EXERCISE 13.2 PAGE NO: 13.31

1. Express the following complex numbers in the standard form a + ib:
(i) $(1 + i)(1 + 2i)$
(ii) $(3 + 2i) / (-2 + i)$
(iii) $1/(2 + i)^2$
(iv) $(1 - i) / (1 + i)$
(v) $(2 + i)^3 / (2 + 3i)$
(vi) $[(1 + i)(1 + \sqrt{3}i)] / (1 - i)$
(vii) $(2 + 3i) / (4 + 5i)$
(viii) $(1 - i)^3 / (1 - i^3)$
(ix) $(1 + 2i)^{-3}$
(x) $(3 - 4i) / [(4 - 2i)(1 + i)]$
(xi)
$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$
(xii) $(5 + \sqrt{2}i) / (1 - \sqrt{2}i)$

Solution:

(i) $(1 + i)(1 + 2i)$

Let us simplify and express in the standard form of (a + ib),

$(1 + i)(1 + 2i) = (1+i)(1+2i)$
$= 1(1+2i)+i(1+2i)$
$= 1+2i+i+2i^2$
$= 1+3i+2(-1)$ [since, $i^2 = -1$]
$= 1+3i-2$
$= -1+3i$

∴ The values of a, b are -1, 3.

(ii) $(3 + 2i) / (-2 + i)$

Let us simplify and express in the standard form of (a + ib),

$(3 + 2i) / (-2 + i) = [(3 + 2i) / (-2 + i)] \times (-2-i) / (-2-i)$ [multiply and divide with (-2-i)]
$= [3(-2-i) + 2i(-2-i)] / [(-2)^2 - (i)^2]$
$= [-6 - 3i - 4i - 2i^2] / (4-i^2)$
$= [-6 - 7i - 2(-1)] / (4 - (-1))$ [since, $i^2 = -1$]
$= [-4 - 7i] / 5$

∴ The values of a, b are -4/5, -7/5

(iii) $1/(2 + i)^2$

Let us simplify and express in the standard form of (a + ib),

$1/(2 + i)^2 = 1/(2^2 + i^2 + 2(2)(i))$

$= 1/(4 – 1 + 4i)$ [since, $i^2 = -1$]

$= 1/(3 + 4i)$ [multiply and divide with $(3 – 4i)$]

$= 1/(3 + 4i) \times (3 – 4i)/(3 – 4i)]$

$= (3-4i)/(3^2 – (4i)^2)$

$= (3-4i)/(9 – 16i^2)$

$= (3-4i)/(9 – 16(-1))$ [since, $i^2 = -1$]

$= (3-4i)/25$

∴ The values of a, b are 3/25, -4/25

(iv) $(1 - i) / (1 + i)$

Let us simplify and express in the standard form of (a + ib),

$(1 - i) / (1 + i) = (1 - i) / (1 + i) \times (1-i)/(1-i)$ [multiply and divide with (1-i)]

$= (1^2 + i^2 – 2(1)(i)) / (1^2 – i^2)$

$= (1 + (-1) -2i) / (1 – (-1))$

$= -2i/2$

$= -i$

∴ The values of a, b are 0, -1

(v) $(2 + i)^3 / (2 + 3i)$

Let us simplify and express in the standard form of (a + ib),

$(2 + i)^3 / (2 + 3i) = (2^3 + i^3 + 3(2)^2(i) + 3(i)^2(2)) / (2 + 3i)$

$= (8 + (i^2.i) + 3(4)(i) + 6i^2) / (2 + 3i)$

$= (8 + (-1)i + 12i + 6(-1)) / (2 + 3i)$

$= (2 + 11i) / (2 + 3i)$

[multiply and divide with (2-3i)]

$= (2 + 11i)/(2 + 3i) \times (2-3i)/(2-3i)$

$= [2(2-3i) + 11i(2-3i)] / (2^2 – (3i)^2)$

$= (4 – 6i + 22i – 33i^2) / (4 – 9i^2)$

$= (4 + 16i – 33(-1)) / (4 – 9(-1))$ [since, $i^2 = -1$]

