Exercise 3.5 Page No: 3.73

In each of the following systems of equation determine whether the system has a unique solution, no solution or infinite solutions. In case there is a unique solution, find it from 1 to 4:

1. $x - 3y = 3$
$3x - 9y = 2$

Solution:

The given system of equations is:
$x - 3y - 3 = 0$
$3x - 9y - 2 = 0$

The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1, b_1 = -3, c_1 = -3$
$a_2 = 3, b_2 = -9, c_2 = -2$
So according to the question, we get
$a_1 / a_2 = 1/3$
$b_1 / b_2 = -3/ -9 = 1/3$ and,
$c_1 / c_2 = -3/ -2 = 3/2$
$\Rightarrow$ $a_1 / a_2 = b_1/ b2 \neq c_1 / c_2$

Hence, we can conclude that the given system of equation has no solution.

2. $2x + y = 5$
$4x + 2y = 10$

Solution:

The given system of equations is:
$2x + y - 5 = 0$
$4x + 2y - 10 = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2, b_1 = 1, c_1 = -5$
$a_2 = 4, b_2 = 2, c_2 = -10$
So according to the question, we get
$a_1 / a_2 = 2/4 = 1/2$
$b_1 / b_2 = 1/ 2$ and,
$c_1 / c_2 = -5/ -10 = 1/2$
$\Rightarrow$ $a_1 / a_2 = b_1/ b2 = c_1 / c_2$

Hence, we can conclude that the given system of equation has infinity many solutions.

3. $3x - 5y = 20$
$6x - 10y = 40$
Solution:

The given system of equations is:

$3x - 5y - 20 = 0$
$6x - 10y - 40 = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 3$, $b_1 = -5$, $c_1 = -20$
$a_2 = 6$, $b_2 = -10$, $c_2 = -40$

So according to the question, we get

$a_1 / a_2 = 3/6 = 1/2$
$b_1 / b_2 = -5/ -10 = 1/2$ and,
$c_1 / c_2 = -20/ -40 = 1/2$
$\Rightarrow \quad a_1 / a_2 = b_1/ b2 = c_1 / c_2$

Hence, we can conclude that the given system of equation has infinity many solutions.

4. $x - 2y = 8$
$5x - 10y = 10$
Solution:

The given system of equations is:

$x - 2y - 8 = 0$
$5x - 10y - 10 = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -8$
$a_2 = 5$, $b_2 = -10$, $c_2 = -10$

So according to the question, we get

$a_1 / a_2 = 1/5$
$b_1 / b_2 = -2/ -10 = 1/5$ and,
$c_1 / c_2 = -8/ -10 = 4/5$
$\Rightarrow \quad a_1 / a_2 = b_1/ b2 \neq c_1 / c_2$

Hence, we can conclude that the given system of equation has no solution.

Find the value of k for which the following system of equations has a unique solution: (5-8)

5. kx + 2y = 5
3x + y = 1
Solution:

The given system of equations is:
kx + 2y - 5 = 0
3x + y - 1 = 0
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = k$, $b_1 = 2$, $c_1 = -5$
$a_2 = 3$, $b_2 = 1$, $c_2 = -1$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 \neq b_1 / b_2$
$k/3 \neq 2/1$
$\Rightarrow k \neq 6$
Hence, the given system of equations will have unique solution for all real values of k other than 6.

6. 4x + ky + 8 = 0
2x + 2y + 2 = 0
Solution:

The given system of equations is:
4x + ky + 8 = 0
2x + 2y + 2 = 0
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 4$, $b_1 = k$, $c_1 = 8$
$a_2 = 2$, $b_2 = 2$, $c_2 = 2$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 \neq b_1 / b_2$
$4/2 \neq k/2$
$\Rightarrow k \neq 4$
Hence, the given system of equations will have unique solution for all real values of k other than 4.

7. 4x – 5y = k
2x – 3y = 12
Solution

The given system of equations is:
$4x - 5y - k = 0$
$2x - 3y - 12 = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 4, b_1 = 5, c_1 = -k$
$a_2 = 2, b_2 = 3, c_2 = 12$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 \neq b_1 / b_2$
$4/2 \neq 5/3$
$\Rightarrow$k can have any real values.
Hence, the given system of equations will have unique solution for all real values of k.

