = BC/AC$$

Substituting the values

$$\cos \theta = 4x/5x = 4/5$$

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{5 \times \frac{3}{5} - 3 \times \frac{4}{5}}{5 \times \frac{3}{5} + 3 \times \frac{4}{5}}$$

By further calculation

$$\begin{aligned} &= \frac{\frac{15}{5} - \frac{12}{5}}{\frac{15}{5} + \frac{12}{5}} \\ &= \frac{\frac{15-12}{5}}{\frac{15+12}{5}} \end{aligned}$$

So we get

$$\begin{aligned} &= \frac{\frac{3}{5}}{\frac{27}{5}} \\ &= \frac{3}{5} \times \frac{5}{27} \\ &= \frac{3}{27} \\ &= \frac{1}{9} \end{aligned}$$

Therefore, $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{1}{9}$

23. (i) If $5 \cos \theta - 12 \sin \theta = 0$, find the value of $(\sin \theta + \cos \theta)/(2 \cos \theta - \sin \theta)$.
 (ii) If $\operatorname{cosec} \theta = 13/12$, find the value of $(2 \sin \theta - 3 \cos \theta)/(4 \sin \theta - 9 \cos \theta)$.

Solution:

(i) It is given that

$$5 \cos \theta - 12 \sin \theta = 0$$

We can write it as

$$5 \cos \theta = 12 \sin \theta$$

$$\sin \theta / \cos \theta = 5/12$$

$$\tan \theta = 5/12$$

Dividing both numerator and denominator by $\cos \theta$

$$\frac{\sin\theta + \cos\theta}{2\cos\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{2\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\tan\theta + 1}{2 - \tan\theta}$$

Substituting the values

$$= \frac{\frac{5}{12} + 1}{2 - \frac{5}{12}}$$

Taking LCM

$$= \frac{\frac{5+12}{12}}{\frac{24-5}{12}}$$
$$= \frac{17}{19}$$

So we get

$$= \frac{17}{12} \times \frac{12}{19}$$
$$= \frac{17}{19}$$

(ii) It is given that

$$\operatorname{cosec} \theta = 13/12$$

We know that $\operatorname{cosec} \theta = 1/\sin \theta$

$$1/\sin \theta = 13/12$$

$$\sin \theta = 12/13$$

$$\text{Here } \cos^2 \theta = 1 - \sin^2 \theta$$

Substituting the values

$$= 1 - (12/13)^2$$

By further calculation

$$= 1 - 144/169$$

Taking LCM

$$= (169 - 144)/169$$

$$= 25/169$$

So we get

$$= (5/13)^2$$

$$\cos \theta = 5/13$$

Here

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2(\frac{12}{13}) - 3(\frac{5}{13})}{4(\frac{12}{13}) - 9(\frac{5}{13})}$$

By further calculation

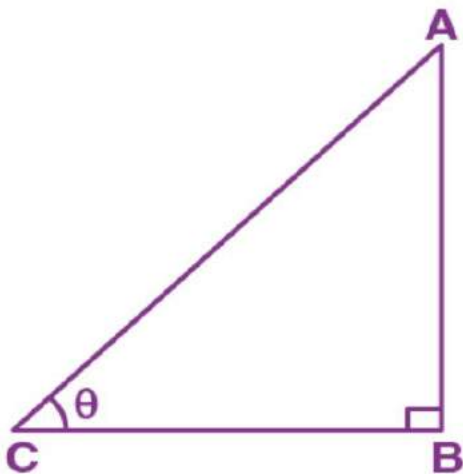
$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

So we get

$$\begin{aligned} &= \frac{24-15}{48-45} \\ &= \frac{9}{13} \\ &= \frac{9}{13} \times \frac{13}{3} \\ &= 3 \end{aligned}$$

24. If $5 \sin \theta = 3$, find the value of $(\sec \theta - \tan \theta) / (\sec \theta + \tan \theta)$.

Solution:



Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$5 \sin \theta = 3$$

$$\sin \theta = AB/AC = 3/5$$

Take $AB = 3x$ then $AC = 5x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

Substituting the values

$$BC^2 = (5x)^2 - (3x)^2$$

So we get

$$BC^2 = 25x^2 - 9x^2 = 16x^2$$

$$BC^2 = (4x)^2$$

$$BC = 4x$$

In right angled ΔABC

$\sec \theta = \text{hypotenuse/base}$

$$\sec \theta = AC/BC = 5x/4x = 5/4$$

In right angled ΔABC

$\tan \theta = \text{perpendicular/base}$

$$\tan \theta = AB/BC = 3x/4x = \frac{3}{4}$$

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$$

By further calculation

$$= \frac{\frac{5-3}{4}}{\frac{5+3}{4}}$$

So we get

$$= \frac{\frac{2}{4}}{\frac{8}{4}}$$

$$= \frac{2}{4} \times \frac{4}{8}$$

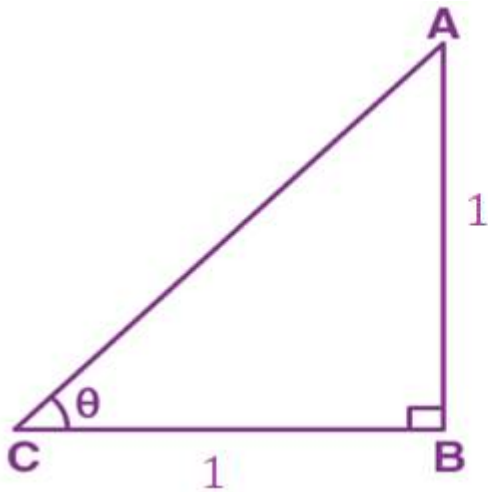
$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

Therefore, $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{1}{4}$

25. If θ is an acute angle and $\sin \theta = \cos \theta$, find the value of $2 \tan^2 \theta + \sin^2 \theta - 1$.

Solution:



Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\sin \theta = \cos \theta$$

$$\sin \theta / \cos \theta = 1$$

$$\tan \theta = AB/BC = 1$$

Take $AB = x$ then $BC = x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = x^2 + x^2 = 2x^2$$

So we get

$$AC = \sqrt{2x^2}$$

$$AC = (\sqrt{2})x$$

In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

$$\sin \theta = AB/AC = x/\sqrt{2}x = 1/\sqrt{2}$$

Here

$$2 \tan^2 \theta + \sin^2 \theta - 1 = 2 \times (1)^2 + (1/\sqrt{2})^2 - 1$$

By further calculation

$$= 2 \times 1 + \frac{1}{2} - 1$$

$$= 2 + \frac{1}{2} - 1$$

$$= 1 + \frac{1}{2}$$

Taking LCM

$$= (2 + 1)/2$$

$$= 3/2$$

Therefore, $2 \tan^2 \theta + \sin^2 \theta - 1 = 3/2$.

