

Exercise 3.11

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1. If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.

Solution:

Let's assume the length and breadth of the rectangle be x units and y units respectively.
Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

Case 1:

Length is increased by 2 units \Rightarrow now, the new length is $x+2$ units

Breadth is reduced by 2 units \Rightarrow now, the new breadth is $y-2$ units

And it's given that the area is reduced by 28 square units i.e. = $xy - 28$

So, the equation becomes

$$\begin{aligned} (x+2)(y-2) &= xy - 28 \\ \Rightarrow xy - 2x + 2y - 4 &= xy - 28 \\ \Rightarrow -2x + 2y - 4 + 28 &= 0 \\ \Rightarrow -2x + 2y + 24 &= 0 \\ \Rightarrow 2x - 2y - 24 &= 0 \dots\dots\dots (i) \end{aligned}$$

Case 2:

Length is reduced by 1 unit \Rightarrow now, the new length is $x-1$ units

Breadth is increased by 2 units \Rightarrow now, the new breadth is $y+2$ units

And, it's given that the area is increased by 33 square units i.e. = $xy + 33$

So, the equation becomes

$$\begin{aligned} (x-1)(y+2) &= xy + 33 \\ \Rightarrow xy + 2x - y - 2 &= xy + 33 \\ \Rightarrow 2x - y - 2 - 33 &= 0 \\ \Rightarrow 2x - y - 35 &= 0 \dots\dots\dots (ii) \end{aligned}$$

Solving (i) and (ii),

By using cross multiplication, we get

$$\begin{aligned} \frac{x}{(-2*-35)-(-1*-24)} &= \frac{y}{(2*-35)-(2*-24)} = \frac{1}{(2*-1)-(2*-2)} \\ \frac{x}{70-24} &= \frac{-y}{-70+48} = \frac{1}{-2+4} \\ \frac{x}{46} &= \frac{-y}{-22} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} x &= 46/2 \\ x &= 23 \end{aligned}$$

And,

$$\begin{aligned} y &= 22/2 \\ y &= 11 \end{aligned}$$

Hence,

The length of the rectangle is 23 units.

The breadth of the rectangle is 11 units.

So, the area of the actual rectangle = length x breadth,

$$= x \times y$$

$$= 23 \times 11$$

$$= 253 \text{ sq. units}$$

Therefore, the area of rectangle is 253 sq. units.

2. The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and the breadth is increased by 5 metres. Find the dimensions of the rectangle.

Solution:

Let's assume the length and breadth of the rectangle be x units and y units respectively.

Hence, the area of rectangle = xy sq. units

From the question we have the following cases,

Case 1

Length is increased by 7 metres \Rightarrow now, the new length is $x+7$

Breadth is decreased by 3 metres \Rightarrow now, the new breadth is $y-3$

And it's given, the area of the rectangle remains same i.e. = xy .

So, the equation becomes

$$xy = (x+7)(y-3)$$

$$xy = xy + 7y - 3x - 21$$

$$3x - 7y + 21 = 0 \dots\dots\dots (i)$$

Case 2:

Length is decreased by 7 metres \Rightarrow now, the new length is $x-7$

Breadth is increased by 5 metres \Rightarrow now, the new breadth is $y+5$

And it's given that, the area of the rectangle still remains same i.e. = xy .

So, the equation becomes

$$xy = (x-7)(y+5)$$

$$xy = xy - 7y + 5x - 35$$

$$5x - 7y - 35 = 0 \dots\dots\dots (ii)$$

Solving (i) and (ii),

By using cross-multiplication, we get,

$$\frac{x}{(-7 \times -5) - (-7 \times 21)} = \frac{y}{(3 \times -35) - (5 \times 21)} = \frac{1}{(3 \times -7) - (5 \times -7)}$$

$$\frac{x}{245 + 147} = \frac{-y}{-105 - 105} = \frac{1}{-21 + 35}$$

$$\frac{x}{392} = \frac{-y}{-210} = \frac{1}{14}$$

$$x = 392/14$$

$$x = 28$$

And,

$$y = 210/14$$

$$y = 15$$

Therefore, the length of the rectangle is 28 m. and the breadth of the actual rectangle is 15 m.

