1. State which of these fractions have a terminating decimal.
(a) (3/ 5)
(b) (5 / 7)
(c) (25 / 49)
(d) (37 / 40)
(e) (57 / 64)
(f) (59 / 75)
(g) (89 / 125)
(h) (125 / 213)
(i) (147 / 160)
Solutions:
(a) (3 / 5)
$5 = 1 \times 5$
$5 = 2^0 \times 5^1$
i.e, 5 can be expressed as $2^m \times 5^n$
Therefore,
(3 / 5) has terminating decimal representation

(b) (5 / 7)
$7 = 1 \times 7$
i.e, 7 cannot be expressed a $2^m \times 5^n$
Therefore,
(5 / 7) does not have terminating decimal representation

(c) (25 / 49)
$49 = 7 \times 7$
i.e, 49 cannot be expressed as $2^m \times 5^n$
Therefore,
(25 / 49) does not have terminating decimal representation

(d) (37 / 40)
$40 = 2 \times 2 \times 2 \times 5$
$40 = 2^3 \times 5^1$
i.e, 40 can be expressed as $2^m \times 5^n$
Therefore,
(37 / 40) has terminating decimal representation

(e) (57 / 64)
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = 2^6 \times 5^0$
i.e, 64 can be expressed as $2^m \times 5^n$
Therefore,
(57 / 64) has terminating decimal representation

(f) (59 / 75)
$75 = 5 \times 5 \times 3$
$75 = 5^2 \times 3^1$
i.e, 75 cannot be expressed as $2^m \times 5^n$
Therefore,
(59 / 75) does not have terminating decimal representation

(g) (89 / 125)
$125 = 5 \times 5 \times 5$
$125 = 2^0 \times 5^3$
i.e, 125 can be expressed as $2^m \times 5^n$
Therefore,
(89 / 125) has terminating decimal representation

(h) (125 / 213)
$213 = 3 \times 71$
i.e, 213 cannot be expressed as $2^m \times 5^n$
Therefore,
(125 / 213) does not have terminating decimal representation

(i) (147 / 160)
$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5$
$160 = 2^5 \times 5^1$
i.e, 160 can be expressed as $2^m \times 5^n$
Therefore,
(147 / 160) has terminating decimal representation

2. Express each of the following decimals as a rational number
(a) 0.93
(b) 4.56
(c) 0.614
(d) 21.025
Solution:
(a) 0.93 = 93 / 100

Hence,
The rational number of decimal 0.93 is (93 / 100)

(b) 4.56 = (456 / 100)
= (456 ÷ 4) / (100 ÷ 4)
We get,
= (114 / 25)
Hence,
The rational number of decimal 4.56 is (114 / 25)

(c) 0.614 = (614 / 1000)
= (614 ÷ 2) / (1000 ÷ 2)
We get,
= (307 / 500)
Hence,
The rational number of decimal 0.614 is (307 / 500)

(d) 21.025 = (21025 / 1000)
= (21025 ÷ 25) / (1000 ÷ 25)
We get,
= (841 / 40)
Hence,
The rational number of decimal 21.025 is (841 / 40)

3. Convert the following fractions into decimals:
(i) (3 / 5)
(ii) (8 / 11)
(iii) (-2 / 7)
(iv) (12 / 21)
(v) (13 / 25)
(vi) (2 / 3)
Solution:
(i) (3 / 5)
(3 / 5) = 0.6
Hence,
The decimal form of (3 / 5) is 0.6
(ii) (8 / 11)
(8 / 11) = 0.72727272…..
(8 / 11) = $0.\overline{72}$

Hence,
The decimal form of (8 / 11) is $0.\overline{72}$
(iii) (-2 / 7)
(-2 / 7) = -0.285714285714….
(-2 / 7) = - $0.\overline{285714}$
Hence,
The decimal form of (-2 / 7) is - $0.\overline{285714}$
(iv) (12 / 21)
(12 / 21) = 0.571428571428…….
(12 / 21) = $0.\overline{571428}$
Hence,
The decimal form of (12 / 21) is $0.\overline{571428}$
(v) (13 / 25)
(13 / 25) = 0.52
Hence,
The decimal form of (13 / 25) is 0.52
(vi) (2 / 3)
(2 / 3) = 0.6666…..
(2 / 3) = 0.6
Hence,
The decimal form of (2 / 3) is 0.6

4. Express each of the following decimals as a rational number.
(a) 0.7
(b) $0.\overline{35}$
(c) $0.\overline{89}$
(d) $0.\overline{057}$
(e) $0.\overline{763}$
(f) $2.\overline{67}$
(g) $4.6\overline{724}$
(h) $0.0\overline{17}$
(i) $17.02\overline{7}$
Solution:
(a) 0.7
Let x = 0.7
Then,
x = 0.7777….. (1)
Here, the number of digits recurring is only 1,
So, we multiply both sides of the equation (1) by 10

We get,
$10x = 10 \times 0.7777\ldots$ (2)
$10x = 7.777\ldots..$
On subtracting (1) from (2),
We get,
$9x = 7$
$x = (7 / 9)$
$0.7 = (7 / 9)$
Therefore,
$0.7 = (7 / 9)$

(b) $0.\overline{35}$
Let $x = 0.\overline{35}$
Then,
$x = 0.353535\ldots\ldots..$ (1)
Here,
The number of digits recurring is 2,
So, we multiply both sides of equation (1) by 100
We get,
$100x = 100 \times 0.353535\ldots.$
$= 35.3535$ ……. (2)
On subtracting (1) from (2),
We get,
$99x = 35$
$x = (35 / 99)$
Therefore,
$0.\overline{35} = (35 / 99)$

(c) $0.\overline{89}$
Let $x = 0.\overline{89}$
Then,
$x = 0.898989\ldots\ldots\ldots.$ (1)
Here,
The number of digits recurring is 2,
So we multiply both sides of the equation (1) by 100
We get,
$100x = 100 \times 0.898989\ldots\ldots\ldots$
$= 89.8989\ldots.$ (2)
On subtracting (1) from (2),