26. Prove the following:

(i) $\cos \theta \tan \theta = \sin \theta$

(ii) $\sin \theta \cot \theta = \cos \theta$

(iii) $\sin^2 \theta / \cos \theta + \cos \theta = 1 / \cos \theta$.

Solution:

(i) $\cos \theta \tan \theta = \sin \theta$

$$\text{LHS} = \cos \theta \tan \theta$$

$$\text{We know that } \tan \theta = \sin \theta / \cos \theta$$

$$= \cos \theta (\sin \theta / \cos \theta)$$

So we get

$$= 1 \times \sin \theta / 1$$

$$= \sin \theta$$

$$= \text{RHS}$$

Therefore, $\text{LHS} = \text{RHS}$.

(ii) $\sin \theta \cot \theta = \cos \theta$

$$\text{LHS} = \sin \theta \cot \theta$$

$$\text{We know that } \cot \theta = \cos \theta / \sin \theta$$

$$= \sin \theta (\cos \theta / \sin \theta)$$

$$= 1 \times \cos \theta / 1$$

$$= \cos \theta$$

$$= \text{RHS}$$

Therefore, LHS = RHS.

$$(iii) \sin^2 \theta / \cos \theta + \cos \theta = 1 / \cos \theta$$

$$\text{LHS} = \sin^2 \theta / \cos \theta + \cos \theta / 1$$

Taking LCM

$$= (\sin^2 \theta + \cos^2 \theta) / \cos \theta$$

$$\text{We know that } \sin^2 \theta + \cos^2 \theta = 1$$

$$= 1 / \cos \theta$$

$$= \text{RHS}$$

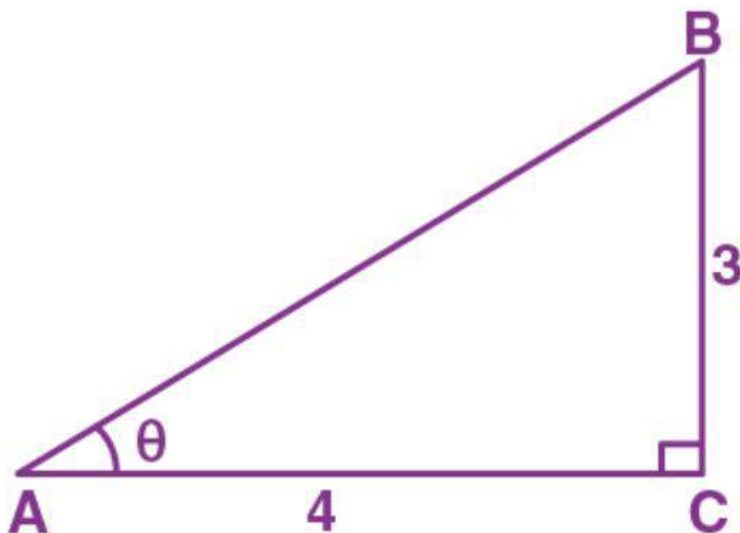
Therefore, LHS = RHS.

27. If in ΔABC , $\angle C = 90^\circ$ and $\tan A = \frac{3}{4}$, prove that $\sin A \cos B + \cos A \sin B = 1$.

Solution:

It is given that

$$\tan A = BC/AC = \frac{3}{4}$$



Using Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

Substituting the values

$$= 4^2 + 3^2$$

$$= 16 + 9$$

$$= 25$$

$$= 5^2$$

So we get $AB = 5$

Here

$$\sin A = BC/AC = 3/5$$

$$\cos A = AC/AB = 4/5$$

$$\cos B = BC/AB = 3/5$$

$$\sin B = AC/AB = 4/5$$

$$\text{LHS} = \sin A \cos B + \cos A \sin B$$

Substituting the values

$$= 3/5 \times 3/5 + 4/5 \times 4/5$$

By further calculation

$$= 9/25 + 16/25$$

$$= (9 + 16)/25$$

$$= 25/25$$

$$= 1$$

$$= \text{RHS}$$

Therefore, $\text{LHS} = \text{RHS}$.

28. (a) In figure (1) given below, ΔABC is right-angled at B and ΔBRS is right-angled at R. If $AB = 18$ cm, $BC = 7.5$ cm, $RS = 5$ cm, $\angle BSR = x^\circ$ and $\angle SAB = y^\circ$, then find:

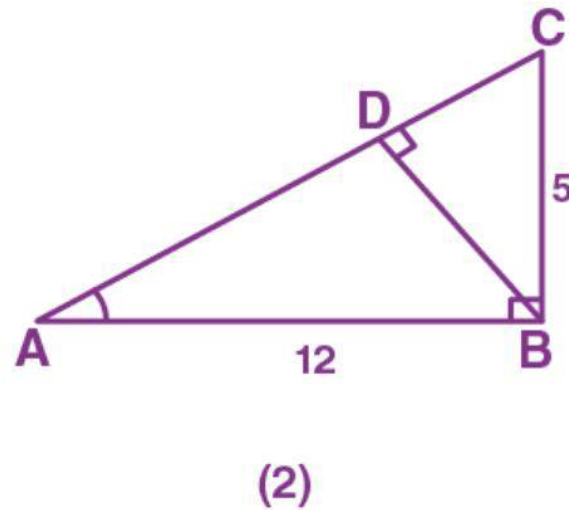
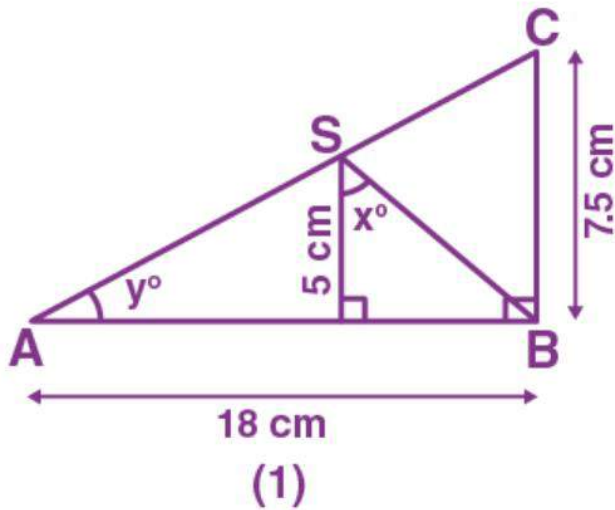
(i) $\tan x^\circ$

(ii) $\sin y^\circ$.