**3. In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the triangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimension of the rectangle.
Solution:**

Let's assume the length and breadth of the rectangle be x units and y units respectively.

Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

According to the question,

Case 1:

Length is increased by 3 metres \Rightarrow now, the new length is $x+3$

Breadth is reduced by 4 metres \Rightarrow now, the new breadth is $y-4$

And it's given, the area of the rectangle is reduced by $67 \text{ m}^2 = xy - 67$.

So, the equation becomes

$$xy - 67 = (x + 3)(y - 4)$$

$$xy - 67 = xy + 3y - 4x - 12$$

$$4xy - 3y - 67 + 12 = 0$$

$$4x - 3y - 55 = 0 \text{ --- (i)}$$

Case 2:

Length is reduced by 1 m \Rightarrow now, the new length is $x-1$

Breadth is increased by 4 metre \Rightarrow now, the new breadth is $y+4$

And it's given, the area of the rectangle is increased by $89 \text{ m}^2 = xy + 89$.

Then, the equation becomes

$$xy + 89 = (x - 1)(y + 4)$$

$$4x - y - 93 = 0 \text{ --- (ii)}$$

Solving (i) and (ii),

Using cross multiplication, we get

$$\frac{x}{(-3 \times -93) - (-1 \times -55)} = \frac{-y}{(4 \times -93) - (4 \times -55)} = \frac{1}{(4 \times -1) - (4 \times -3)}$$

$$\frac{x}{279 - 55} = \frac{-y}{-372 + 220} = \frac{1}{-4 + 12}$$

$$\frac{x}{224} = \frac{-y}{-152} = \frac{1}{8}$$

$$x = 224/8$$

$$x = 28$$

And,

$$y = 152/8$$

$$y = 19$$

Therefore, the length of rectangle is 28 m and the breadth of rectangle is 19 m.

4. The income of X and Y are in the ratio of 8: 7 and their expenditures are in the ratio 19: 16. If each saves ₹ 1250, find their incomes.

Solution:

Let the income be denoted by x and the expenditure be denoted by y .

Then, from the question we have

The income of X is ₹ $8x$ and the expenditure of X is $19y$.

The income of Y is ₹ $7x$ and the expenditure of Y is $16y$.

So, on calculating the savings, we get

$$\text{Saving of X} = 8x - 19y = 1250$$

$$\text{Saving of Y} = 7x - 16y = 1250$$

Hence, the system of equations formed are

$$8x - 19y - 1250 = 0 \text{ --- (i)}$$

$$7x - 16y - 1250 = 0 \text{ --- (ii)}$$

Using cross-multiplication method, we have

$$\frac{x}{(-19 \times -1250) - (-16 \times -1250)} = \frac{-y}{(8 \times -1250) - (7 \times -1250)} = \frac{1}{(8 \times -16) - (7 \times -19)}$$

$$\frac{x}{23750 - 20000} = \frac{-y}{-10000 + 8750} = \frac{1}{-128 + 133}$$

$$\frac{x}{3750} = \frac{y}{1250} = \frac{1}{5}$$

$$x = 3750/5$$

$$x = 750$$

If, $x = 750$, then

The income of X = $8x$

$$= 8 \times 750$$

$$= 6000$$

The income of Y = $7x$

$$\begin{aligned} &= 7 \times 750 \\ &= 5250 \end{aligned}$$

Therefore, the income of X is ₹ 6000 and the income of Y is ₹ 5250

5. A and B each has some money. If A gives ₹ 30 to B, then B will have twice the money left with A. But, if B gives ₹ 10 to A, then A will have thrice as much as is left with B. How much money does each have?