8. x + 2y = 3
5x + ky + 7 = 0
Solution:

The given system of equations is:
$x + 2y - 3 = 0$
$5x + ky + 7 = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = -3$
$a_2 = 5, b_2 = k, c_2 = 7$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 \neq b_1 / b_2$
$1/5 \neq 2/k$
$\Rightarrow k \neq 10$
Hence, the given system of equations will have unique solution for all real values of k other than 10.

Find the value of k for which each of the following system of equations having infinitely many solution: (9-19)
9. 2x + 3y – 5 = 0
6x + ky – 15 = 0
Solution:

The given system of equations is:
$2x + 3y - 5 = 0$
$6x + ky - 15 = 0$

The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$
$a_2 = 6$, $b_2 = k$, $c_2 = -15$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$2/6 = 3/k$
$\Rightarrow k = 9$
Hence, the given system of equations will have infinitely many solutions, if k = 9.

10. 4x + 5y = 3
kx + 15y = 9
Solution:

The given system of equations is:
$4x + 5y - 3 = 0$
$kx + 15y - 9 = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 4$, $b_1 = 5$, $c_1 = -3$
$a_2 = k$, $b_2 = 15$, $c_2 = -9$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$4/ k = 5/ 15 = -3/ -9$
$4/ k = 1/ 3$
$\Rightarrow k = 12$

Hence, the given system of equations will have infinitely many solutions, if k = 12.

11. kx – 2y + 6 = 0
4x – 3y + 9 = 0
Solution:

The given system of equations is:
$kx - 2y + 6 = 0$
$4x - 3y + 9 = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$

$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = k$, $b_1 = -2$, $c_1 = 6$
$a_2 = 4$, $b_2 = -3$, $c_2 = 9$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$k/ 4 = -2/ -3 = 2/ 3$
$\Rightarrow k = 8/ 3$

Hence, the given system of equations will have infinitely many solutions, if $k = 8/3$.

12. 8x + 5y = 9
kx + 10y = 18
Solution:

The given system of equations is:
$8x + 5y - 9 = 0$
$kx + 10y - 18 = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$
$a_2 = k$, $b_2 = 10$, $c_2 = -18$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$8/k = 5/10 = -9/ -18 = 1/2$
$\Rightarrow k=16$

Hence, the given system of equations will have infinitely many solutions, if $k = 16$.

13. 2x – 3y = 7
(k+2)x – (2k+1)y = 3(2k-1)
Solution:

The given system of equations is:
$2x - 3y - 7 = 0$
$(k+2)x - (2k+1)y - 3(2k-1) = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$
$a_2 = (k+2)$, $b_2 = -(2k+1)$, $c_2 = -3(2k-1)$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$2/ (k+2) = -3/ -(2k+1) = -7/ -3(2k-1)$
$2/(k+2) = -3/ -(2k+1)$ and $-3/ -(2k+1)= -7/ -3(2k-1$
$\Rightarrow 2(2k+1) = 3(k+2)$ and $3\times3(2k-1) = 7(2k+1)$
$\Rightarrow 4k+2 = 3k+6$ and $18k - 9 = 14k + 7$
$\Rightarrow k=4$ and $4k = 16 \Rightarrow k=4$
Hence, the given system of equations will have infinitely many solutions, if k = 4.

14. 2x + 3y = 2
(k+2)x + (2k+1)y = 2(k-1)
Solution:

The given system of equations is:
$2x + 3y - 2= 0$
$(k+2)x + (2k+1)y - 2(k-1) = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$
$a_2 = (k+2)$, $b_2 = (2k+1)$, $c_2 = -2(k-1)$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$2/ (k+2) = 3/ (2k+1) = -2/ -2(k-1)$
$2/ (k+2) = 3/ (2k+1)$ and $3/(2k+1) = 2/2(k-1)$
$\Rightarrow 2(2k+1) = 3(k+2)$ and $3(k-1) = (2k+1)$
$\Rightarrow 4k+2 = 3k+6$ and $3k-3 = 2k+1$
$\Rightarrow k = 4$ and $k = 4$
Hence, the given system of equations will have infinitely many solutions, if k = 4.