(b) In the figure (2) given below, ΔABC is right angled at B and BD is perpendicular to AC. Find

(i) $\cos \angle CBD$

(ii) $\cot \angle ABD$.



Solution:

(a) $\triangle ABC$ is right-angled at B, $\triangle BSC$ is right-angled at S and $\triangle BRS$ is right-angled at R
It is given that

$AB = 18$ cm, $BC = 7.5$ cm, $RS = 5$ cm, $\angle BSR = x^\circ$ and $\angle SAB = y^\circ$

By Geometry $\triangle ARS$ and $\triangle ABC$ are similar

$$AR/AB = RS/BC$$

Substituting the values

$$AR/18 = 5/7.5$$

By further calculation

$$AR = (5 \times 18)/7.5 = (1 \times 18)/1.5$$

Multiply both numerator and denominator by 10

$$AR = (18 \times 10)/15$$

$$AR = (10 \times 6)/5$$

$$AR = (2 \times 6)/1 = 12$$

So we get

$$RB = AB - AR$$

$$RB = 18 - 12 = 6$$

In right angled $\triangle ABC$

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 18^2 + 7.5^2$$

By further calculation

$$AC^2 = 324 + 56.25 = 380.25$$

$$AC = \sqrt{380.25} = 19.5 \text{ cm}$$

(i) In right angled $\triangle BSR$

$\tan x^\circ = \text{perpendicular/base}$

$$\tan x^\circ = RB/RS = 6/5$$

(ii) In right angled $\triangle ASR$

$\sin y^\circ = \text{perpendicular/hypotenuse}$

Using Pythagoras theorem

$$AS^2 = 12^2 + 5^2$$

By further calculation

$$AS^2 = 144 + 25 = 169$$

$$AS = \sqrt{169} = 13 \text{ cm}$$

So we get

$$\sin y^\circ = RS/AS = 5/13$$

(b) We know that

$\triangle ABC$ is right angled at B and BD is perpendicular to AC

In right angled $\triangle ABC$

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 12^2 + 5^2$$

By further calculation

$$AC^2 = 144 + 25 = 169$$

So we get

$$AC^2 = (13)^2$$

$$AC = 13$$

By Geometry $\angle CBD = \angle A$ and $\angle ABD = \angle C$

(i) $\cos \angle CBD = \cos \angle A = \text{base/hypotenuse}$

In right angled $\triangle ABC$

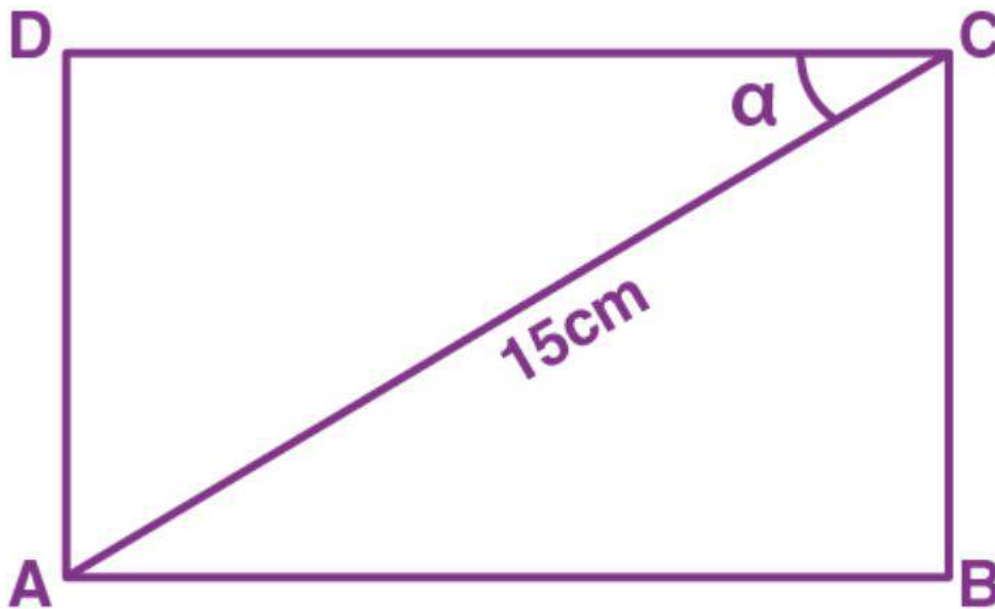
$$\cos \angle CBD = \cos \angle A = AB/AC = 12/13$$

(ii) $\cos \angle ABD = \cos \angle C = \text{base/perpendicular}$

In right angled ΔABC

$$\cos \angle ABD = \cos \angle C = BC/AB = 5/12$$

29. In the adjoining figure, ABCD is a rectangle. Its diagonal AC = 15 cm and $\angle ACD = \alpha$. If $\cot \alpha = 3/2$, find the perimeter and the area of the rectangle.



Solution:

In right ΔADC

$$\cot \alpha = CD/AD = 3/2$$

Take $CD = 3x$ then $AD = 2x$

Using Pythagoras theorem

$$AC^2 = CD^2 + AD^2$$

Substituting the values

$$(15)^2 = (3x)^2 + (2x)^2$$

By further calculation

$$13x^2 = 225$$

$$x^2 = 225/13$$

So we get

$$x = \sqrt{225/13} = 15/\sqrt{13}$$

Length of rectangle (l) = $3x = (3 \times 15)/\sqrt{13} = 45/\sqrt{13}$ cm

Breadth of rectangle (b) = $2x = (2 \times 15)/\sqrt{13} = 30/\sqrt{13}$ cm

(i) Perimeter of rectangle = $2(l + b)$

Substituting the values of l and b

$$= 2(45/\sqrt{13} + 30/\sqrt{13})$$

So we get

$$= 2 \times 75/\sqrt{13}$$

$$= 150/\sqrt{13} \text{ cm}$$

(ii) Area of rectangle = $l \times b$

Substituting the values of l and b

$$= 45/\sqrt{13} \times 30/\sqrt{13}$$

So we get

$$= 1350/13$$

$$= 103 \frac{11}{13} \text{ cm}^2$$

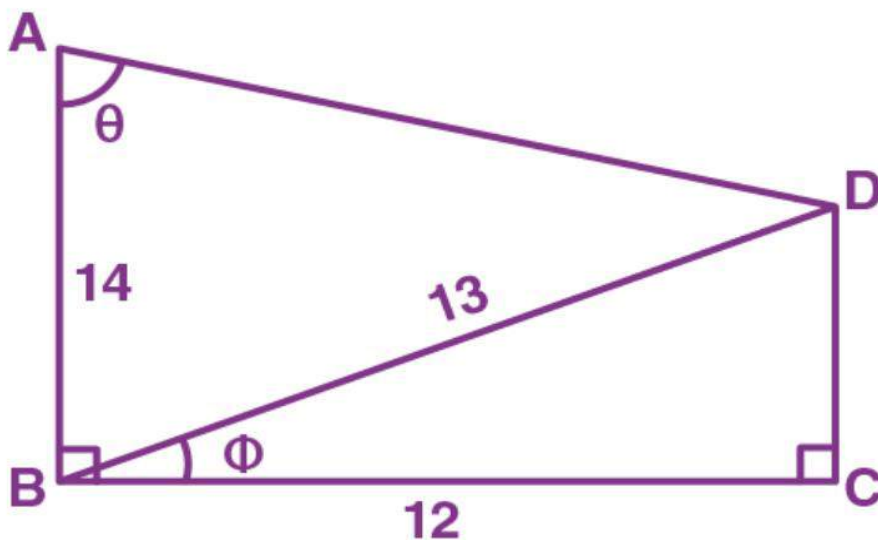
30. Using the measurements given in the figure alongside,

(a) Find the values of:

(i) $\sin \varphi$

(ii) $\tan \theta$.

(b) Write an expression for AD in terms of θ .



Solution:

From the figure

$$BC = 12, BD = 13$$

In right angled $\triangle BCD$

Using Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

It can be written as

$$CD^2 = BD^2 - BC^2$$

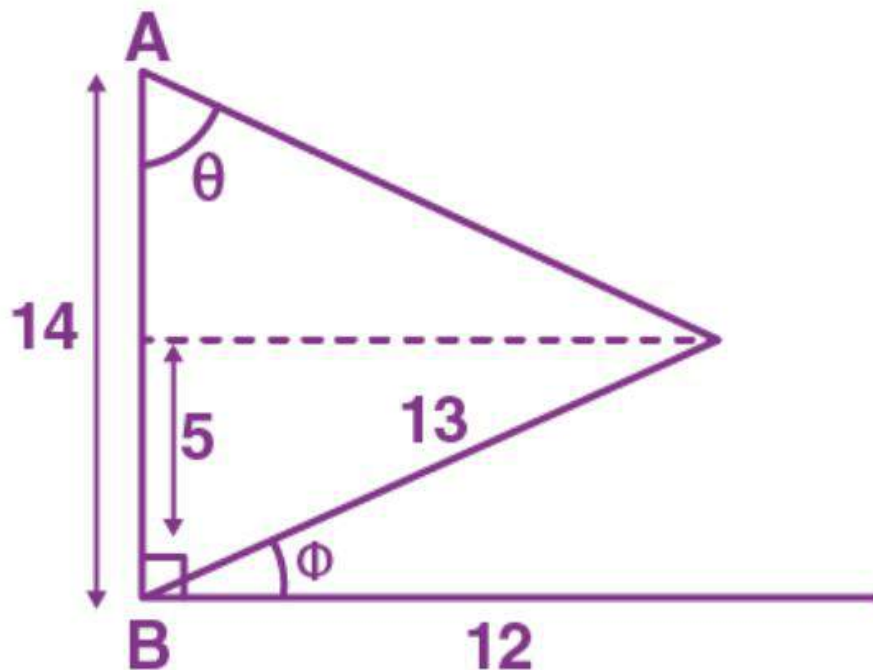
Substituting the values

$$CD^2 = (13)^2 - (12)^2$$

$$CD^2 = 169 - 144 = 25$$

So we get

$$CD = \sqrt{25} = 5$$



Construct BE perpendicular to AC

$$CD = BE = 5 \text{ and } EA = AE = 14 - 5 = 9$$

(a) (i) $\sin \phi = \text{perpendicular/hypotenuse}$

In right angled $\triangle BCD$

$$\sin \phi = CD/BD = 5/13$$

(ii) $\tan \theta = \text{perpendicular/hypotenuse}$

In right angled ΔAED

$$\tan \theta = ED/AE = BC/AE = 12/9 = 4/3 \text{ (Since } ED = BC\text{)}$$

(b) In right angled ΔAED

$\sin \theta = \text{perpendicular/hypotenuse}$

$\cos \theta = \text{base/perpendicular}$

We can write it as

$$\sin \theta = ED/AD \text{ or } \cos \theta = AE/AD$$

$$AD = ED/\sin \theta \text{ or } AD = AE/\cos \theta$$

Substituting the values

$$AD = 12/\sin \theta \text{ or } AD = 9/\cos \theta$$

$$\text{Therefore, } AD = 12/\sin \theta \text{ or } AD = 9/\cos \theta.$$

31. Prove the following:

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

(ii) $\cot^2 A - 1/\sin^2 A + 1 = 0$

(iii) $1/(1 + \tan^2 A) + 1/(1 + \cot^2 A) = 1$

Solution:

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

$$\text{LHS} = (\sin A + \cos A)^2 + (\sin A - \cos A)^2$$

Using the formula

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and } (a - b)^2 = a^2 + b^2 - 2ab$$

$$= [(\sin A)^2 + (\cos A)^2 + 2 \sin A \cos A] + [(\sin A)^2 + (\cos A)^2 - 2 \sin A \cos A]$$

By further calculation

$$= \sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A$$

$$= \sin^2 A + \cos^2 A + \sin^2 A + \cos^2 A$$

$$= 2 \sin^2 A + 2 \cos^2 A$$

$$\text{We know that } \sin^2 A + \cos^2 A = 1$$

$$= 2 (\sin^2 A + \cos^2 A)$$

$$= 2 (1)$$

$$= 2$$

$$= \text{RHS}$$

Therefore, LHS = RHS.

$$(ii) \cot^2 A - 1/\sin^2 A + 1 = 0$$

$$\text{LHS} = \cot^2 A - 1/\sin^2 A + 1$$

We know that

$$1/\sin A = \operatorname{cosec} A$$

$$= \cot^2 A - \operatorname{cosec}^2 A + 1$$

$$= (1 + \cot^2 A) - \operatorname{cosec}^2 A$$

$$\text{We know that } 1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$= \operatorname{cosec}^2 A - \operatorname{cosec}^2 A$$

$$= 0$$

$$= \text{RHS}$$

Therefore, LHS = RHS.

$$(iii) 1/(1 + \tan^2 A) + 1/(1 + \cot^2 A) = 1$$

$$\text{LHS} = 1/(1 + \tan^2 A) + 1/(1 + \cot^2 A)$$

We know that

$$\sec^2 A - \tan^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

So we get

$$= 1/\sec^2 A + 1/\operatorname{cosec}^2 A$$

$$\text{Here } 1/\sec A = \cos A \text{ and } 1/\operatorname{cosec} A = \sin A$$

$$= \cos^2 A + \sin^2 A$$

$$= 1$$

$$= \text{RHS}$$

Therefore, LHS = RHS.