Solution:

Let's assume the money with A be ₹ x and the money with B be ₹ y.
Then, from the question we have the following cases

Case 1: If A gives ₹ 30 to B, then B will have twice the money left with A.

So, the equation becomes

$$\begin{aligned} y + 30 &= 2(x - 30) \\ y + 30 &= 2x - 60 \\ 2x - y - 60 - 30 &= 0 \\ 2x - y - 90 &= 0 \text{ — (i)} \end{aligned}$$

Case 2: If B gives ₹ 10 to A, then A will have thrice as much as is left with B.

$$\begin{aligned} x + 10 &= 3(y - 10) \\ x + 10 &= 3y - 10 \\ x - 3y + 10 + 30 &= 0 \\ x - 3y + 40 &= 0 \text{ — (ii)} \end{aligned}$$

Solving (i) and (ii),

On multiplying equation (ii) with 2, we get,

$$2x - 6y + 80 = 0$$

Subtract equation (ii) from (i), we get,

$$\begin{aligned} 2x - y - 90 - (2x - 6y + 80) &= 0 \\ 5y - 170 &= 0 \\ y &= 34 \end{aligned}$$

Now, on using $y = 34$ in equation (i), we find,

$$x = 62$$

Hence, the money with A is ₹ 62 and the money with B be ₹ 34

7. 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?

Solution:

Assuming that the time required for a man alone to finish the work be x days and also the time required for a boy alone to finish the work be y days.

Then, we know

The work done by a man in one day = $1/x$

The work done by a boy in one day = $1/y$

Similarly,

The work done by 2 men in one day = $2/x$

The work done by 7 boys in one day = $7/y$

So, the condition given in the question states that,

2 men and 7 boys together can finish the work in 4 days

$$4(2/x + 7/y) = 1$$

$$8/x + 28/y = 1 \text{ ——(i)}$$

And, the second condition from the question states that,

4 men and 4 boys can finish the work in 3 days

For this, the equation so formed is

$$3(4/x + 4/y) = 1$$

$$12/x + 12/y = 1 \text{ ——(ii)}$$

Hence, solving (i) and (ii) \Rightarrow

Taking, $1/x = u$ and $1/y = v$

So, the equations (i) and (ii) becomes,

$$8u + 28v = 1$$

$$12u + 12v = 1$$

$$8u + 28v - 1 = 0 \text{ —— (iii)}$$

$$12u + 12v - 1 = 0 \text{ —— (iv)}$$

By using cross multiplication, we get,

$$u = 1/15$$

$$1/x = 1/15$$

$$x = 15$$

And,

$$v = 1/60$$

$$1/y = 1/60$$

$$y = 60$$

Therefore,

The time required for a man alone to finish the work is 15 days and the time required for a boy alone to finish the work is 60 days.

8. In a $\triangle ABC$, $\angle A = x^\circ$, $\angle B = (3x - 2)^\circ$, $\angle C = y^\circ$. Also, $\angle C - \angle B = 9^\circ$. Find the three angles.

Solution:

It's given that,

$$\angle A = x^\circ,$$

$$\angle B = (3x - 2)^\circ,$$

$$\angle C = y^\circ$$

Also given that,

$$\begin{aligned}\angle C - \angle B &= 9^\circ \\ \Rightarrow \angle C &= 9^\circ + \angle B \\ \Rightarrow \angle C &= 9^\circ + 3x^\circ - 2^\circ \\ \Rightarrow \angle C &= 7^\circ + 3x^\circ\end{aligned}$$

Substituting the value for

$$\begin{aligned}\angle C &= y^\circ \text{ in above equation we get,} \\ y^\circ &= 7^\circ + 3x^\circ\end{aligned}$$