We get,
99x = 89
x = (89 / 99)
$0.\overline{89}$ = (89 / 99)
Therefore,
$0.\overline{89}$ = (89 / 99)

(d) $0.\overline{057}$
Let x = $0.\overline{057}$
Then,
x = 0.057057…… (1)
Here,
The number of digits recurring is 3,
So, we multiply both sides of the equation (1) by 1000,
We get,
1000x = 1000 × 0.057057……
= 57.057……. (2)
On subtracting (1) from (2),
We get,
999x = 57
x = (57 / 999)
x = (19 / 333)
$0.\overline{057}$ = (19 / 333)
Therefore,
$0.\overline{057}$ = (19 / 333)

(e) $0.\overline{763}$
Let x = $0.\overline{763}$
Then,
x = 0.763763……. (1)
Here,
The number of digits recurring is 3,
So, we multiply both sides of the equation (1) by 1000,
We get,
1000x = 1000 × 0.763763……
= 763.763…….. (2)
On subtracting (1) from (2),
We get,
999x = 763

x = (763 / 999)
$0.\overline{763}$ = (763 / 999)
Therefore,
$0.\overline{763}$ = (763 / 999)

(f) $2.\overline{67}$
Let x = $2.\overline{67}$
Then,
x = 2.676767……. (1)
Here,
The number of digits recurring is 2,
So, we multiply both sides of the equation (1) by 100,
We get,
100x = 100 × 2.676767……..
= 267.6767…….. (2)
On subtracting (1) from (2),
We get,
99x = 265
x = (265 / 99)
$2.\overline{67}$ = (265 / 99)
Therefore,
$2.\overline{67}$ = (265 / 99)

(g) $4.6\overline{724}$
Let x = $4.6\overline{724}$ = 4.6724724…….
Here,
Only numbers 724 is being repeated, so first we need to remove 6 which proceeds 724
We multiply by 10 so that only the recurring digits remain after decimal
Thus,
10x = 10 x 4.6724724…….
10x = 46. 724724……. (1)
The number of digits recurring in equation (1) is 3
Hence, we multiply both sides of the equation (1) by 1000
10000x = 1000 x 46.724724 = 46724.724.. …(2)
On subtracting (1) from (2), we get,
9990x = 46678
x = 46678 / 9990
We get,
x = 23339 / 4995

(h) $0.0\overline{17}$
Let x = $0.0\overline{17}$
= 0.01717…….
Here, only numbers 17 is being repeated, so first we need to remove 0 which proceeds 17
We multiply by 10 so that only the recurring digits remain after decimal,
Hence,
10x = 0.1717…… (1)
The number of digits recurring in equation (1) is 2,
So we multiply both sides of the equation (1) by 100,
Hence,
1000x = 100 × 0.1717……..
= 17.1717….. (2)
On subtracting (1) from (2),
We get,
990x = 17
x = (17 / 990)
$0.0\overline{17}$ = (17 / 990)
Therefore,
$0.0\overline{17}$ = (17 / 990)

(i) $17.02\overline{7}$
Let x = $17.02\overline{7}$
= 17.027777……
Here, only number 7 is being repeated, so first we need to remove 02 which proceeds 7
We multiply by 100 so that only the recurring digits remain after decimal
Hence,
100x = 1702.7777…… (1)
The number of digits recurring in equation (1) is 1,
So we multiply both sides of the equation (1) by 10
Hence,
1000x = 10 × 1702.7777…….
= 17027.777……. (2)
On subtracting (1) from (2),
We get,
900x = 15325
x = (15325 / 900)
x = (613 / 36)
$17.02\overline{7}$ = (613 / 36)
Therefore,

$17.02\overline{7}$ = (613 / 36)

5. Insert a rational number between:
(a) (2 / 5) and (3 / 4)
(b) (3 / 4) and (5 / 7)
(c) (4 / 3) and (7 / 5)
(d) (5 / 9) and (6 / 7)
Solution:
(a) (2 / 5) and (3 / 4)
= [{(2 / 5) + (3 / 4)} / 2]
On further calculation, we get,
= [{(8 + 15) / 20} / 2]
= {(23 / 20) / 2}
We get,
= (23 / 40)
Therefore,
A rational number lying between (2 / 5) and (3 / 4) is (23 / 40)

(b) (3 / 4) and (5 / 7)
= [{(3 / 4) + (5 / 7)} / 2]
On further calculation, we get,
= [{(21 + 20) / 28} / 2]
= {(41 / 28) / 2}
We get,
= (41 / 56)
Therefore,
A rational number lying between (3 / 4) and (5 / 7) is (41 / 56)

(c) (4 / 3) and (7 / 5)
= [{(4 / 3) + (7 / 5)} / 2]
On further calculation, we get,
= [{(20 + 21) / 15} / 2]
= {(41 / 15) / 2}
We get,
= (41 / 30)
Therefore,
A rational number lying between (4 / 3) and (7 / 5) is (41 / 30)

(d) (5 / 9) and (6 / 7)

= [{(5 / 9) + (6 / 7)} / 2]
On further calculation, we get,
= [{(35 + 54) / 63} / 2]
= {(89 / 63) / 2}
We get,
= (89 / 126)
Therefore,
A rational number lying between (5 / 9) and (6 / 7) is (89 / 126)

6. State, whether the following numbers are rational or irrational:
(a) $(3 + \sqrt{3})^2$
(b) $(5 - \sqrt{5})^2$
(c) $(2 + \sqrt{2})(2 - \sqrt{2})$
(d) $\{(\sqrt{5}) / (3\sqrt{2})\}^2$
Solution:
(a) $(3 + \sqrt{3})^2$
$= (3)^2 + (\sqrt{3})^2 + 2 \times 3 \times \sqrt{3}$
On calculating further, we get,
$= 9 + 3 + 6\sqrt{3}$
$= 12 + 6\sqrt{3}$
- which is a rational number
Therefore,
$(3 + \sqrt{3})^2$ is a rational number