$= (37 + 16i) / 13$

∴ The values of a, b are 37/13, 16/13

(vi) $[(1 + i)(1 +\sqrt{3}i)] / (1 - i)$

Let us simplify and express in the standard form of (a + ib),

$[(1 + i)(1 +\sqrt{3}i)] / (1 - i) = [1(1+\sqrt{3}i) + i(1+\sqrt{3}i)] / (1-i)$

$= (1 + \sqrt{3}i + i + \sqrt{3}i^2) / (1 - i)$

$= (1 + (\sqrt{3}+1)i + \sqrt{3}(-1)) / (1-i)$ [since, $i^2 = -1$]

$= [(1-\sqrt{3}) + (1+\sqrt{3})i] / (1-i)$

[multiply and divide with (1+i)]
$= [(1-\sqrt{3}) + (1+\sqrt{3})i] / (1-i) \times (1+i)/(1+i)$
$= [(1-\sqrt{3})(1+i) + (1+\sqrt{3})i(1+i)] / (1^2 - i^2)$
$= [1-\sqrt{3} + (1-\sqrt{3})i + (1+\sqrt{3})i + (1+\sqrt{3})i^2] / (1-(-1))$ [since, $i^2 = -1$]
$= [(1-\sqrt{3})+(1-\sqrt{3}+1+\sqrt{3})i+(1+\sqrt{3})(-1)] / 2$
$= (-2\sqrt{3} + 2i) / 2$
$= -\sqrt{3} + i$

∴ The values of a, b are $-\sqrt{3}$, 1

(vii) $(2 + 3i) / (4 + 5i)$

Let us simplify and express in the standard form of (a + ib),

$(2 + 3i) / (4 + 5i)$ = [multiply and divide with (4-5i)]
$= (2 + 3i) / (4 + 5i) \times (4-5i)/(4-5i)$
$= [2(4-5i) + 3i(4-5i)] / (4^2 - (5i)^2)$
$= [8 - 10i + 12i - 15i^2] / (16 - 25i^2)$
$= [8+2i-15(-1)] / (16 - 25(-1))$ [since, $i^2 = -1$]
$= (23 + 2i) / 41$

∴ The values of a, b are 23/41, 2/41

(viii) $(1 - i)^3 / (1 - i^3)$

Let us simplify and express in the standard form of (a + ib),

$(1 - i)^3 / (1 - i^3) = [1^3 - 3(1)^2 i + 3(1)(i)^2 - i^3] / (1-i^2.i)$
$= [1 - 3i + 3(-1)-i^2.i] / (1 - (-1)i)$ [since, $i^2 = -1$]
$= [-2 - 3i - (-1)i] / (1+i)$
$= [-2-4i] / (1+i)$
[Multiply and divide with (1-i)]
$= [-2-4i] / (1+i) \times (1-i)/(1-i)$
$= [-2(1-i)-4i(1-i)] / (1^2 - i^2)$
$= [-2+2i-4i+4i^2] / (1 - (-1))$
$= [-2-2i+4(-1)] /2$
$= (-6-2i)/2$
$= -3 - i$

∴ The values of a, b are -3, -1

(ix) $(1 + 2i)^{-3}$

Let us simplify and express in the standard form of (a + ib),

$(1 + 2i)^{-3} = 1/(1 + 2i)^3$
$= 1/(1^3+3(1)^2 (2i)+2(1)(2i)^2 + (2i)^3)$
$= 1/(1+6i+4i^2+8i^3)$

= 1/(1+6i+4(-1)+8i^2.i) [since, i^2 = -1]
= 1/(-3+6i+8(-1)i) [since, i^2 = -1]
= 1/(-3-2i)
= -1/(3+2i)
[Multiply and divide with (3-2i)]
= -1/(3+2i) × (3-2i)/(3-2i)
= (-3+2i)/(3^2 – $(2i)^2$)
= (-3+2i) / (9-4i^2)
= (-3+2i) / (9-4(-1))
= (-3+2i) /13
∴ The values of a, b are -3/13, 2/13