We know that, $\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property of a triangle)

$$\begin{aligned}\Rightarrow x^\circ + (3x^\circ - 2^\circ) + (7^\circ + 3x^\circ) &= 180^\circ \\ \Rightarrow 7x^\circ + 5^\circ &= 180^\circ \\ \Rightarrow 7x^\circ &= 175^\circ \\ \Rightarrow x^\circ &= 25^\circ\end{aligned}$$

Hence, calculating for the individual angles we get,

$$\begin{aligned}\angle A &= x^\circ = 25^\circ \\ \angle B &= (3x - 2)^\circ = 73^\circ \\ \angle C &= (7 + 3x)^\circ = 82^\circ\end{aligned}$$

Therefore,

$$\angle A = 25^\circ, \angle B = 73^\circ \text{ and } \angle C = 82^\circ .$$

9. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^\circ$, $\angle B = (y + 3)^\circ$, $\angle C = (2y + 10)^\circ$, $\angle D = (4x - 5)^\circ$. Find the four angles.

Solution:

We know that,

The sum of the opposite angles of cyclic quadrilateral should be 180° .

And, in the cyclic quadrilateral ABCD,

Angles $\angle A$ and $\angle C$ & angles $\angle B$ and $\angle D$ are the pairs of opposite angles.

So,

$$\begin{aligned}\angle A + \angle C &= 180^\circ \text{ and} \\ \angle B + \angle D &= 180^\circ\end{aligned}$$

Substituting the values given to the above two equations, we have

For $\angle A + \angle C = 180^\circ$

$$\begin{aligned}\Rightarrow \angle A &= (2x + 4)^\circ \text{ and } \angle C = (2y + 10)^\circ \\ 2x + 4 + 2y + 10 &= 180^\circ \\ 2x + 2y + 14 &= 180^\circ \\ 2x + 2y &= 180^\circ - 14^\circ \\ 2x + 2y &= 166 \text{ —— (i)}\end{aligned}$$

And for, $\angle B + \angle D = 180^\circ$, we have

$$\begin{aligned}\Rightarrow \angle B &= (y+3)^\circ \text{ and } \angle D = (4x - 5)^\circ \\ y + 3 + 4x - 5 &= 180^\circ\end{aligned}$$

$$\begin{aligned}4x + y - 5 + 3 &= 180^\circ \\4x + y - 2 &= 180^\circ \\4x + y &= 180^\circ + 2^\circ \\4x + y &= 182^\circ \text{ —— (i)}\end{aligned}$$

Now for solving (i) and (ii), we perform

Multiplying equation (ii) by 2 to get,

$$8x + 2y = 364 \text{ —— (iii)}$$

And now, subtract equation (iii) from (i) to get

$$\begin{aligned}-6x &= -198 \\x &= -198 / -6 \\ \Rightarrow x &= 33^\circ\end{aligned}$$

Now, substituting the value of $x = 33^\circ$ in equation (i) to find y

$$\begin{aligned}4x + y &= 182 \\132 + y &= 182 \\y &= 182 - 132 \\ \Rightarrow y &= 50\end{aligned}$$

Thus, calculating the angles of a cyclic quadrilateral we get:

$$\begin{aligned}\angle A &= 2x + 4 \\ &= 66 + 4 \\ &= 70^\circ \\ \angle B &= y + 3 \\ &= 50 + 3 \\ &= 53^\circ \\ \angle C &= 2y + 10 \\ &= 100 + 10 \\ &= 110^\circ \\ \angle D &= 4x - 5 \\ &= 132 - 5 \\ &= 127^\circ\end{aligned}$$

Therefore, the angles of the cyclic quadrilateral ABCD are

$$\angle A = 70^\circ, \angle B = 53^\circ, \angle C = 110^\circ \text{ and } \angle D = 127^\circ$$

10. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Solution:

Let's assume that the total number of correct answers be x and the total number of incorrect answers be y .

Hence, their sum will give the total number of questions in the test i.e. $x + y$

Further from the question, we have two type of marking scheme:

1) When 3 marks is awarded for every right answer and 1 mark deducted for every wrong answer.