(b) $(5 - \sqrt{5})^2$
$= (5)^2 + (\sqrt{5})^2 – 2 \times 5 \times \sqrt{5}$
On further calculation, we get,
$= 25 + 5 - 10\sqrt{5}$
$= 30 - 10\sqrt{5}$
-which is a irrational number
Therefore,
$(5 - \sqrt{5})^2$ is an irrational number

(c) $(2 + \sqrt{2})(2 - \sqrt{2})$
$= (2)^2 – (\sqrt{2})^2$
$= 4 – 2$
$= 2$
-which is a rational number
Therefore,

$(2 + \sqrt{2})(2 - \sqrt{2})$ is a rational number

(d) $\{(\sqrt{5}) / (3\sqrt{2})\}^2$
$= \{(5) / (9 \times 2)\}$
We get,
$= (5 / 18)$
-which is a rational number
Therefore,
$\{(\sqrt{5}) / (3\sqrt{2})\}^2$ is a rational number

7. Check whether the square of the following is rational or irrational:
(a) $3\sqrt{2}$
(b) $3 + \sqrt{2}$
(c) $(3\sqrt{2}) / 2$
(d) $\sqrt{2} + \sqrt{3}$
Solution:
(a) $3\sqrt{2}$
$(3\sqrt{2})^2$
$= 9 \times 2$
$= 18$
-which is a rational number
Hence,
The square of $(3\sqrt{2})$ is a rational number

(b) $3 + \sqrt{2}$
$(3 + \sqrt{2})^2$
$= (3)^2 + (\sqrt{2})^2 + 2 \times 3 \times \sqrt{2}$
On further calculation, we get,
$= 9 + 2 + 6\sqrt{2}$
$= 11 + 6\sqrt{2}$
-which is irrational number
Hence,
The square of $(3 + \sqrt{2})$ is an irrational number

(c) $(3\sqrt{2}) / 2$
$\{(3\sqrt{2}) / 2\}^2$
$= (9 \times 2) / 4$
$= (9 / 2)$
-which is a rational number

Hence,
The square of $\{(3\sqrt{2}) / 2\}$ is a rational number

(d) $\sqrt{2} + \sqrt{3}$
$(\sqrt{2} + \sqrt{3})^2$
$= (\sqrt{2})^2 + (\sqrt{3})^2 + 2 \times \sqrt{2} \times \sqrt{3}$
$= 2 + 3 + 2\sqrt{6}$
We get,
$= 5 + 2\sqrt{6}$
-which is irrational number
Hence,
The square of $(\sqrt{2} + \sqrt{3})$ is an irrational number

8. Show that $\sqrt{5}$ is an irrational number. (Use division method)
Solution:

	2.23606.....
2	5.0000000000....
	-4
42	100
	-84
443	1600
	-1329
4466	27100
	-26796
447206	3040000
	-2683236
	356764.....

Here,
Clearly, $\sqrt{5}$ = 2.23606…… is an irrational number
Therefore,
$\sqrt{5}$ is an irrational number

9. Without using division method show that $\sqrt{7}$ is an irrational number
Solution:
Let $\sqrt{7}$ be a rational number
Hence,

$\sqrt{7} = (a / b)$
On squaring both sides, we get,
$7 = (a^2 / b^2)$
$a^2 = 7b^2$
Since, a^2 is divisible by 7, a is also divisible by 7……. (1)
Let a = 7c
On squaring both sides, we get,
$a^2 = 49c^2$
Substituting $a^2 = 7b^2$
We get,
$7b^2 = 49c^2$
$b^2 = 7c^2$
Since, b^2 is divisible by 7, b is also divisible by 7…… (2)
From (1) and (2) we can observe that both a and b are divisible by 7
i.e, a and b have a common factor 7
This contradicts our assumption that (a / b) is rational number
i.e, a and b do not have any common factor other than unity (1)
Hence,
(a / b) is not rational number
$\sqrt{7}$ is not rational number
Therefore,
$\sqrt{7}$ is an irrational number

10. Write a pair of irrational numbers
(a) ($\sqrt{3}$ + 5) and ($\sqrt{5}$ – 3) whose sum is irrational
(b) ($\sqrt{3}$ + 5) and (4 - $\sqrt{3}$) whose sum is rational
(c) ($\sqrt{3}$ + 2) and ($\sqrt{2}$ – 3) whose difference is irrational
(d) ($\sqrt{5}$ – 3) and ($\sqrt{5}$ + 3) whose difference is rational
(e) (5 + $\sqrt{2}$) and ($\sqrt{5}$ – 2) whose product is irrational
(f) ($\sqrt{3}$ + $\sqrt{2}$) and ($\sqrt{3}$ - $\sqrt{2}$) whose product is rational
Solution:
(a) Given
($\sqrt{3}$ + 5) and ($\sqrt{5}$ – 3) are irrational numbers whose sum is irrational
Thus,
We have,
($\sqrt{3}$ + 5) + ($\sqrt{5}$ – 3)
= $\sqrt{3}$ + 5 + $\sqrt{5}$ – 3
We get,
= $\sqrt{3}$ + $\sqrt{5}$ + 2

-which is irrational number

(b) Given
$(\sqrt{3} + 5)$ and $(4 - \sqrt{3})$ are irrational numbers whose sum is rational
Thus,
We have,
$(\sqrt{3} + 5) + (4 - \sqrt{3})$
$= \sqrt{3} + 5 + 4 - \sqrt{3}$
We get,
$= 9$
-which is a rational number

(c) Given
$(\sqrt{3} + 2)$ and $(\sqrt{2} – 3)$ are irrational numbers whose difference is irrational
Thus,
We have,
$(\sqrt{3} + 2) – (\sqrt{2} – 3)$
$= \sqrt{3} + 2 - \sqrt{2} + 3$
We get,
$= \sqrt{3} - \sqrt{2} + 5$
-which is irrational