(x) (3 – 4i) / [(4 – 2i) (1 + i)]
Let us simplify and express in the standard form of (a + ib),
(3 – 4i) / [(4 – 2i) (1 + i)] = (3-4i)/ [4(1+i)-2i(1+i)]
= (3-4i)/ [4+4i-2i-2i^2]
= (3-4i)/ [4+2i-2(-1)] [since, i^2 = -1]
= (3-4i)/ (6+2i)
[Multiply and divide with (6-2i)]
= (3-4i)/ (6+2i) × (6-2i)/(6-2i)
= [3(6-2i)-4i(6-2i)] / (6^2 – $(2i)^2$)
= [18 – 6i – 24i + 8i^2] / (36 – 4i^2)
= [18 – 30i + 8 (-1)] / (36 – 4 (-1)) [since, i^2 = -1]
= [10-30i] / 40
= (1 – 3i) / 4
∴ The values of a, b are 1/4, -3/4

(xi)
$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$
Let us simplify and express in the standard form of (a + ib),
$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)=\left(\frac{1+i-2(1-4i)}{(1-4i)(1+i)}\right)\left(\frac{3-4i}{5+i}\right)$$
$$=\left(\frac{1+i-2+8i}{1(1+i)-4i(1+i)}\right)\left(\frac{3-4i}{5+i}\right)$$
$$=\left(\frac{-1+9i}{1+i-4i-4i^2}\right)\left(\frac{3-4i}{5+i}\right)$$
$$=\left(\frac{-1+9i}{1-3i-4(-1)}\right)\left(\frac{3-4i}{5+i}\right)$$

$$= \frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}$$

$$= \frac{-1(3-4i)+9i(3-i)}{5(5+i)-3i(5+i)}$$

$$= \frac{-3+4i+27i-9i^2}{25+5i-15i-3i^2}$$

$$= \frac{-3+31i-9(-1)}{25-10i-3(-1)}$$

$$= \frac{6+31i}{28-10i}$$

[Multiply and divide with (28+10i)]

$$= \frac{6+31i}{28-10i} \times \frac{28+10i}{28+10i}$$

$$= \frac{6(28+10i)+31i(28+10i)}{28^2-(10i)^2}$$

$$= \frac{168+60i+868i+310i^2}{784-100i^2}$$

$$= \frac{168+928i+310(-1)}{784-100(-1)}$$

$$= \frac{478+928i}{884}$$

Therefore value of a, b are 478/884, 928/884.

(xii) (5 +√2i) / (1-√2i)

Let us simplify and express in the standard form of (a + ib),

(5 +√2i) / (1-√2i) = [Multiply and divide with (1+√2i)]

= (5 +√2i) / (1-√2i) × (1+√2i)/(1+√2i)

= [5(1+√2i) + √2i(1+√2i)] / (1^2 – $(\sqrt{2})^2$)

= [5+5√2i + √2i + $2i^2$] / (1 – $2i^2$)

= [5 + 6√2i + 2(-1)] / (1-2(-1)) [since, i^2 = -1]

= [3+6√2i]/3

= 1+ 2√2i

∴ The values of a, b are 1, 2√2

2. Find the real values of x and y, if

(i) (x + iy) (2 – 3i) = 4 + i

(ii) (3x – 2i y) $(2 + i)^2$ = 10(1 + i)

(iii) $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

(iv) (1 + i) (x + iy) = 2 – 5i
Solution:
(i) (x + iy) (2 – 3i) = 4 + i
Given:
(x + iy) (2 - 3i) = 4 + i
Let us simplify the expression we get,
x(2 - 3i) + iy(2 - 3i) = 4 + i
2x - 3xi + 2yi - 3yi^2 = 4 + i
2x + (-3x+2y)i - 3y (-1) = 4 + i [since, i^2 = -1]
2x + (-3x+2y)i + 3y = 4 + i [since, i^2 = -1]
(2x+3y) + i(-3x+2y) = 4 + i
Equating Real and Imaginary parts on both sides, we get
2x+3y = 4… (i)
And -3x+2y = 1… (ii)
Multiply (i) by 3 and (ii) by 2 and add
On solving we get,
6x – 6x – 9y + 4y = 12 + 2
13y = 14
y = 14/13
Substitute the value of y in (i) we get,
2x+3y = 4
2x + 3(14/13) = 4
2x = 4 – (42/13)
= (52-42)/13
2x = 10/13
x = 5/13
x = 5/13, y = 14/13
∴ The real values of x and y are 5/13, 14/13