32. Simplify

$$\sqrt{\frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}}$$

Solution:

$$\sqrt{\frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}} = \sqrt{\frac{\sin^2 \theta + \cos^2 \theta - \sin^2 \theta}{\sin^2 \theta + \cos^2 \theta - \cos^2 \theta}}$$

$$\text{We know that } 1 = \sin^2 \theta + \cos^2 \theta$$

$$\begin{aligned} &= \sqrt{\cos^2 \theta / \sin^2 \theta} \\ &= \cos \theta / \sin \theta \\ \text{Here } \cos \theta / \sin \theta &= \cot \theta \\ &= \cot \theta \end{aligned}$$

Therefore, $\sqrt{\frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}} = \cot \theta.$

33. If $\sin \theta + \operatorname{cosec} \theta = 2$, find the value of $\sin^2 \theta + \operatorname{cosec}^2 \theta$.

Solution:

It is given that

$$\sin \theta + \operatorname{cosec} \theta = 2$$

$$\sin \theta + 1/\sin \theta = 2$$

By further calculation

$$\sin^2 \theta + 1 = 2 \sin \theta$$

$$\sin^2 \theta - 2 \sin \theta + 1 = 0$$

So we get

$$(\sin \theta - 1)^2 = 0$$

$$\sin \theta - 1 = 0$$

$$\sin \theta = 1$$

Here

$$\sin^2 \theta + \operatorname{cosec}^2 \theta = \sin^2 \theta + 1/\sin^2 \theta$$

Substituting the values

$$= 1^2 + 1/1^2$$

$$= 1 + 1/1$$

$$= 1 + 1$$

$$= 2$$

34. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, prove that $x^2 + y^2 = a^2 + b^2$.

Solution:

It is given that

$$x = a \cos \theta + b \sin \theta \dots (1)$$

$$y = a \sin \theta - b \cos \theta \dots (2)$$

By squaring and adding both the equations

$$x^2 + y^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$$

Using the formula

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and } (a - b)^2 = a^2 + b^2 - 2ab$$

$$= [(a \cos \theta)^2 + (b \sin \theta)^2 + 2(a \cos \theta)(b \sin \theta)] + [(a \sin \theta)^2 + (b \cos \theta)^2 - 2(a \sin \theta)(b \cos \theta)]$$

By further calculation

$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta$$

$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta$$

So we get

$$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$\text{Here } \sin^2 \theta + \cos^2 \theta = 1$$

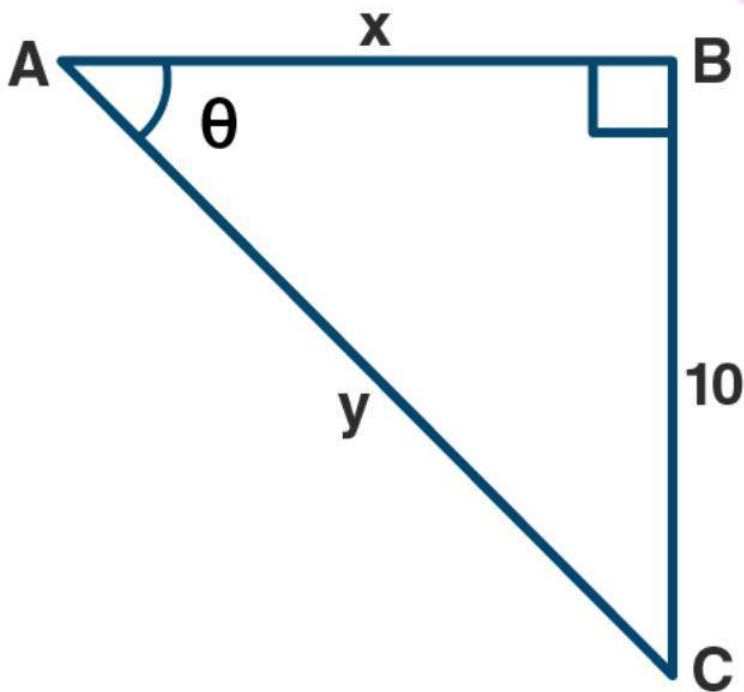
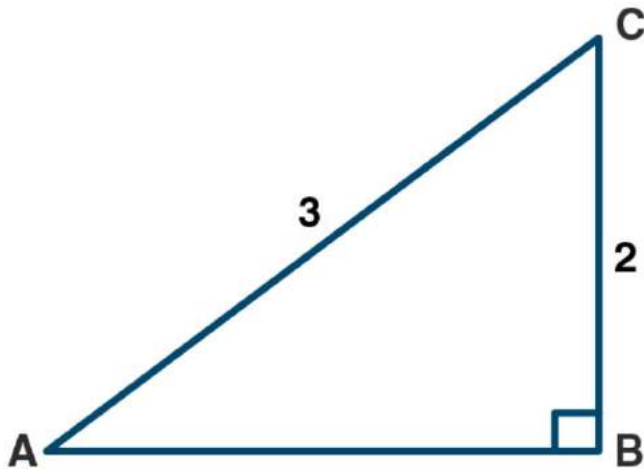
$$= a^2 (1) + b^2 (1)$$

$$= a^2 + b^2$$

Therefore, $x^2 + y^2 = a^2 + b^2$.

CHAPTER TEST

1. (a) From the figure (i) given below, calculate all the six t-ratios for both acute.....
 (b) From the figure (ii) given below, find the values of x and y in terms of t-ratios



Solution:

- (a) From right angled triangle ABC,
 By Pythagoras theorem, we get
 $AC^2 = AB^2 + BC^2$
 $AB^2 = AC^2 - BC^2$

$$AB^2 = (3)^2 - (2)^2$$

$$AB^2 = 9 - 4$$

$$AB^2 = 5$$

$$AB = \sqrt{5}$$

(i) $\sin A = \text{perpendicular} / \text{hypotenuse}$

$$= BC/AC$$

$$= 2/3$$

(ii) $\cos A = \text{base} / \text{hypotenuse}$

$$= AB/AC$$

$$= \sqrt{5}/3$$

(iii) $\tan A = \text{perpendicular} / \text{base}$

$$= BC/AB$$

$$= 2/\sqrt{5}$$

(iv) $\cot A = \text{base} / \text{perpendicular}$

$$= AB/BC$$

$$= \sqrt{5}/2$$

(v) $\sec A = \text{hypotenuse} / \text{base}$

$$= AC/AB$$

$$= 3/\sqrt{5}$$

(vi) $\text{cosec } A = \text{hypotenuse} / \text{perpendicular}$

$$= AC/BC$$

$$= 3/2$$

(b) From right angled triangle ABC,

$$\angle BAC = \theta$$

Then we know that,

$$\cot \theta = \text{base} / \text{perpendicular}$$

$$= AB/BC$$

$$= x/10$$

$$x = 10 \cot \theta$$

also, $\operatorname{cosec} \theta = \text{hypotenuse} / \text{perpendicular}$
 $= AC / BC$
 $= y / 10$
 $y = 10 \operatorname{cosec} \theta$