(d) Given
$(\sqrt{5} – 3)$ and $(\sqrt{5} + 3)$ are irrational numbers whose difference is rational
Thus,
We have,
$(\sqrt{5} – 3) – (\sqrt{5} + 3)$
$= \sqrt{5} – 3 - \sqrt{5} – 3$
We get,
$= -6$
-which is a rational number

(e) Given
$(5 + \sqrt{2})$ and $(\sqrt{5} – 2)$ are irrational numbers whose product is irrational
Thus,
We have,
$(5 + \sqrt{2}) (\sqrt{5} – 2)$
$= 5 (\sqrt{5} – 2) + \sqrt{2} (\sqrt{5} – 2)$
We get,

$= 5\sqrt{5} – 10 + \sqrt{10} - 2\sqrt{2}$
-which is irrational numbers

(f) Given
$(\sqrt{3} + \sqrt{2})$ and $(\sqrt{3} - \sqrt{2})$ are irrational numbers whose product is rational
Thus,
We have,
$(\sqrt{3} + \sqrt{2}) (\sqrt{3} - \sqrt{2})$
$= (\sqrt{3})^2 – (\sqrt{2})^2$
$= 3 – 2$
We get,
$= 1$
-which is a rational number

11. Simplify by rationalizing the denominator in each of the following:
(a) $(3\sqrt{2}) / \sqrt{5}$
(b) $\{1 / (5 + \sqrt{2})\}$
(c) $\{1 / (\sqrt{3} + \sqrt{2})\}$
(d) $\{2 / (3 + \sqrt{7})\}$
(e) $\{5 / (\sqrt{7} - \sqrt{2})\}$
(f) $\{42 / (2\sqrt{3} + 3\sqrt{2})\}$
(g) $\{(\sqrt{3} + 1) / (\sqrt{3} – 1)\}$
(h) $(\sqrt{5} - \sqrt{7}) / \sqrt{3}$
(i) $(3 - \sqrt{3}) / (2 + \sqrt{2})$
Solution:
(a) $(3\sqrt{2}) / \sqrt{5}$
$= \{(3\sqrt{2}) / \sqrt{5}\} \times (\sqrt{5}) / (\sqrt{5})$
On simplification, we get,
$= \{(3\sqrt{2}) \times \sqrt{5}\} / (\sqrt{5})^2$
We get,
$= (3\sqrt{10}) / 5$

(b) $\{1 / (5 + \sqrt{2})\}$
$= \{1 / (5 + \sqrt{2})\} \times \{(5 - \sqrt{2}) / (5 - \sqrt{2})\}$
On simplification, we get,
$= (5 - \sqrt{2}) / (5)^2 – (\sqrt{2})^2$
$= (5 - \sqrt{2}) / (25 – 2)$
We get,
$= (5 - \sqrt{2}) / 23$

(c) $\{1 / (\sqrt{3} + \sqrt{2})\}$
$= \{1 / (\sqrt{3} + \sqrt{2})\} \times (\sqrt{3} - \sqrt{2}) / (\sqrt{3} - \sqrt{2})$
On simplification, we get,
$= (\sqrt{3} - \sqrt{2}) / (\sqrt{3})^2 – (\sqrt{2})^2$
$= (\sqrt{3} - \sqrt{2}) / (3 – 2)$
$= (\sqrt{3} - \sqrt{2}) / 1$
We get,
$= (\sqrt{3} - \sqrt{2})$

(d) $\{2 / (3 + \sqrt{7})\}$
$= \{2 / (3 + \sqrt{7})\} \times (3 - \sqrt{7}) / (3 - \sqrt{7})$
On further calculation, we get,
$= \{2 (3 - \sqrt{7})\} / (3)^2 – (\sqrt{7})^2$
$= \{2(3 - \sqrt{7})\} / (9 – 7)$
$= \{2(3 - \sqrt{7})\} / 2$
We get,
$= (3 - \sqrt{7})$

(e) $\{5 / (\sqrt{7} - \sqrt{2})\}$
$= \{5 / (\sqrt{7} - \sqrt{2})\} \times (\sqrt{7} + \sqrt{2}) / (\sqrt{7} + \sqrt{2})$
On further calculation, we get,
$= \{5(\sqrt{7} + \sqrt{2})\} / (\sqrt{7})^2 – (\sqrt{2})^2$
$= \{5(\sqrt{7} + \sqrt{2})\} / (7 – 2)$
$= \{5(\sqrt{7} + \sqrt{2})\} / 5$
We get,
$= (\sqrt{7} + \sqrt{2})$

(f) $\{42 / (2\sqrt{3} + 3\sqrt{2})\}$
$= \{42 / (2\sqrt{3} + 3\sqrt{2})\} \times \{(2\sqrt{3} - 3\sqrt{2}) / (2\sqrt{3} - 3\sqrt{2})\}$
$= \{42 (2\sqrt{3} - 3\sqrt{2})\} / (2\sqrt{3})^2 – (3\sqrt{2})^2$
On further calculation, we get,
$= (84\sqrt{3} - 126\sqrt{2}) / (4 \times 3) – (9 \times 2)$
$= (84\sqrt{3} - 126\sqrt{2}) / (12 – 18)$
$= (84\sqrt{3} - 126\sqrt{2}) / -6$
$= -14\sqrt{3} + 21\sqrt{2}$
$= 21\sqrt{2} - 14\sqrt{3}$
We get,
$= 7(3\sqrt{2} - 2\sqrt{3})$

(g) $\{(\sqrt{3} + 1) / (\sqrt{3} – 1)\}$
$= (\sqrt{3} + 1) / (\sqrt{3} – 1) \times (\sqrt{3} + 1) / (\sqrt{3} + 1)$
On further calculation, we get,
$= (\sqrt{3} + 1)^2 / (\sqrt{3})^2 – (1)^2$
$= \{(\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + (1)^2\} / (3 – 1)$
$= (3 + 2\sqrt{3} + 1) / 2$
$= (4 + 2\sqrt{3}) / 2$
We get,
$= 2 + \sqrt{3}$