(ii) (3x – 2i y) (2 + i)^2 = 10(1 + i)
Given:
(3x - 2iy) (2+i)^2 = 10(1+i)
(3x - 2yi) (2^2+i^2+2(2)(i)) = 10+10i
(3x - 2yi) (4 + (-1)+4i) = 10+10i [since, i^2 = -1]
(3x - 2yi) (3+4i) = 10+10i
Let us divide with 3+4i on both sides we get,
(3x - 2yi) = (10+10i)/(3+4i)
= Now multiply and divide with (3-4i)
= [10(3-4i) + 10i(3-4i)] / (3^2 – (4i)^2)

= [30-40i+30i-40i^2] / (9 – 16i^2)
= [30-10i-40(-1)] / (9-16(-1))
= [70-10i]/25
Now, equating Real and Imaginary parts on both sides we get
3x = 70/25 and -2y = -10/25
x = 70/75 and y = 1/5
x = 14/15 and y = 1/5
∴ The real values of x and y are 14/15, 1/5

(iii) $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$

Given:

$$\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

$$\frac{\left(((1+i)x-2i)(3-i)\right)+\left(((2-3i)y+i)(3+i)\right)}{(3+i)(3-i)}=i$$

$$\frac{((1+i)(3-i)x)-(2i)(3-i)+((2-3i)(3+i)y)+(i)(3+i)}{3^2-i^2}=i$$

$$\frac{(3-i+3i-i^2)x-6i+2i^2+(6+2i-9i-3i^2)y+3i+i^2}{9-(-1)}=i$$

$$\frac{(3+2i-(-1))x-6i+2(-1)+(6-7i-3(-1))y+3i+(-1)}{10}=i \text{ [since, } i^2=-1]$$

(4+2i) x-3i-3 + (9-7i)y = 10i
(4x+9y-3) + i(2x-7y-3) = 10i
Now, equating Real and Imaginary parts on both sides we get,
4x+9y-3 = 0 … (i)
And 2x-7y-3 = 10
2x-7y = 13 … (ii)
Multiply (i) by 7 and (ii) by 9 and add
On solving these equations we get
28x + 18x + 63y – 63y = 117 + 21
46x = 117 + 21
46x = 138
x = 138/46
= 3
Substitute the value of x in (i) we get,
4x+9y-3 = 0

$9y = -9$
$y = -9/9$
$= -1$
$x = 3$ and $y = -1$
∴ The real values of x and y are 3 and -1

(iv) $(1 + i)(x + iy) = 2 – 5i$
Given:
$(1 + i)(x + iy) = 2 – 5i$
Divide with (1+i) on both the sides we get,
$(x + iy) = (2 – 5i)/(1+i)$
Multiply and divide by (1-i)
$= (2 – 5i)/(1+i) \times (1-i)/(1-i)$
$= [2(1-i) – 5i(1-i)] / (1^2 – i^2)$
$= [2 – 7i + 5(-1)] / 2$ [since, $i^2 = -1$]
$= (-3-7i)/2$
Now, equating Real and Imaginary parts on both sides we get
$x = -3/2$ and $y = -7/2$
∴ The real values of x and y are -3/2, -7/2

3. Find the conjugates of the following complex numbers:
(i) $4 – 5i$
(ii) $1 / (3 + 5i)$
(iii) $1 / (1 + i)$
(iv) $(3 - i)^2 / (2 + i)$
(v) $[(1 + i)(2 + i)] / (3 + i)$
(vi) $[(3 – 2i)(2 + 3i)] / [(1 + 2i)(2 - i)]$
Solution:
(i) $4 – 5i$
Given:
$4 – 5i$
We know the conjugate of a complex number (a + ib) is (a - ib)
So,
∴ The conjugate of (4 – 5i) is (4 + 5i)

(ii) $1 / (3 + 5i)$
Given:
$1 / (3 + 5i)$
Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form by multiplying and dividing with (3 – 5i)
We get,

$$\frac{1}{3+5i} = \frac{1}{3+5i} \times \frac{3-5i}{3-5i}$$
$$= \frac{3-5i}{3^2-(5i)^2}$$
$$= \frac{3-5i}{9-25i^2}$$
$$= \frac{3-5i}{9-25(-1)} \text{ [Since, } i^2 = -1]$$
$$= \frac{3-5i}{34}$$

We know the conjugate of a complex number (a + ib) is (a - ib)
So,
∴ The conjugate of (3 – 5i)/34 is (3 + 5i)/34