According to this type, the total marks scored by Yash is 40. (Given)

So, the equation formed will be

$$3x - 1y = 40 \dots (i)$$

Next,

2) When 4 marks is awarded for every right answer and 2 marks deducted for every wrong answer.

According to this type, the total marks scored by Yash is 50. (Given)

So, the equation formed will be

$$4x - 2y = 50 \dots (ii)$$

Thus, by solving (i) and (ii) we obtained the values of x and y.

From (i), we get

$$y = 3x - 40 \dots (iii)$$

Using (iii) in (ii) we get,

$$4x - 2(3x - 40) = 50$$

$$4x - 6x + 80 = 50$$

$$2x = 30$$

$$x = 15$$

Putting $x = 15$ in (iii) we get,

$$y = 3(15) - 40$$

$$y = 5$$

So, $x + y = 15 + 5 = 20$

Therefore, the number of questions in the test were 20.

11. In a ΔABC , $\angle A = x^\circ$, $\angle B = 3x^\circ$, $\angle C = y^\circ$. If $3y - 5x = 30$, prove that the triangle is right angled.

Solution:

We need to prove that ΔABC is right angled.

Given:

$$\angle A = x^\circ, \angle B = 3x^\circ \text{ and } \angle C = y^\circ$$

Sum of the three angles in a triangle is 180° (Angle sum property of a triangle)

$$\text{i.e., } \angle A + \angle B + \angle C = 180^\circ$$

$$x + 3x + y = 180^\circ$$

$$4x + y = 180 \text{ --- (i)}$$

From question it's given that, $3y - 5x = 30$ --- (ii)

To solve (i) and (ii), we perform
Multiplying equation (i) by 3 to get,
 $12x + 36y = 540$ — (iii)

Now, subtracting equation (ii) from equation (iii) we get

$$\begin{aligned}17x &= 510 \\x &= 510/17 \\ \Rightarrow x &= 30^\circ\end{aligned}$$

Substituting the value of $x = 30^\circ$ in equation (i) to find y

$$\begin{aligned}4x + y &= 180 \\120 + y &= 180 \\y &= 180 - 120 \\ \Rightarrow y &= 60^\circ\end{aligned}$$

Thus the angles $\angle A$, $\angle B$ and $\angle C$ are calculated to be

$$\begin{aligned}\angle A &= x^\circ = 30^\circ \\ \angle B &= 3x^\circ = 90^\circ \\ \angle C &= y^\circ = 60^\circ\end{aligned}$$

A right angled triangle is a triangle with any one side right angled to other, i.e., 90° to other.

And here we have,

$$\angle B = 90^\circ.$$

Therefore, the triangle ABC is right angled. Hence proved.

12. The car hire charges in a city comprise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is ₹ 89 and for a journey of 20 km, the charge paid is ₹ 145. What will a person have to pay for travelling a distance of 30 km?

Solution:

Let the fixed charge of the car be ₹ x and,
Let the variable charges of the car be ₹ y per km.
So according to the question, we get 2 equations

$$\begin{aligned}x + 12y &= 89 \text{ — (i) and,} \\ x + 20y &= 145 \text{ — (ii)}\end{aligned}$$

Now, by solving (i) and (ii) we can find the charges.

On subtraction of (i) from (ii), we get,

$$\begin{aligned}-8y &= -56 \\ y &= -56 - 8 \\ \Rightarrow y &= 7\end{aligned}$$

So, substituting the value of $y = 7$ in equation (i) we get

$$\begin{aligned}x + 12y &= 89 \\ x + 84 &= 89 \\ x &= 89 - 84 \\ \Rightarrow x &= 5\end{aligned}$$

Thus, the total charges for travelling a distance of 30 km can be calculated as: $x + 30y$
 $\Rightarrow x + 30y = 5 + 210 = ₹ 215$

Therefore, a person has to pay ₹ 215 for travelling a distance of 30 km by the car.