(h) $(\sqrt{5} - \sqrt{7}) / \sqrt{3}$
$= \{(\sqrt{5} - \sqrt{7}) / \sqrt{3}\} \times (\sqrt{3}) / (\sqrt{3})$
$= (\sqrt{5} \times \sqrt{3} - \sqrt{7} \times \sqrt{3}) / (\sqrt{3})^2$
We get,
$= (\sqrt{15} - \sqrt{21}) / 3$

(i) $(3 - \sqrt{3}) / (2 + \sqrt{2})$
$= (3 - \sqrt{3}) / (2 + \sqrt{2}) \times (2 - \sqrt{2}) / (2 - \sqrt{2})$
$= \{3(2 - \sqrt{2}) - \sqrt{3}(2 - \sqrt{2})\} / (2)^2 – (\sqrt{2})^2$
On further calculation, we get,
$= (6 - 3\sqrt{2} - 2\sqrt{3} + \sqrt{6}) / (4 – 2)$
$= (6 - 3\sqrt{2} - 2\sqrt{3} + \sqrt{6}) / 2$

12. Simplify by rationalizing the denominator in each of the following:
(i) $(5 + \sqrt{6}) / (5 - \sqrt{6})$
(ii) $(4 + \sqrt{8}) / (4 - \sqrt{8})$
(iii) $(\sqrt{15} + 3) / (\sqrt{15} – 3)$
(iv) $(\sqrt{7} - \sqrt{5}) / (\sqrt{7} + \sqrt{5})$
(v) $(3\sqrt{5} + \sqrt{7}) / (3\sqrt{5} - \sqrt{7})$
(vi) $(2\sqrt{3} - \sqrt{6}) / (2\sqrt{3} + \sqrt{6})$
(vii) $(5\sqrt{3} - \sqrt{15}) / (5\sqrt{3} + \sqrt{15})$
(viii) $(2\sqrt{6} - \sqrt{5}) / (3\sqrt{5} - 2\sqrt{6})$
(ix) $(7\sqrt{3} - 5\sqrt{2}) / (\sqrt{48} + \sqrt{18})$
(x) $(\sqrt{12} + \sqrt{18}) / (\sqrt{75} - \sqrt{50})$
Solution:
(i) $(5 + \sqrt{6}) / (5 - \sqrt{6})$
$= (5 + \sqrt{6}) / (5 - \sqrt{6}) \times (5 + \sqrt{6}) / (5 + \sqrt{6})$
On further calculation, we get,

$= (5 + \sqrt{6})^2 / (5)^2 - (\sqrt{6})^2$
$= \{(5)^2 + 2 \times 5 \times \sqrt{6} + (\sqrt{6})^2\} / (25 - 6)$
$= (25 + 10\sqrt{6} + 6) /19$
We get,
$= (31 + 10\sqrt{6}) / 19$

(ii) $(4 + \sqrt{8}) / (4 - \sqrt{8})$
$= (4 + \sqrt{8}) / (4 - \sqrt{8}) \times (4 + \sqrt{8}) / (4 + \sqrt{8})$
On further calculation, we get,
$= (4 + \sqrt{8})^2 / (4)^2 – (\sqrt{8})^2$
$= \{(4)^2 + 2 \times 4 \times \sqrt{8} + (\sqrt{8})^2\} / (16 – 8)$
$= (16 + 8\sqrt{8} + 8) / (16 – 8)$
$= (24 + 8\sqrt{8}) / 8$
We get,
$= 3 + \sqrt{8}$

(iii) $(\sqrt{15} + 3) / (\sqrt{15} – 3)$
$= (\sqrt{15} + 3) / (\sqrt{15} – 3) \times (\sqrt{15} + 3) / (\sqrt{15} + 3)$
$= (\sqrt{15} + 3)^2 / (\sqrt{15})^2 – (3)^2$
$= \{(\sqrt{15})^2 + 2 \times \sqrt{15} \times 3 + (3)^2\} / (15 – 9)$
On further calculation, we get,
$= (15 + 6\sqrt{15} + 9) / 6$
$= (24 + 6\sqrt{15}) / 6$
We get,
$= 4 + \sqrt{15}$

(iv) $(\sqrt{7} - \sqrt{5}) / (\sqrt{7} + \sqrt{5})$
$= (\sqrt{7} - \sqrt{5}) / (\sqrt{7} + \sqrt{5}) \times (\sqrt{7} - \sqrt{5}) / (\sqrt{7} - \sqrt{5})$
$= (\sqrt{7} - \sqrt{5})^2 / (\sqrt{7})^2 – (\sqrt{5})^2$
On further calculation, we get,
$= (7 + 5 - 2\sqrt{35}) / (7 – 5)$
$= (12 - 2\sqrt{35}) / 2$
We get,
$= 6 - \sqrt{35}$

(v) $(3\sqrt{5} + \sqrt{7}) / (3\sqrt{5} - \sqrt{7})$
$= (3\sqrt{5} + \sqrt{7}) / 3\sqrt{5} - \sqrt{7}) \times (3\sqrt{5} + \sqrt{7}) / (3\sqrt{5} + \sqrt{7})$
$= (3\sqrt{5} + \sqrt{7})^2 / (3\sqrt{5})^2 – (\sqrt{7})^2$
On further calculation, we get,

$= \{(3\sqrt{5})^2 + (\sqrt{7})^2 + 2 \times 3\sqrt{5} \times \sqrt{7}\} / (45 - 7)$
$= (45 + 7 + 6\sqrt{35}) / 38$
$= (52 + 6\sqrt{35}) / 38$
We get,
$= (26 + 3\sqrt{35}) / 19$

