

EXERCISE 19.4

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1. Find the sum of the following arithmetic progressions:**(i) 50, 46, 42, to 10 terms****(ii) 1, 3, 5, 7, ... to 12 terms****(iii) 3, $9/2$, 6, $15/2$, ... to 25 terms****(iv) 41, 36, 31, ... to 12 terms****(v) $a+b$, $a-b$, $a-3b$, ... to 22 terms****(vi) $(x - y)^2$, $(x^2 + y^2)$, $(x + y)^2$, ... to n terms****(vii) $(x - y)/(x + y)$, $(3x - 2y)/(x + y)$, $(5x - 3y)/(x + y)$, ... to n terms****Solution:****(i) 50, 46, 42, to 10 terms**

$$n = 10$$

$$\text{First term, } a = a_1 = 50$$

$$\text{Common difference, } d = a_2 - a_1 = 46 - 50 = -4$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = 10/2 (100 + (9) (-4))$$

$$= 5 (100 - 36)$$

$$= 5 (64)$$

$$= 320$$

 \therefore The sum of the given AP is 320.**(ii) 1, 3, 5, 7, ... to 12 terms**

$$n = 12$$

$$\text{First term, } a = a_1 = 1$$

$$\text{Common difference, } d = a_2 - a_1 = 3 - 1 = 2$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = 12/2 (2(1) + (12-1) (2))$$

$$= 6 (2 + (11) (2))$$

$$= 6 (2 + 22)$$

$$= 6 (24)$$

$$= 144$$

 \therefore The sum of the given AP is 144.**(iii) 3, $9/2$, 6, $15/2$, ... to 25 terms**

$$n = 25$$

$$\text{First term, } a = a_1 = 3$$

$$\text{Common difference, } d = a_2 - a_1 = 9/2 - 3 = (9 - 6)/2 = 3/2$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = 25/2 (2(3) + (25-1) (3/2))$$

$$= 25/2 (6 + (24) (3/2))$$

$$= 25/2 (6 + 36)$$

$$= 25/2 (42)$$

$$= 25 (21)$$

$$= 525$$

∴ The sum of the given AP is 525.

(iv) 41, 36, 31, ... to 12 terms

$$n = 12$$

$$\text{First term, } a = a_1 = 41$$

$$\text{Common difference, } d = a_2 - a_1 = 36 - 41 = -5$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = 12/2 (2(41) + (12-1) (-5))$$

$$= 6 (82 + (11) (-5))$$

$$= 6 (82 - 55)$$

$$= 6 (27)$$

$$= 162$$

∴ The sum of the given AP is 162.

(v) $a+b$, $a-b$, $a-3b$, ... to 22 terms

$$n = 22$$

$$\text{First term, } a = a_1 = a+b$$

$$\text{Common difference, } d = a_2 - a_1 = (a-b) - (a+b) = a-b-a-b = -2b$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = 22/2 (2(a+b) + (22-1) (-2b))$$

$$= 11 (2a + 2b + (21) (-2b))$$

$$= 11 (2a + 2b - 42b)$$

$$= 11 (2a - 40b)$$

$$= 22a - 440b$$

∴ The sum of the given AP is $22a - 440b$.

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \dots$ to n terms

$$n = n$$

$$\text{First term, } a = a_1 = (x-y)^2$$

$$\text{Common difference, } d = a_2 - a_1 = (x^2 + y^2) - (x-y)^2 = 2xy$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = n/2 (2(x-y)^2 + (n-1) (2xy))$$

$$= n/2 (2 (x^2 + y^2 - 2xy) + 2xyn - 2xy)$$

$$= n/2 \times 2 ((x^2 + y^2 - 2xy) + xyn - xy)$$

$$= n (x^2 + y^2 - 3xy + xyn)$$

∴ The sum of the given AP is $n (x^2 + y^2 - 3xy + xyn)$.

(vii) $(x - y)/(x + y), (3x - 2y)/(x + y), (5x - 3y)/(x + y), \dots$ to n terms

$$n = n$$

$$\text{First term, } a = a_1 = (x-y)/(x+y)$$

$$\text{Common difference, } d = a_2 - a_1 = (3x - 2y)/(x + y) - (x-y)/(x+y) = (2x - y)/(x+y)$$

By using the formula,

$$S = n/2 (2a + (n - 1) d)$$

Substitute the values of 'a' and 'd', we get

$$S = n/2 (2((x-y)/(x+y)) + (n-1) ((2x - y)/(x+y)))$$

$$= n/2(x+y) \{n (2x-y) - y\}$$

∴ The sum of the given AP is $n/2(x+y) \{n (2x-y) - y\}$

2. Find the sum of the following series:

(i) $2 + 5 + 8 + \dots + 182$

(ii) $101 + 99 + 97 + \dots + 47$

(iii) $(a - b)^2 + (a^2 + b^2) + (a + b)^2 + s\dots + [(a + b)^2 + 6ab]$

Solution:

(i) $2 + 5 + 8 + \dots + 182$

$$\text{First term, } a = a_1 = 2$$

$$\text{Common difference, } d = a_2 - a_1 = 5 - 2 = 3$$

a_n term of given AP is 182

$$a_n = a + (n-1) d$$

$$182 = 2 + (n-1) 3$$

$$182 = 2 + 3n - 3$$

$$182 = 3n - 1$$

$$3n = 182 + 1$$

$$n = 183/3$$

$$= 61$$

Now,

By using the formula,

$$S = n/2 (a + l)$$

$$= 61/2 (2 + 182)$$

$$= 61/2 (184)$$

$$= 61 (92)$$

$$= 5612$$

∴ The sum of the series is 5612

(ii) $101 + 99 + 97 + \dots + 47$

First term, $a = a_1 = 101$

Common difference, $d = a_2 - a_1 = 99 - 101 = -2$

a_n term of given AP is 47

$$a_n = a + (n-1)d$$

$$47 = 101 + (n-1)(-2)$$

$$47 = 101 - 2n + 2$$

$$2n = 103 - 47$$

$$2n = 56$$

$$n = 56/2 = 28$$

Then,

$$S = n/2 (a + l)$$

$$= 28/2 (101 + 47)$$

$$= 28/2 (148)$$

$$= 14 (148)$$

$$= 2072$$

∴ The sum of the series is 2072

(iii) $(a - b)^2 + (a^2 + b^2) + (a + b)^2 + s \dots + [(a + b)^2 + 6ab]$

First term, $a = a_1 = (a-b)^2$

Common difference, $d = a_2 - a_1 = (a^2 + b^2) - (a - b)^2 = 2ab$

a_n term of given AP is $[(a + b)^2 + 6ab]$

$$a_n = a + (n-1)d$$

$$[(a + b)^2 + 6ab] = (a-b)^2 + (n-1)2ab$$

$$a^2 + b^2 + 2ab + 6ab = a^2 + b^2 - 2ab + 2abn - 2ab$$

$$a^2 + b^2 + 8ab - a^2 - b^2 + 2ab + 2ab = 2abn$$

$$12ab = 2abn$$

$$n = 12ab / 2ab$$

$$= 6$$

Then,

$$S = n/2 (a + 1)$$

$$= 6/2 ((a-b)^2 + [(a + b)^2 + 6ab])$$

$$= 3 (a^2 + b^2 - 2ab + a^2 + b^2 + 2ab + 6ab)$$

$$= 3 (2a^2 + 2b^2 + 6ab)$$

$$= 3 \times 2 (a^2 + b^2 + 3ab)$$

$$= 6 (a^2 + b^2 + 3ab)$$

\therefore The sum of the series is $6 (a^2 + b^2 + 3ab)$

3. Find the sum of first n natural numbers.

Solution:

Let AP be 1, 2, 3, 4, ..., n

Here,

First term, $a = a_1 = 1$

Common difference, $d = a_2 - a_1 = 2 - 1 = 1$

$l = n$

So, the sum of n terms = $S = n/2 [2a + (n-1) d]$

$$= n/2 [2(1) + (n-1) 1]$$

$$= n/2 [2 + n - 1]$$

$$= n[n + 1]/2$$

\therefore The sum of the first n natural numbers is $n(n+1)/2$

4. Find the sum of all - natural numbers between 1 and 100, which are divisible by 2 or 5

Solution:

The natural numbers which are divisible by 2 or 5 are:

$$2 + 4 + 5 + 6 + 8 + 10 + \dots + 100 = (2 + 4 + 6 + \dots + 100) + (5 + 15 + 25 + \dots + 95)$$

Now, $(2 + 4 + 6 + \dots + 100) + (5 + 15 + 25 + \dots + 95)$ are AP with common difference of 2 and 10.

So, for the 1st sequence $\Rightarrow (2 + 4 + 6 + \dots + 100)$

$$a = 2, d = 4-2 = 2, a_n = 100$$

By using the formula,

$$a_n = a + (n-1)d$$

$$100 = 2 + (n-1)2$$

$$100 = 2 + 2n - 2$$

$$2n = 100$$

$$n = 100/2$$
$$= 50$$

$$\text{So now, } S = n/2 (2a + (n-1)d)$$
$$= 50/2 (2(2) + (50-1)2)$$
$$= 25 (4 + 49(2))$$
$$= 25 (4 + 98)$$
$$= 2550$$

Again, for the 2nd sequence, $(5 + 15 + 25 + \dots + 95)$

$$a = 5, d = 15 - 5 = 10, a_n = 95$$

By using the formula,

$$a_n = a + (n-1)d$$

$$95 = 5 + (n-1)10$$

$$95 = 5 + 10n - 10$$

$$10n = 95 + 10 - 5$$

$$10n = 100$$

$$n = 100/10$$

$$= 10$$

$$\text{So now, } S = n/2 (2a + (n-1)d)$$
$$= 10/2 (2(5) + (10-1)10)$$
$$= 5 (10 + 9(10))$$
$$= 5 (10 + 90)$$
$$= 500$$