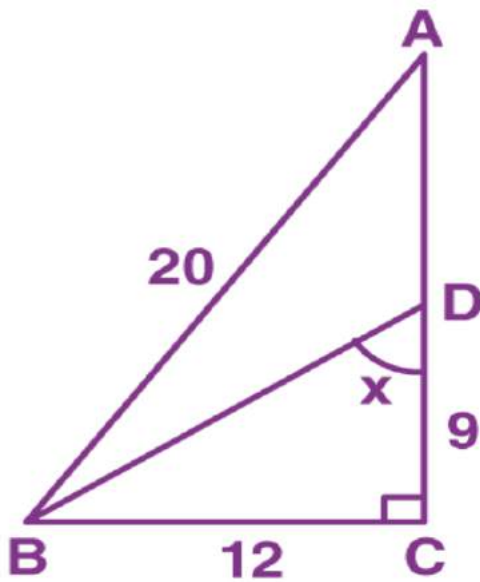
Therefore, $x = 10 \cot \theta$ and $y = 10 \operatorname{cosec} \theta$.

2. (a) From the figure (1) given below, find the values of:

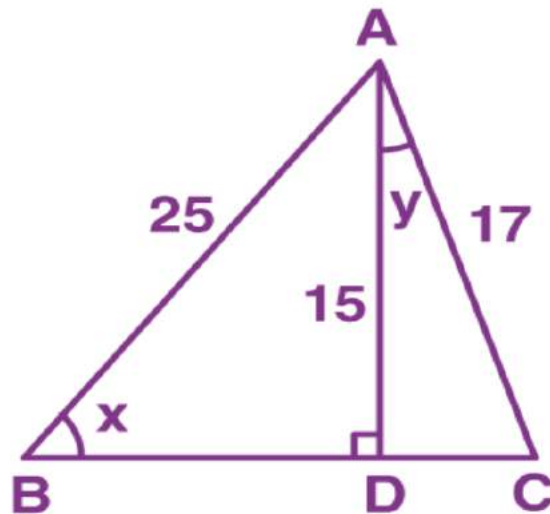
- (i) $\sin \angle ABC$
- (ii) $\tan x - \cos x + 3 \sin x$.

(b) From the figure (2) given below, find the values of:

- (i) $5 \sin x$
- (ii) $7 \tan x$
- (iii) $5 \cos x - 17 \sin y - \tan x$.



(i)



(ii)

Solution:

(a) From the figure
 $BC = 12$, $CD = 9$ and $BC = 20$

In right angled $\triangle ABC$,
 Using Pythagoras theorem
 $AB^2 = AC^2 + BC^2$

It can be written as

$$AC^2 = AB^2 - BC^2$$

Substituting the values

$$AC^2 = (20)^2 - (12)^2$$

By further calculation

$$AC^2 = 400 - 144 = 256$$

So we get

$$AC^2 = (16)^2$$

$$AC = 16$$

In right angled $\triangle BCD$

Using Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

Substituting the values

$$BD^2 = 12^2 + 9^2$$

By further calculation

$$BD^2 = 144 + 81 = 225$$

So we get

$$BD^2 = (15)^2$$

$$BD = 15$$

(i) In right angled $\triangle ABC$

$\sin \angle ABC = \text{perpendicular/hypotenuse}$

So we get

$$\sin \angle ABC = AC/AB = 16/20 = 4/5$$

(ii) In right angled $\triangle BCD$

$\tan x = \text{perpendicular/base}$

So we get

$$\tan x = BC/CD = 12/9 = 4/3$$

In right angled $\triangle BCD$

$\cos x = \text{base/hypotenuse}$

So we get

$$\cos x = CD/BD = 9/15 = 3/5$$

In right angled $\triangle BCD$

$\sin x = \text{perpendicular/hypotenuse}$

So we get

$$\sin x = BC/BD = 12/15 = 4/5$$

$$\tan x - \cos x + 3 \sin x = 4/3 - 3/5 + 3 \times 4/5$$

By further calculation

$$= 4/3 - 3/5 + 12/5$$

Taking LCM

$$= (4 \times 5 - 3 \times 3 + 12 \times 3) / 15$$

So we get

$$= (20 - 9 + 36) / 15$$

$$= (56 - 9) / 15$$

$$= 27/15$$

$$= 3 \frac{2}{15}$$

Therefore, $\tan x - \cos x + 3 \sin x = 3 \frac{2}{15}$.

(b) In the figure

$$AC = 17, AB = 25, AD = 15$$

In right angled $\triangle ACD$

Using Pythagoras theorem

$$AC^2 = AD^2 + CD^2$$

Substituting the values

$$(17)^2 = (15)^2 + (CD)^2$$

By further calculation

$$CD^2 = (17)^2 - (15)^2$$

$$CD^2 = 289 - 225 = 64$$

So we get

$$CD^2 = 8^2$$

$$CD = 8$$

In right angled $\triangle ABD$

Using Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

Substituting the values

$$(25)^2 = (15)^2 + BD^2$$

By further calculation

$$BD^2 = (25)^2 - (15)^2$$

$$BD^2 = 625 - 225 = 400$$

So we get

$$BD^2 = (20)^2$$

$$BD = 20$$

(i) In right angled $\triangle ABD$

$$5 \sin x = 5 \text{ (perpendicular/hypotenuse)}$$

So we get

$$= 5 (AD/AB)$$

$$= 5 \times 15/25$$

$$= 15/5$$

$$= 3$$

(ii) In right angled $\triangle ABD$

$$7 \tan x = 7 \text{ (perpendicular/base)}$$

So we get

$$= 7 (AD/AB)$$

$$= 7 \times 15/20$$

$$= 7 \times \frac{3}{4}$$

$$= 21/4$$

$$= 5 \frac{1}{4}$$

(iii) In right angled $\triangle ABD$

$$\cos x = \text{base/hypotenuse}$$

So we get

$$\cos x = BD/AB = 20/25 = 4/5$$

In right angled $\triangle ACD$

$$\sin y = \text{perpendicular/hypotenuse}$$

So we get

$$\sin y = CD/AC = 8/17$$

In right angled $\triangle ABD$

$$\tan x = \text{perpendicular/base}$$

So we get

$$\tan x = AD/BD = 15/20 = \frac{3}{4}$$

$$5 \cos x - 17 \sin y - \tan x = 5 \times \frac{4}{5} - 17 \times \frac{8}{17} - \frac{3}{4}$$