(vi) $(2\sqrt{3} - \sqrt{6}) / (2\sqrt{3} + \sqrt{6})$
$= (2\sqrt{3} - \sqrt{6}) / (2\sqrt{3} + \sqrt{6}) \times (2\sqrt{3} - \sqrt{6}) / (2\sqrt{3} - \sqrt{6})$
$= (2\sqrt{3} - \sqrt{6})^2 / (2\sqrt{3})^2 - (\sqrt{6})^2$
On further calculation, we get,
$= \{(2\sqrt{3})^2 + (\sqrt{6})^2 - 2 \times 2\sqrt{3} \times \sqrt{6}\} / (4\times 3 - 6)$
$= (12 + 6 - 4\sqrt{18}) / (12 - 6)$
$= (18 - 4\sqrt{18}) / 6$
$= (9 - 2\sqrt{18}) / 3$
$= (9 - 6\sqrt{2}) / 3$
We get,
$= 3 - 2\sqrt{2}$

(vii) $(5\sqrt{3} - \sqrt{15}) / (5\sqrt{3} + \sqrt{15})$
$= (5\sqrt{3} - \sqrt{15}) / 5\sqrt{3} + \sqrt{15}) \times (5\sqrt{3} - \sqrt{15}) / (5\sqrt{3} - \sqrt{15})$
$= (5\sqrt{3} - \sqrt{15})^2 / (5\sqrt{3})^2 - (\sqrt{15})^2$
On further calculation, we get,
$= (75 + 15 - 10\sqrt{45}) / (75 - 15)$
$= (90 - 10\sqrt{45}) / 60$
$= (9 - 1\sqrt{45}) / 6$
$= (9 - 3\sqrt{5}) / 6$
We get,
$= (3 - \sqrt{5}) / 2$

(viii) $(2\sqrt{6} - \sqrt{5}) / (3\sqrt{5} - 2\sqrt{6})$
$= (2\sqrt{6} - \sqrt{5}) / (3\sqrt{5} - 2\sqrt{6}) \times (3\sqrt{5} + 2\sqrt{6}) / (3\sqrt{5} + 2\sqrt{6})$
On simplification, we get,
$= (6\sqrt{30} + 24 - 15 - 2\sqrt{30}) / (3\sqrt{5})^2 - (2\sqrt{6})^2$
$= (6\sqrt{30} + 9 - 2\sqrt{30}) / (45 - 24)$
We get,
$= (4\sqrt{30} + 9) / 21$

(ix) $(7\sqrt{3} - 5\sqrt{2}) / (\sqrt{48} + \sqrt{18})$
$= (7\sqrt{3} - 5\sqrt{2}) / (\sqrt{48} + \sqrt{18}) \times (\sqrt{48} - \sqrt{18}) / (\sqrt{48} - \sqrt{18})$

On simplification, we get,
$= (7\sqrt{144} - 7\sqrt{54} - 5\sqrt{96} + 5\sqrt{36}) / (\sqrt{48})^2 – (\sqrt{18})^2$
$= (84 - 21\sqrt{6} - 20\sqrt{6} + 30) / (48 – 18)$
We get,
$= (114 - 41\sqrt{6}) / 30$

(x) $(\sqrt{12} + \sqrt{18}) / (\sqrt{75} - \sqrt{50})$
$= (\sqrt{12} + \sqrt{18}) / (\sqrt{75} - \sqrt{50}) \times (\sqrt{75} + \sqrt{50}) / (\sqrt{75} + \sqrt{50})$
On further calculation, we get,
$= \{(2\sqrt{3} + 3\sqrt{2}) (5\sqrt{3} + 5\sqrt{2})\} / (\sqrt{75})^2 - (\sqrt{50})^2$
$= (30 + 10\sqrt{6} + 15\sqrt{6} + 30) / (75 – 50)$
$= (60 + 25\sqrt{6}) / 25$
We get,
$= (12 + 5\sqrt{6}) / 5$

13. Simplify each of the following:
(i) $3 / (5 - \sqrt{3}) + 2 / (5 + \sqrt{3})$
(ii) $(4 + \sqrt{5}) / (4 - \sqrt{5}) + (4 - \sqrt{5}) / (4 + \sqrt{5})$
(iii) $(\sqrt{5} – 2) / (\sqrt{5} + 2) – (\sqrt{5} + 2) / (\sqrt{5} – 2)$
(iv) $(\sqrt{7} - \sqrt{3}) / (\sqrt{7} + \sqrt{3}) – (\sqrt{7} + \sqrt{3}) / (\sqrt{7} - \sqrt{3})$
(v) $(\sqrt{5} + \sqrt{3}) / \sqrt{5} - \sqrt{3}) + (\sqrt{5} - \sqrt{3}) / \sqrt{5} + \sqrt{3})$
Solution:
(i) $3 / (5 - \sqrt{3}) + 2 / (5 + \sqrt{3})$
$= \{3(5 + \sqrt{3}) + 2(5 - \sqrt{3})\} / (5 - \sqrt{3}) (5 + \sqrt{3})$
On simplification, we get,
$= (15 + 3\sqrt{3} + 10 - 2\sqrt{3}) / (5)^2 – (\sqrt{3})^2$
$= (25 + \sqrt{3}) / (25 – 3)$
We get,
$= (25 + \sqrt{3}) / 22$

(ii) $(4 + \sqrt{5}) / (4 - \sqrt{5}) + (4 - \sqrt{5}) / (4 + \sqrt{5})$
$= \{(4 + \sqrt{5})^2 + (4 - \sqrt{5})^2\} / (4 - \sqrt{5}) (4 + \sqrt{5})$
On simplification, we get,
$= (16 + 5 + 8\sqrt{5} + 16 + 5 - 8\sqrt{5}) / (4)^2 – (\sqrt{5})^2$
$= (21 + 21) / (16 – 5)$
We get,
$= (42 / 11)$

(iii) $(\sqrt{5} – 2) / (\sqrt{5} + 2) – (\sqrt{5} + 2) / (\sqrt{5} – 2)$

$= (\sqrt{5} – 2)^2 – (\sqrt{5} + 2)^2\} / (\sqrt{5} + 2) (\sqrt{5} – 2)$
On simplification, we get,
$= (5 + 4 - 4\sqrt{5} – 5 - 4 - 4\sqrt{5}) / (\sqrt{5})^2 – (2)^2$
$= - 8\sqrt{5} / (5 – 4)$
We get,
$= - 8\sqrt{5}$

