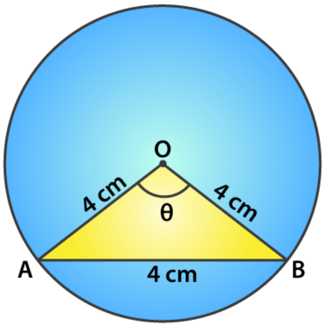
Exercise 15.3 Page No: 15.32

1. AB is a chord of a circle with centre O and radius 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment.
Solution:



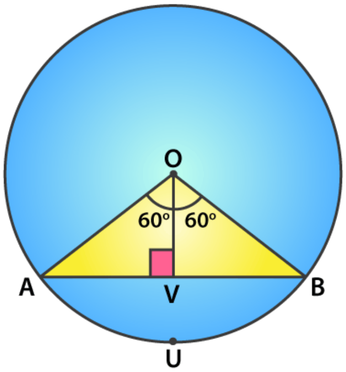
Given,
Radius of the circle with centre 'O', r = 4 cm = OA = OB
Length of the chord AB = 4 cm
So, OAB is an equilateral triangle and angle AOB = 60°
Thus, the angle subtended at centre θ = 60°
Area of the minor segment = (Area of sector) - (Area of triangle AOB)

$$= \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{60}{360} \times \pi 4^2 - \frac{\sqrt{3}}{4}(4)^2$$

= (8π/3 - 4√3) = 8.37 - 6.92 = 1.45 cm^2
Therefore, the required area of the segment is (8π/3 - 4√3) cm^2

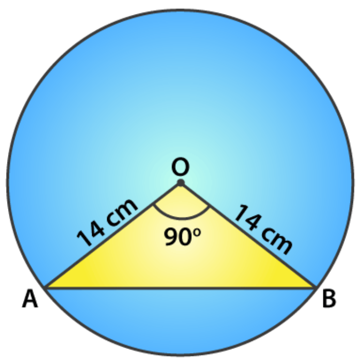
2. A chord PQ of length 12 cm subtends an angle 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ.
Solution:



Given, ∠POQ = 120° and PQ = 12 cm
Draw OV ⊥ PQ,

PV = PQ × (0.5) = 12 × 0.5 = 6 cm
Since, ∠POV = 120°
∠POV = ∠QOV = 60°
In triangle OPQ, we have
sin θ = PV/ OA
sin 60° = 6/ OA
√3/2 = 6/ OA
OA = 12/ √3 = 4√3 = r
Now, using the above we shall find the area of the minor segment
We know that,
Area of the segment = area of sector OPUQO – area of △OPQ
= $\theta/360 \times \pi r^2 - ½ \times PQ \times OV$
= $120/360 \times \pi(4\sqrt{3})^2 - ½ \times 12 \times 2\sqrt{3}$
= $16\pi - 12\sqrt{3} = 4(4\pi - 3\sqrt{3})$
Therefore, the area of the minor segment = $4(4\pi - 3\sqrt{3})$ cm^2

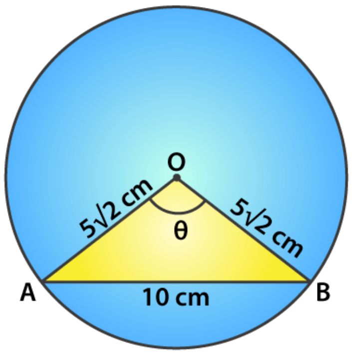
3. A chord of a circle of radius 14 cm makes a right angle at the centre. Find the areas of the minor and major segments of the circle.
Solution:



Given,
Radius (r) = 14 cm
Angle subtended by the chord with the centre of the circle, θ = 90°
Area of minor segment = $\theta/360 \times \pi r^2 - ½ \times r^2 \sin\theta$
= $90/360 \times \pi(14)^2 - ½ \times 14^2 \sin(90)$
= $¼ \times (22/7)(14)^2 - 7 \times 14$
= 56 cm^2
Area of circle = πr^2
= $22/7 \times (14)^2 = 616$ cm^2
Thus,
The area of the major segment = Area of circle – Area of the minor segment
= 616 – 56
= 560 cm^2

4. A chord 10 cm long is drawn in a circle whose radius is 5√2 cm. Find the area of both the segments.

Solution:



Given,
Radius of the circle, r = $5\sqrt{2}$ cm = OA = OB
Length of the chord AB = 10 cm
In triangle OAB,

$= OA^2 + OB^2$

$= (5\sqrt{2})^2 + (5\sqrt{2})^2$

$= 50 + 50$

$= 100$

$AB^2 = 100$

We see that the Pythagoras theorem is satisfied.
So, OAB is a right angle triangle.
Angle subtended by the chord with the centre of the circle, θ = 90°
Area of minor segment = area of sector - area of triangle
$= \theta/360 \times \pi r^2 - ½ \times r^2 \sin\theta$
$= 90/360 \times (3.14)\ 5\sqrt{2}^2 - ½ \times (5\sqrt{2})^2 \sin 90$
= [¼ x 3.14 x 25 x 2] – [½ x 25 x 2 x 1]
= 25(1.57 - 1)
= 14.25 cm^2

Area of circle = $\pi r^2 = 3.14 \times (5\sqrt{2})^2 = 3.14 \times 50 = 157\ cm^2$

Thus, Area of major segment = Area of circle – Area of minor segment
= 157 – 14.25
= 142.75 cm^2

5. A chord AB of circle of radius 14 cm makes an angle of 60° at the centre of a circle. Find the area of the minor segment of the circle.
Solution:

Given,
Radius of the circle (r) = 14 cm = OA = OB
Angle subtended by the chord with the centre of the circle, $\theta = 60°$
In triangle AOB, angle A = angle B [angle opposite to equal sides OA and OB = x]
By angle sum property,
$\angle A + \angle B + \angle O = 180$
$x + x + 60° = 180°$
$2x = 120°, x = 60°$
All angles are 60° so the triangle OAB is an equilateral with OA = OB = AB
Area of the minor segment = area of sector - area of triangle OAB
$= \theta/360 \times \pi r^2 - \sqrt{3}/4\ (\text{side})^2$
$= 60/360 \times (22/7)\ 14^2 - \sqrt{3}/4\ (14)^2$
$= [1/6 \times 22 \times 2 \times 14] - \sqrt{3} \times (7)^2$
$= 102.7 - 84.9 = 17.8$
Therefore, the area of the minor segment = 17.80 cm^2