It can be written as

$$= \frac{4}{1} - \frac{8}{1} - \frac{3}{4}$$

Taking LCM

$$= \frac{(16 - 32 - 3)}{4}$$

$$= \frac{(16 - 35)}{4}$$

So we get

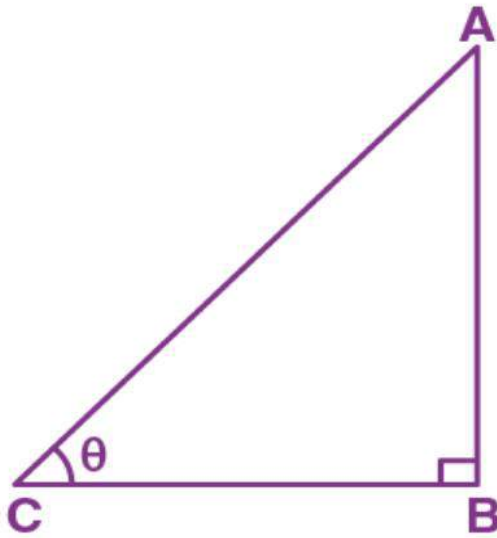
$$= -\frac{19}{4}$$

$$= -4\frac{3}{4}$$

Therefore, $5 \cos x - 17 \sin y - \tan x = -4\frac{3}{4}$.

3. If $q \cos \theta = p$, find $\tan \theta - \cot \theta$ in terms of p and q .

Solution:



Consider ABC as a triangle right angled at B and $\angle ACB = \theta$

It is given that

$$q \cos \theta = p$$

$$\cos \theta = \frac{BC}{AC} = \frac{p}{q}$$

Take $BC = px$ then $AC = qx$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

It can be written as

$$AB^2 = AC^2 - BC^2$$

Substituting the values

$$AB^2 = (qx)^2 - (px)^2$$

$$AB^2 = q^2x^2 - p^2x^2$$

Taking out the common terms

$$AB^2 = (q^2 - p^2)x^2$$

So we get

$$AB = \sqrt{(q^2 - p^2)} x$$

$$AB = (\sqrt{q^2 - p^2})x$$

In right angled ΔABC

$$\tan \theta = \text{perpendicular/base}$$

So we get

$$\tan \theta = AB/BC = [(\sqrt{q^2 - p^2})x]/px$$

$$\tan \theta = (\sqrt{q^2 - p^2})/p$$

In right angled ΔABC

$$\cot \theta = \text{base/perpendicular}$$

So we get

$$\cot \theta = BC/AB = px/[(\sqrt{q^2 - p^2})x]$$

$$\cot [(\sqrt{q^2 - p^2})x] = p/(\sqrt{q^2 - p^2})$$

$$\tan \theta - \cot \theta = \frac{\sqrt{q^2 - p^2}}{p} - \frac{p}{\sqrt{q^2 - p^2}}$$

Taking LCM

$$= \frac{\sqrt{q^2 - p^2}(q^2 - p^2) - p \times p}{p(\sqrt{q^2 - p^2})}$$

So we get

$$= \frac{q^2 - p^2 - p^2}{p(\sqrt{q^2 - p^2})}$$

$$= \frac{q^2 - 2p^2}{p\sqrt{q^2 - p^2}}$$

$$\text{Therefore, } \tan \theta - \cot \theta = \frac{q^2 - 2p^2}{p\sqrt{q^2 - p^2}}$$

4. Given $4 \sin \theta = 3 \cos \theta$, find the values of:

(i) $\sin \theta$

(ii) $\cos \theta$

(iii) $\cot^2 \theta - \operatorname{cosec}^2 \theta$.

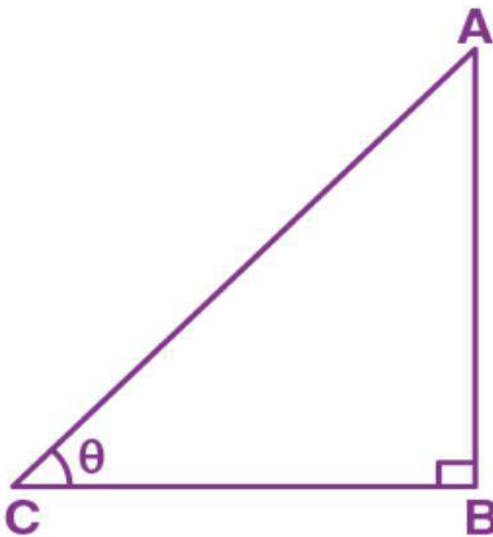
Solution:

It is given that

$$4 \sin \theta = 3 \cos \theta$$

$$\sin \theta / \cos \theta = \frac{3}{4}$$

$$\tan \theta = \frac{3}{4}$$



Consider $\triangle ABC$ right angled at B and $\angle ACB = \theta$

$\tan \theta = \text{perpendicular/base}$

Substituting the values

$$\frac{3}{4} = \frac{AB}{BC}$$

$$\frac{AB}{BC} = \frac{3}{4}$$

Take $AB = 3x$ then $BC = 4x$

In right angled $\triangle ABC$

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (3x)^2 + (4x)^2$$

By further calculation

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

So we get

$$AC^2 = (5x)^2$$

$$AC = 5x$$

(i) In right angled ΔABC

$\sin \theta = \text{perpendicular/hypotenuse}$

So we get

$$\sin \theta = AB/AC = 3x/5x = 3/5$$

(ii) In right angled ΔABC

$\cos \theta = \text{base/hypotenuse}$

So we get

$$\cos \theta = BC/AC = 4x/5x = 4/5$$

(iii) In right angled ΔABC

$\cot \theta = \text{base/perpendicular}$

So we get

$$\cot \theta = BC/AB = 4x/3x = 4/3$$

In right angled ΔABC

$\text{cosec } \theta = \text{hypotenuse/perpendicular}$

So we get

$$\text{cosec } \theta = AC/AB = 5x/3x = 5/3$$

Here

$$\cot^2 \theta - \text{cosec}^2 \theta = (4/3)^2 - (5/3)^2$$

By further calculation

$$= 16/9 - 25/9$$

$$= (16 - 25)/9$$

$$= -9/9$$

$$= -1$$

Therefore, $\cot^2 \theta - \text{cosec}^2 \theta = -1$.