(iv) $(\sqrt{7} - \sqrt{3}) / (\sqrt{7} + \sqrt{3}) – (\sqrt{7} + \sqrt{3}) / (\sqrt{7} - \sqrt{3})$
$= (\sqrt{7} - \sqrt{3})^2 - (\sqrt{7} + \sqrt{3})^2 / (\sqrt{7} + \sqrt{3}) (\sqrt{7} - \sqrt{3})$
On simplification, we get,
$= (7 + 3 - 2\sqrt{21} – 7 – 3 - 2\sqrt{21}) / (\sqrt{7})^2 – (\sqrt{3})^2$
$= - 4\sqrt{21} / (7 – 3)$
$= (- 4\sqrt{21}) / 4$
We get,
$= - \sqrt{21}$

(v) $(\sqrt{5} + \sqrt{3}) / \sqrt{5} - \sqrt{3}) + (\sqrt{5} - \sqrt{3}) / \sqrt{5} + \sqrt{3})$
$= (\sqrt{5} + \sqrt{3})^2 + (\sqrt{5} - \sqrt{3})^2 / (\sqrt{5} - \sqrt{3}) (\sqrt{5} + \sqrt{3})$
On simplification, we get,
$= (5 + 3 + 2\sqrt{15} + 5 + 3 - 2\sqrt{15}) / (5 – 3)$
$= 16 / 2$
We get,
$= 8$

14. Simplify the following:
(i) $\sqrt{6} / (\sqrt{2} + \sqrt{3}) + 3\sqrt{2} / (\sqrt{6} + \sqrt{3}) - 4\sqrt{3} / (\sqrt{6} + \sqrt{2})$
(ii) $3\sqrt{2} / (\sqrt{6} - \sqrt{3}) - 4\sqrt{3} / (\sqrt{6} - \sqrt{2}) + 2\sqrt{3} / (\sqrt{6} + 2)$
(iii) $6 / (2\sqrt{3} - \sqrt{6}) + \sqrt{6} / (\sqrt{3} + \sqrt{2}) - 4\sqrt{3} / (\sqrt{6} - \sqrt{2})$
(iv) $7\sqrt{3} / (\sqrt{10} + \sqrt{3}) - 2\sqrt{5} / (\sqrt{6} + \sqrt{5}) - 3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})$
(v) $4\sqrt{3} / (2 - \sqrt{2}) – 30 / (4\sqrt{3} - 3\sqrt{2}) - 3\sqrt{2} / (3 + 2\sqrt{3})$
Solution:
(i) $\sqrt{6} / (\sqrt{2} + \sqrt{3}) + 3\sqrt{2} / (\sqrt{6} + \sqrt{3}) - 4\sqrt{3} / (\sqrt{6} + \sqrt{2})$
Rationalizing the denominator of each term, we have
$= \{\sqrt{6} (\sqrt{2} - \sqrt{3}) / (\sqrt{2} + \sqrt{3}) (\sqrt{2} - \sqrt{3})\} + \{(3\sqrt{2} (\sqrt{6} - \sqrt{3}) / (\sqrt{6} + \sqrt{3}) (\sqrt{6} - \sqrt{3})\} - \{(4\sqrt{3} (\sqrt{6} - \sqrt{2}) / (\sqrt{6} + \sqrt{2}) (\sqrt{6} - \sqrt{2})\}$
On further calculation, we get,
$= \{(\sqrt{12} - \sqrt{18}) / (2 – 3)\} + \{(3\sqrt{12} - 3\sqrt{6}) / (6 – 3)\} – \{(4\sqrt{18} - 4\sqrt{6}) / (6 – 2)\}$
$= \{(\sqrt{12} - \sqrt{18}) / -1\} + \{(3\sqrt{12} - 3\sqrt{6}) / 3\} – \{(4\sqrt{18} - 4\sqrt{6}) / 4\}$
$= \sqrt{18} - \sqrt{12} + \sqrt{12} - \sqrt{6} - \sqrt{18} + \sqrt{6}$

We get,
= 0

(ii) $3\sqrt{2} / (\sqrt{6} - \sqrt{3}) - 4\sqrt{3} / (\sqrt{6} - \sqrt{2}) + 2\sqrt{3} / (\sqrt{6} + 2)$
Rationalizing the denominator of each term, we have,
$= \{3\sqrt{2} (\sqrt{6} + \sqrt{3}) / (\sqrt{6} - \sqrt{3}) (\sqrt{6} + \sqrt{3})\} – \{(4\sqrt{3} (\sqrt{6} + \sqrt{2}) / (\sqrt{6} - \sqrt{2}) (\sqrt{6} + \sqrt{2})\} + \{(2\sqrt{3} (\sqrt{6} – 2) / (\sqrt{6} + 2) (\sqrt{6} – 2)\}$
On further calculation, we get,
$= \{(3\sqrt{12} + 3\sqrt{6}) / (6 – 3)\} – \{(4\sqrt{18} + 4\sqrt{6}) / (6 – 2)\} + \{(2\sqrt{18} - 4\sqrt{3}) / (6 – 4)\}$
$= \{(3\sqrt{12} + 3\sqrt{6}) / 3\} – \{(4\sqrt{18} + 4\sqrt{6}) / 4\} + \{(2\sqrt{18} - 4\sqrt{3}) / 2\}$
$= \sqrt{12} + \sqrt{6} - \sqrt{18} - \sqrt{6} + \sqrt{18} - 2\sqrt{3}$
$= \sqrt{12} - 2\sqrt{3}$
$= 2\sqrt{3} - 2\sqrt{3}$
We get,
= 0