\therefore The sum of the numbers divisible by 2 or 5 is: $2550 + 500 = 3050$

5. Find the sum of first n odd natural numbers.

Solution:

Given an AP of first n odd natural numbers whose first term a is 1, and common difference d is 2

The sequence is 1, 3, 5, 7,n

$$a = 1, d = 3 - 1 = 2, n = n$$

By using the formula,

$$S = n/2 [2a + (n-1)d]$$
$$= n/2 [2(1) + (n-1)2]$$
$$= n/2 [2 + 2n - 2]$$
$$= n/2 [2n]$$
$$= n^2$$

\therefore The sum of the first n odd natural numbers is n^2 .

6. Find the sum of all odd numbers between 100 and 200

Solution:

The series is 101, 103, 105, ..., 199

Let the number of terms be n

So, $a = 101$, $d = 103 - 101 = 2$, $a_n = 199$

$$a_n = a + (n-1)d$$

$$199 = 101 + (n-1)2$$

$$199 = 101 + 2n - 2$$

$$2n = 199 - 101 + 2$$

$$2n = 100$$

$$n = 100/2$$

$$= 50$$

By using the formula,

$$\text{The sum of } n \text{ terms} = S = n/2[a + l]$$

$$= 50/2 [101 + 199]$$

$$= 25 [300]$$

$$= 7500$$

\therefore The sum of the odd numbers between 100 and 200 is 7500.

7. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667**Solution:**

The odd numbers between 1 and 1000 divisible by 3 are 3, 9, 15, ..., 999

Let the number of terms be ' n ', so the n th term is 999

$$a = 3, d = 9 - 3 = 6, a_n = 999$$

$$a_n = a + (n-1)d$$

$$999 = 3 + (n-1)6$$

$$999 = 3 + 6n - 6$$

$$6n = 999 + 6 - 3$$

$$6n = 1002$$

$$n = 1002/6$$

$$= 167$$

By using the formula,

$$\text{Sum of } n \text{ terms, } S = n/2 [a + l]$$

$$= 167/2 [3 + 999]$$

$$= 167/2 [1002]$$

$$= 167 [501]$$

$$= 83667$$

\therefore The sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Hence proved.

8. Find the sum of all integers between 84 and 719, which are multiples of 5**Solution:**

The series is 85, 90, 95, ..., 715

Let there be 'n' terms in the AP

So, $a = 85$, $d = 90 - 85 = 5$, $a_n = 715$

$$a_n = a + (n-1)d$$

$$715 = 85 + (n-1)5$$

$$715 = 85 + 5n - 5$$

$$5n = 715 - 85 + 5$$

$$5n = 635$$

$$n = 635/5$$

$$= 127$$

By using the formula,

$$\text{Sum of } n \text{ terms, } S = n/2 [a + l]$$

$$= 127/2 [85 + 715]$$

$$= 127/2 [800]$$

$$= 127 [400]$$

$$= 50800$$

\therefore The sum of all integers between 84 and 719, which are multiples of 5 is 50800.

9. Find the sum of all integers between 50 and 500 which are divisible by 7**Solution:**

The series of integers divisible by 7 between 50 and 500 are 56, 63, 70, ..., 497

Let the number of terms be 'n'

So, $a = 56$, $d = 63 - 56 = 7$, $a_n = 497$

$$a_n = a + (n-1)d$$

$$497 = 56 + (n-1)7$$

$$497 = 56 + 7n - 7$$

$$7n = 497 - 56 + 7$$

$$7n = 448$$

$$n = 448/7$$

$$= 64$$

By using the formula,

$$\text{Sum of } n \text{ terms, } S = n/2 [a + l]$$

$$= 64/2 [56 + 497]$$

$$= 32 [553]$$

$$= 17696$$

\therefore The sum of all integers between 50 and 500 which are divisible by 7 is 17696.

10. Find the sum of all even integers between 101 and 999**Solution:**

We know that all even integers will have a common difference of 2.

So, AP is 102, 104, 106, ..., 998

We know, $a = 102$, $d = 104 - 102 = 2$, $a_n = 998$

By using the formula,

$$a_n = a + (n-1)d$$

$$998 = 102 + (n-1)2$$

$$998 = 102 + 2n - 2$$

$$2n = 998 - 102 + 2$$

$$2n = 898$$

$$n = 898/2$$

$$= 449$$

By using the formula,

$$\text{Sum of } n \text{ terms, } S = n/2 [a + l]$$

$$= 449/2 [102 + 998]$$

$$= 449/2 [1100]$$

$$= 449 [550]$$

$$= 246950$$

\therefore The sum of all even integers between 101 and 999 is 246950.