5. If $2 \cos \theta = \sqrt{3}$, prove that $3 \sin \theta - 4 \sin^3 \theta = 1$.

Solution:

It is given that

$$2 \cos \theta = \sqrt{3}$$
$$\cos \theta = \sqrt{3}/2$$

We know that

$$\sin^2 \theta = 1 - \cos^2 \theta$$

Substituting the values

$$= 1 - (\sqrt{3}/2)^2$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

$$\sin \theta = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Consider

$$\text{LHS} = 3 \sin \theta - 4 \sin^3 \theta$$

It can be written as

$$= \sin \theta (3 - 4 \sin^2 \theta)$$

Substituting the values

$$= \frac{1}{2} (3 - 4 \times \frac{1}{4})$$

$$= \frac{1}{2} (3 - 1)$$

$$= \frac{1}{2} \times 2$$

$$= 1$$

$$= \text{RHS}$$

Therefore, proved.

6. If $(\sec \theta - \tan \theta) / (\sec \theta + \tan \theta) = \frac{1}{4}$, find $\sin \theta$.

Solution:

We know that

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

Taking LCM

$$= \frac{\frac{1 - \sin\theta}{\cos\theta}}{\frac{1 + \sin\theta}{\cos\theta}}$$

So we get

$$= \frac{1 - \sin\theta}{\cos\theta} \times \frac{\cos\theta}{1 + \sin\theta}$$
$$= \frac{1 - \sin\theta}{1 + \sin\theta}$$

Here

$$\frac{1 - \sin\theta}{1 + \sin\theta} = \frac{1}{4}$$

By cross multiplication

$$4 - 4 \sin\theta = 1 + \sin\theta$$

We get

$$4 - 1 = \sin\theta + 4 \sin\theta$$

$$3 = 5 \sin\theta$$

$$\sin\theta = 3/5$$

7. If $\sin\theta + \operatorname{cosec}\theta = 3 \frac{1}{3}$, find the value of $\sin^2\theta + \operatorname{cosec}^2\theta$.

Solution:

It is given that

$$\sin\theta + \operatorname{cosec}\theta = 3 \frac{1}{3} = 10/3$$

By squaring on both sides

$$(\sin\theta + \operatorname{cosec}\theta)^2 = (10/3)^2$$

Expanding using formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$\sin^2\theta + \operatorname{cosec}^2\theta + 2 \sin\theta \operatorname{cosec}\theta = 100/9$$

We know that $\sin\theta = 1/\operatorname{cosec}\theta$

$$\sin^2\theta + \operatorname{cosec}^2\theta + 2 \sin\theta \times 1/\sin\theta = 100/9$$

By further calculation

$$\sin^2\theta + \operatorname{cosec}^2\theta + 2 = 100/9$$

$$\sin^2 \theta + \operatorname{cosec}^2 \theta = 100/9 - 2$$

Taking LCM

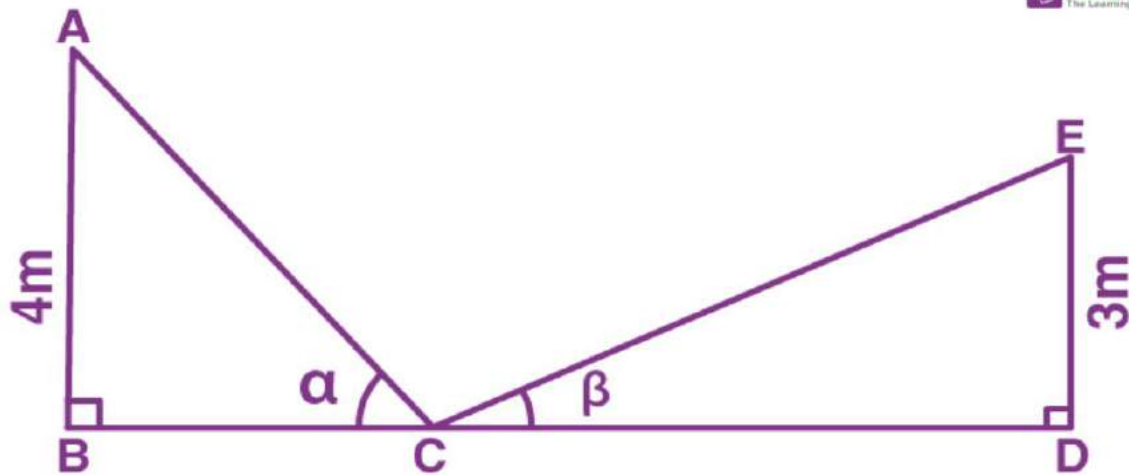
$$\sin^2 \theta + \operatorname{cosec}^2 \theta = (100 - 18)/9 = 82/9$$

So we get

$$\sin^2 \theta + \operatorname{cosec}^2 \theta = 9 \frac{1}{9}$$

8. In the adjoining figure, AB = 4 m and ED = 3 m.

If $\sin \alpha = 3/5$ and $\cos \beta = 12/13$, find the length of BD.



Solution:

It is given that

$$\sin \alpha = AB/AC = 3/5$$

$$AB = 3 \text{ and } AC = 5$$

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$5^2 = 3^2 + BC^2$$

By further calculation

$$25 = 9 + BC^2$$

$$BC^2 = 25 - 9 = 16$$

So we get

$$BC^2 = 4^2$$

$$BC = 4$$

We know that

$$\tan \alpha = AB/BC = 4/5$$
$$\cos \beta = CD/CE = 12/13$$
$$CD = 12 \text{ and } CE = 13$$

Using Pythagoras theorem

$$CE^2 = CD^2 + ED^2$$

Substituting the values

$$13^2 = 12^2 + ED^2$$

By further calculation

$$ED^2 = 13^2 - 12^2$$

$$ED^2 = 169 - 144 = 25$$

So we get

$$ED^2 = (5)^2$$

$$ED = 5$$

$$\tan \beta = ED/CD = 5/12$$

From the figure

$$\tan \alpha = AB/BC = 4/BC$$

So we get

$$\frac{4}{3} = 4/BC$$

$$BC = (4 \times 4)/3 = 16/3 \text{ m}$$

$$\tan \beta = ED/CD = 3/CD$$

$$5/12 = 3/CD$$

So we get

$$CD = (12 \times 3)/5 = 36/5 \text{ m}$$

Here

$$BD = BC + CD$$

Substituting the values

$$= 16/3 + 36/5$$

Taking LCM

$$= (80 + 108)/15$$

$$= 188/15 \text{ m}$$

$$= 12 \frac{8}{15} \text{ m}$$