(iii) $6 / (2\sqrt{3} - \sqrt{6}) + \sqrt{6} / (\sqrt{3} + \sqrt{2}) - 4\sqrt{3} / (\sqrt{6} - \sqrt{2})$
Rationalizing the denominator of each term, we have
$= \{6 (2\sqrt{3} + \sqrt{6}) / (2\sqrt{3} - \sqrt{6}) (2\sqrt{3} + \sqrt{6})\} + \{(\sqrt{6} (\sqrt{3} - \sqrt{2}) / (\sqrt{3} + \sqrt{2}) (\sqrt{3} - \sqrt{2})\} – \{(4\sqrt{3} (\sqrt{6} + \sqrt{2}) / (\sqrt{6} - \sqrt{2}) (\sqrt{6} + \sqrt{2})$
On simplification, we get,
$= (12\sqrt{3} + 6\sqrt{6}) / (12 – 6) + (\sqrt{18} - \sqrt{12}) / (3 – 2) – (4\sqrt{18} + 4\sqrt{6}) / (6 – 2)$
$= \{(12\sqrt{3} + 6\sqrt{6}) / 6\} + \{(\sqrt{18} - \sqrt{12}) / 1\} – \{(4\sqrt{18} + 4\sqrt{6}) / 4\}$
$= 2\sqrt{3} + \sqrt{6} + \sqrt{18} - \sqrt{12} - \sqrt{18} - \sqrt{6}$
$= 2\sqrt{3} - \sqrt{12}$
$= 2 \sqrt{3} - 2\sqrt{3}$
We get,
= 0

(iv) $7\sqrt{3} / (\sqrt{10} + \sqrt{3}) – 2 \sqrt{5} / (\sqrt{6} + \sqrt{5}) - 3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})$
Rationalizing the denominator of each term, we have,
$= \{7\sqrt{3} (\sqrt{10} - \sqrt{3}) / (\sqrt{10} + \sqrt{3}) (\sqrt{10} - \sqrt{3})\} – \{2\sqrt{5} (\sqrt{6} - \sqrt{5}) / (\sqrt{6} + \sqrt{5}) (\sqrt{6} - \sqrt{5})\} – \{3\sqrt{2} (\sqrt{15} - 3\sqrt{2}) / (\sqrt{15} + 3\sqrt{2}) (\sqrt{15} - 3\sqrt{2})$
$= \{(7\sqrt{30} – 21) / (10 – 3)\} – \{(2\sqrt{30} – 10) / (6 – 5)\} – \{(3\sqrt{30} – 18)/ (15 – 18)\}$
$= (7\sqrt{30} – 21) / 7 – (2\sqrt{30} – 10) / 1 – (3\sqrt{30} – 18) / -3$
$= (7\sqrt{30} – 21) / 7 – (2\sqrt{30} – 10) / 1 + (3\sqrt{30} – 18) / 3$
$= \sqrt{30} – 3 - 2\sqrt{30} + 10 + \sqrt{30} – 6$
We get,
= 1

(v) $4\sqrt{3} / (2 - \sqrt{2}) – 30 / (4\sqrt{3} - 3\sqrt{2}) - 3\sqrt{2} / (3 + 2\sqrt{3})$
Rationalizing the denominator of each term, we have,
$= \{(4\sqrt{3} (2 + \sqrt{2}) / (2 - \sqrt{2}) (2 + \sqrt{2})\} – \{30 (4\sqrt{3} + 3\sqrt{2}) / (4\sqrt{3} - 3\sqrt{2}) (4\sqrt{3} + 3\sqrt{2})\} – \{(3\sqrt{2} (3 - 2\sqrt{3}) / (3 + 2\sqrt{3}) (3 - 2\sqrt{3})$
$= \{(8\sqrt{3} + 4\sqrt{6}) / (4 – 2)\} – \{(120\sqrt{3} + 90\sqrt{2}) / (48 – 18)\} – \{(9\sqrt{2} - 6\sqrt{6}) / (9 – 12)\}$
$= \{(8\sqrt{3} + 4\sqrt{6}) / 2\} – \{120\sqrt{3} + 90\sqrt{2}) / 30\} – \{(9\sqrt{2} - 6\sqrt{6}) / -3\}$
$= \{(8\sqrt{3} + 4\sqrt{6}) / 2\} – \{(120\sqrt{3} + 90\sqrt{2}) / 30\} + \{(9\sqrt{2} - 6\sqrt{6}) / 3\}$
$= 4\sqrt{3} + 2\sqrt{6} - 4\sqrt{3} - 3\sqrt{2} + 3\sqrt{2} - 2\sqrt{6}$
We get,
$= 0$

15. If $(\sqrt{2.5} - \sqrt{0.75}) / (\sqrt{2.5} + \sqrt{0.75}) = p + q\sqrt{30}$, find the values of p and q.
Solution:
Given
$(\sqrt{2.5} - \sqrt{0.75}) / (\sqrt{2.5} + \sqrt{0.75})$
$= \{(\sqrt{2.5} - \sqrt{0.75}) / (\sqrt{2.5} + \sqrt{0.75})\} \times \{(\sqrt{2.5} - \sqrt{0.75}) / (\sqrt{2.5} - \sqrt{0.75})\}$
$= (\sqrt{2.5} - \sqrt{0.75})^2 / (\sqrt{2.5})^2 – (\sqrt{0.75})^2$
On simplification, we get,
$= (2.5 – 2 \times \sqrt{2.5} \times \sqrt{0.75} + 0.75) / (2.5 – 0.75)$
$= (3.25 – 2 \times \sqrt{0.25} \times 10 \times \sqrt{0.25} \times 3) / 1.75$
$= (3.25 – 2 \times 0.25\sqrt{30}) / 1.75$
$= (3.25 – 0.5\sqrt{30}) / 1.75$
$= (3.25) / (1.75) – (0.5) / (1.75) \sqrt{30}$
$= (325 / 175) – (50 / 175) \sqrt{30}$
$= (13 / 7) – (2 / 7) \sqrt{30}$
$= (13 / 7) + (- 2 / 7) \sqrt{30}$
$= p + q\sqrt{30}$
Therefore,
The value of $p = (13 / 7)$ and $q = (- 2 / 7)$