15. x + (k+1)y = 4
(k+1)x + 9y = 5k + 2
Solution:

The given system of equations is:
$x + (k+1)y - 4= 0$
$(k+1)x + 9y - (5k + 2) = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1$, $b_1 = (k+1)$, $c_1 = -4$
$a_2 = (k+1)$, $b_2 = 9$, $c_2 = -(5k+2)$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$1/ k+1 = (k+1)/ 9 = -4/ -(5k+2)$
$1/ k+1 = k+1/ 9$ and $k+1/ 9 = 4/ 5k+2$
$\Rightarrow 9 = (k+1)^2$ and $(k+1)(5k+2) = 36$
$\Rightarrow 9 = k^2 + 2k + 1$ and $5k^2+2k+5k+2 = 36$
$\Rightarrow k^2+2k-8 = 0$ and $5k^2+7k-34 = 0$
$\Rightarrow k^2+4k-2k-8 = 0$ and $5k^2+17k-10k-34 = 0$
$\Rightarrow k(k+4)-2(k+4) = 0$ and $(5k+17)-2(5k+17) = 0$
$\Rightarrow (k+4)(k-2) = 0$ and $(5k+17) (k-2) = 0$
$\Rightarrow k = -4$ or $k = 2$ and $k = -17/5$ or $k = 2$
Its seen that k=2 satisfies both the condition.

Hence, the given system of equations will have infinitely many solutions, if k = 9.

16. kx + 3y = 2k + 1
2(k+1)x + 9y = 7k + 1
Solution:

The given system of equations is:
$kx + 3y - (2k + 1) = 0$
$2(k+1)x + 9y - (7k + 1) = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = - (2k+1)$
$a_2 = 2(k+1)$, $b_2 = 9$, $c_2 = - (7k+1)$
So according to the question,
For unique solution, the condition is
$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$
$k/ 2(k+1) = 3/ 9$ and $3/ 9 = -(2k+1)/ -(7k+1)$
$3k = 2k +2$ and $7k+1 = 3(2k+1) = 6k + 3$
$k = 2$ and $k = 2$

Hence, the given system of equations will have infinitely many solutions, if k = 2.

17. 2x + (k-2)y = k
6x + (2k-1)y = 2k + 5
Solution:

The given system of equations is:

$2x + (k-2)y - k = 0$
$6x + (2k-1)y – (2k+5) = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = k-2$, $c_1 = - k$
$a_2 = 6$, $b_2 = 2k-1$, $c_2 = -2k-5$
So according to the question,
For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/6 = (k-2)/ (2k-1)$ and $(k-2)/ (2k-1) = - k/ -2k-5$

$4k -2 = 6k -12$ and $(k-2)(2k+5) = k(2k-1)$

$2k = 10$ and $2k^2 - 4k + 5k – 10 = 2k^2 – k$

$\Rightarrow k = 5$ and $2k = 10 \Rightarrow k = 5$

Hence, the given system of equations will have infinitely many solutions, if k = 5.

18. 2x + 3y = 7
(k+1)x + (2k-1)y = 4k+1
Solution:

The given system of equations is:
$2x + 3y - 7= 0$
$(k+1)x + (2k-1)y – (4k+1) = 0$
The above equations are of the form
$a_1 x + b_1 y - c_1 = 0$
$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = - 7$
$a_2 = (k+1)$, $b_2 = 2k-1$, $c_2 = - (4k+1)$
So according to the question,
For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/ (k+1) = 3/ (2k-1) = -7/ -(4k+1)$

$2/ (k+1) = 3/(2k-1)$ and $3/ (2k-1) = 7/(4k+1)$

$2(2k-1) = 3(k+1)$ and $3(4k+1) = 7(2k-1)$

$\Rightarrow 4k-2 = 3k+3$ and $12k + 3 = 14k - 7$

$\Rightarrow k = 5$ and $2k = 10$

$\Rightarrow k = 5$ and $k = 5$

Hence, the given system of equations will have infinitely many solutions, if k = 5.