(iii) 1 / (1 + i)
Given:
1 / (1 + i)
Since the given complex number is not in the standard form of (a + ib)
Let us convert to standard form by multiplying and dividing with (1 – i)
We get,

$$\frac{1}{1+i} = \frac{1}{1+i} \times \frac{1-i}{1-i}$$
$$= \frac{1-i}{1^2-i^2}$$
$$= \frac{1-i}{1-(-1)} \text{ [since, } i^2 = -1]$$
$$= \frac{1-i}{2}$$

We know the conjugate of a complex number (a + ib) is (a - ib)
So,
∴ The conjugate of (1-i)/2 is (1+i)/2

(iv) $(3 - i)^2 / (2 + i)$
Given:
$(3 - i)^2 / (2 + i)$
Since the given complex number is not in the standard form of (a + ib)
Let us convert to standard form,

$\frac{(3-i)^2}{2+i} = \frac{3^2+i^2-2(3)(i)}{2+i}$

$= \frac{9+(-1)-6i}{2+i}$ [Since, $i^2 = -1$]

$= \frac{8-6i}{2+i}$

Now, let us multiply and divide with (2 - i) we get,

$\frac{8-6i}{2+i} = \frac{8-6i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{8(2-i)-6i(2-i)}{2^2-i^2}$

$= \frac{16-8i-12i+6i^2}{4-(-1)}$ [Since, $i^2 = -1$]

$= \frac{16-20i+6(-1)}{5}$

$= \frac{10-20i}{5}$

$= 10/5 – 20i/5$

$= 2 – 4i$

We know the conjugate of a complex number (a + ib) is (a - ib)

So,

∴ The conjugate of (2 – 4i) is (2 + 4i)

(v) [(1 + i) (2 + i)] / (3 + i)

Given:

[(1 + i) (2 + i)] / (3 + i)

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form,

$\frac{(1+i)(2+i)}{3+i} = \frac{1(2+i)+i(2+i)}{3+i}$

$= \frac{2+i+2i+i^2}{3+i}$

$= \frac{2+3i+(-1)}{3+i}$ [Since, $i^2 = -1$]

$= \frac{1+3i}{3+i}$

Now, let us multiply and divide with (3 - i) we get,

$\frac{1+3i}{3+i} = \frac{1+3i}{3+i} \times \frac{3-i}{3-i}$

$$= \frac{1(3-i)+3i(3-i)}{3^2-i^2}$$

$$= \frac{3-i+9i-3i^2}{9-(-1)}$$ [Since, $i^2 = -1$]

$$= \frac{3+8i-3(-1)}{10}$$

$$= \frac{6+8i}{10}$$

$$= \frac{3}{5} + \frac{4i}{5}$$

We know the conjugate of a complex number (a + ib) is (a - ib)
So,
∴ The conjugate of (3 + 4i)/5 is (3 – 4i)/5

(vi) [(3 – 2i) (2 + 3i)] / [(1 + 2i) (2 - i)]
Given:
[(3 – 2i) (2 + 3i)] / [(1 + 2i) (2 - i)]
Since the given complex number is not in the standard form of (a + ib)
Let us convert to standard form,

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{3(2+3i)-2i(2+3i)}{1(2-i)+2i(2-i)}$$

$$= \frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$$

$$= \frac{6+5i-6(-1)}{2+3i-2(-1)}$$

$$= \frac{12+5i}{4+3i}$$

Now, let us multiply and divide with (4 - 3i) we get,

$$\frac{12+5i}{4+3i} = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{12(4-3i)+5i(4-3i)}{4^2-(3i)^2}$$

$$= \frac{48-36i+20i-15i^2}{16-9i^2}$$

$$= \frac{48-16i-15(-1)}{16-9(-1)}$$

$$= \frac{63-16i}{25}$$

We know the conjugate of a complex number (a + ib) is (a - ib)

So,

∴ The conjugate of (63 - 16i)/25 is (63 + 16i)/25

4. Find the multiplicative inverse of the following complex numbers:

(i) 1 – i

(ii) $(1 + i\sqrt{3})^2$

(iii) 4 – 3i

(iv) $\sqrt{5}$ + 3i

Solution:

(i) 1 – i

Given:

1 – i

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

Z = 1 - i

$$Z^{-1} = \frac{1}{1-i}$$

Let us multiply and divide by (1 + i) we get,

$$= \frac{1}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1+i}{1^2-(i)^2}$$

$$= \frac{1+i}{1-(-1)} \text{ [Since, } i^2 = -1]$$

$$= \frac{1+i}{2}$$

∴ The multiplicative inverse of (1 - i) is (1 + i)/2

(ii) $(1 + i\sqrt{3})^2$

Given:

$(1 + i\sqrt{3})^2$

$Z = (1 + i\sqrt{3})^2$

$= 1^2 + (i\sqrt{3})^2 + 2(1)(i\sqrt{3})$

$= 1 + 3i^2 + 2i\sqrt{3}$

$= 1 + 3(-1) + 2i\sqrt{3}$ [since, $i^2 = -1$]

$= 1 – 3 + 2i\sqrt{3}$

$= -2 + 2i\sqrt{3}$

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

$Z = -2 + 2i\sqrt{3}$

$$Z^{-1} = \frac{1}{-2 + 2i\sqrt{3}}$$

Let us multiply and divide by -2 - 2 i√3, we get

$$= \frac{1}{-2+2\sqrt{3}i} \times \frac{-2-2\sqrt{3}i}{-2-2\sqrt{3}i}$$

$$= \frac{-2-2\sqrt{3}i}{(-2)^2-\left(2\sqrt{3}i\right)^2}$$

$$= \frac{-2-2\sqrt{3}i}{4-12i^2}$$

$$= \frac{-2-2\sqrt{3}i}{4-12(-1)}$$

$$= \frac{-2-2\sqrt{3}i}{16}$$

$$= \frac{-1}{8}\frac{-i\sqrt{3}}{8}$$

∴ The multiplicative inverse of $(1 + i\sqrt{3})^2$ is $(-1-i\sqrt{3})/8$

(iii) 4 – 3i

Given:

4 – 3i

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

Z = 4 – 3i

$$Z^{-1} = \frac{1}{4 - 3i}$$

Let us multiply and divide by (4 + 3i), we get

$$= \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$$

$$= \frac{4+3i}{4^2-(3i)^2}$$

$$= \frac{4+3i}{16-9i^2}$$

$$= \frac{4+3i}{16-9(-1)}$$

$$= \frac{4+3i}{25}$$

∴ The multiplicative inverse of (4 – 3i) is (4 + 3i)/25

(iv) √5 + 3i
Given:
√5 + 3i
We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z
So,
Z = √5 + 3i

$$Z^{-1} = \frac{1}{\sqrt{5}+3i}$$

Let us multiply and divide by (√5 – 3i)

$$= \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$

$$= \frac{\sqrt{5}-3i}{\left(\sqrt{5}\right)^2-(3i)^2}$$

$$= \frac{\sqrt{5}-3i}{5-9i^2}$$

$$= \frac{\sqrt{5}-3i}{5-9(-1)}$$

$$= \frac{\sqrt{5}-3i}{14}$$

∴ The multiplicative inverse of (√5 + 3i) is (√5 - 3i)/14

5. If $z_1 = 2 - i$, $z_2 = 1 + i$, find

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$$

Solution:
Given:
$z_1 = (2 - i)$ and $z_2 = (1 + i)$
We know that, |a/b| = |a| / |b|
So,

$$\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{|z_1+z_2+1|}{|z_1-z_2+i|}$$

$$= \frac{|2-i+1+i+1|}{|2-i-(1+i)+i|}$$

$$= \frac{|4|}{|1-i|}$$

We know, |a + ib| is $\sqrt{a^2+b^2}$
So now,

$$= \frac{\sqrt{4^2+0^2}}{\sqrt{1^2+(-1)^2}}$$
$$= \frac{4}{\sqrt{2}}$$
$$= 2\sqrt{2}$$

∴ The value of $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ is $2\sqrt{2}$

6. If $z_1 = (2 - i)$, $z_2 = (-2 + i)$, find

(i) $\mathrm{Re}\left(\frac{z_1 z_2}{\bar{z_1}}\right)$

(ii) $\mathrm{Im}\left(\frac{1}{z_1\bar{z_1}}\right)$

Solution:

Given:

$z_1 = (2 - i)$ and $z_2 = (-2 + i)$

(i) $\mathrm{Re}\left(\frac{z_1 z_2}{\bar{z_1}}\right)$

We shall rationalise the denominator, we get

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{z_1 z_2}{z_1} \times \frac{z_1}{z_1}$$
$$= \frac{(z_1)^2 z_2}{z_1 z_1}$$
$$= \frac{(2-i)^2(-2+i)}{|z_1|^2} \quad [since, z\bar{z} = |z|^2]$$
$$= \frac{(2^2 + i^2 - 2 \times 2 \times i)(-2+i)}{|2-i|^2}$$
$$= \frac{(4-1-4i)(-2+i)}{2^2+(-1)^2}$$
$$= \frac{(3-4i)(-2+i)}{4+i}$$
$$= \frac{3(-2+i)-4i(-2+i)}{4+i}$$
$$= \frac{-6+3i+8i+4}{5}$$
$$= \frac{-2+11i}{5}$$

$\therefore$ *The real value of* $\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ *is* $\frac{-2}{5}$

$$\text{(ii)}\ \mathrm{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$$

$$\frac{1}{z_1\bar{z}_1} = \frac{1}{|z_1|^2}$$
$$= \frac{1}{|2-i|^2}$$
$$= \frac{1}{2^2+(-1)^2}$$
$$= \frac{1}{4+1}$$
$$= \frac{1}{5}$$

$\therefore$ *The imaginary value of* $\left(\frac{1}{z_1\bar{z}_1}\right)$ *is* 0

7. Find the modulus of [(1 + i)/(1 - i)] – [(1 - i)/(1 + i)]
Solution:
Given:
[(1 + i)/(1 - i)] – [(1 - i)/(1 + i)]
So,
Z = [(1 + i)/(1 - i)] – [(1 - i)/(1 + i)]
Let us simplify, we get
= [(1+i) (1+i) – (1-i) (1-i)] / (1^2 – i^2)
= [1^2 + i^2 + 2(1)(i) – (1^2 + i^2 – 2(1)(i))] / (1 – (-1)) [Since, i^2 = -1]
= 4i/2
= 2i
We know that for a complex number Z = (a+ib) it's magnitude is given by |z| = $\sqrt{(a^2 + b^2)}$
So,
|Z| = $\sqrt{(0^2 + 2^2)}$
= 2
∴ The modulus of [(1 + i)/(1 - i)] – [(1 - i)/(1 + i)] is 2.

8. If x + iy = (a+ib)/(a-ib), prove that $x^2 + y^2 = 1$
Solution:
Given:

x + iy = (a+ib)/(a-ib)
We know that for a complex number Z = (a+ib) it's magnitude is given by |z| = √(a^2 + b^2)
So,
|a/b| is |a| / |b|
Applying Modulus on both sides we get,

$$|x+iy| = \left|\frac{a+ib}{a-ib}\right|$$

$$\sqrt{x^2+y^2} = \frac{|a+ib|}{|a-ib|}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+(-b)^2}}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

= 1
Squaring on both sides we get,

$$\left(\sqrt{x^2+y^2}\right)^2 = 1^2$$

$x^2+y^2=1$
∴ Hence Proved.

9. Find the least positive integral value of n for which [(1+i)/(1-i)]^n is real.
Solution:
Given:
[(1+i)/(1-i)]^n
Z = [(1+i)/(1-i)]^n
Now let us multiply and divide by (1+i), we get

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{1^2-i^2}$$

$$= \frac{1^2+i^2+2(1)(i)}{1-(-1)}$$

$$= \frac{1-1+2i}{2}$$

$$= \frac{2i}{2}$$

= i [which is not real]
For n = 2, we have

$[(1+i)/(1-i)]^2 = i^2$
$= -1$ [which is real]
So, the smallest positive integral ‘n’ that can make $[(1+i)/(1-i)]^n$ real is 2.
∴ The smallest positive integral value of ‘n’ is 2.

10. Find the real values of θ for which the complex number (1 + i cos θ) / (1 – 2i cos θ) is purely real.
Solution:
Given:
(1 + i cos θ) / (1 – 2i cos θ)
Z = (1 + i cos θ) / (1 – 2i cos θ)
Let us multiply and divide by (1 + 2i cos θ)

$$= \frac{1+icos\theta}{1-2icos\theta} \times \frac{1+2icos\theta}{1+2icos\theta}$$

$$= \frac{1(1+2icos\theta)+icos\theta(1+2icos\theta)}{1^2-(2icos\theta)^2}$$

$$= \frac{1+2icos\theta+icos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$$

$$= \frac{1+3icos\theta+2(-1)\cos^2\theta}{1-4(-1)\cos^2\theta}$$

$$= \frac{1-2\cos^2\theta+3icos\theta}{1+4\cos^2\theta}$$

For a complex number to be purely real, the imaginary part should be equal to zero.
So,

$$\frac{3cos\theta}{1+4\cos^2\theta} = 0$$

3cos θ = 0 (since, $1 + 4\cos^2\theta \geq 1$)
cos θ = 0
cos θ = cos π/2
θ = [(2n+1)π] / 2, for n ∈ Z
= 2nπ ± π/2, for n ∈ Z
∴ The values of θ to get the complex number to be purely real is 2nπ ± π/2, for n ∈ Z

11. Find the smallest positive integer value of n for which $(1+i)^n / (1-i)^{n-2}$ is a real number.
Solution:
Given:
$(1+i)^n / (1-i)^{n-2}$
$Z = (1+i)^n / (1-i)^{n-2}$

Let us multiply and divide by $(1 - i)^2$

$= \frac{(1+i)^n}{(1-i)^{n-2}} \times \frac{(1-i)^2}{(1-i)^2}$

$= \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2$

$= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n \times \left(1^2 + i^2 - 2(1)(i)\right)$

$= \left(\frac{(1+i)^2}{1^2-i^2}\right)^n \times (1 + i^2 - 2i)$

$= \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^n \times (1 + (-1) - 2i)$

$= \left(\frac{1-1+2i}{2}\right)^n \times (-2i)$

$= \left(\frac{2i}{2}\right)^n \times (-2i)$

$= i^n \times (-2i)$

$= -2i^{n+1}$

For n = 1,

$Z = -2i^{1+1}$

$= -2i^2$

= 2, which is a real number.

∴ The smallest positive integer value of n is 1.

12. If $[(1+i)/(1-i)]^3 – [(1-i)/(1+i)]^3 = x + iy$, find (x, y)

Solution:

Given:

$[(1+i)/(1-i)]^3 – [(1-i)/(1+i)]^3 = x + iy$

Let us rationalize the denominator, we get

$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^3 - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^3 = x + iy$

$\left(\frac{(1+i)^2}{1^2-i^2}\right)^3 - \left(\frac{(1-i)^2}{1^2-i^2}\right)^3 = x + iy$

$\left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^3 - \left(\frac{1^2+i^2-2(1)(i)}{1-(-1)}\right)^3 = x + iy$

$\left(\frac{1-1+2i}{2}\right)^3 - \left(\frac{1-1-2i}{2}\right)^3 = x + iy$

$\left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 = x + iy$

$i^3 – (-i)^3 = x + iy$
$2i^3 = x + iy$
$2i^2.i = x + iy$
$2(-1)I = x + iy$
$-2i = x + iy$
Equating Real and Imaginary parts on both sides we get
$x = 0$ and $y = -2$
∴ The values of x and y are 0 and -2.

13. If $(1+i)^2 / (2-i) = x + iy$, find $x + y$
Solution:
Given:
$(1+i)^2 / (2-i) = x + iy$
Upon expansion we get,

$$\frac{1^2+i^2+2(1)(i)}{2-i} = x + iy$$

$$\frac{1+(-1)+2i}{2-i} = x + iy$$

$$\frac{2i}{2-i} = x + iy$$

Now, let us multiply and divide by (2+i), we get

$$\frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$$

$$\frac{4i+2i^2}{2^2-i^2} = x + iy$$

$$\frac{2(-1)+4i}{4-(-1)} = x + iy$$

$$\frac{-2+4i}{5} = x + iy$$

Let us equate real and imaginary parts on both sides we get,
$x = -2/5$ and $y = 4/5$
so,
$x + y = -2/5 + 4/5$
$= (-2+4)/5$
$= 2/5$
∴ The value of $(x + y)$ is 2/5