1. Prove the following identities:

(i) $(1 - \sin^2\theta)\sec^2\theta = 1$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (1 - \sin^2\theta)\sec^2\theta$

We know that, $(1 - \sin^2\theta) = \cos^2\theta$

$= \cos^2\theta \sec^2\theta$

Also we know that, $\sec^2\theta = 1/\cos^2\theta$

$= \cos^2\theta \times (1/\cos^2\theta)$

$= 1$

Then, Right Hand Side (RHS) = 1

Therefore, LHS = RHS

(ii) $(1 - \cos^2\theta)\sec^2\theta = \tan^2\theta$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (1 - \cos^2\theta)\sec^2\theta$

We know that, $(1 - \cos^2\theta) = \sin^2\theta$

$= \sin^2\theta \sec^2\theta$

Also we know that, $\sec^2\theta = 1/\cos^2\theta$

$= \sin^2\theta \times (1/\cos^2\theta)$

$= \sin^2\theta/\cos^2\theta$

$= \tan^2\theta$

Then, Right Hand Side (RHS) $= \tan^2\theta$

Therefore, LHS = RHS

(iii) tan A + cot A = sec A cosec A

Solution:-

From the question first we consider Left Hand Side (LHS),

= tan A + cot A

We know that, tan A = sin A/cos A, cot A = cos A/ sin A

Then,

= (sin A/cos A) + (cos A/sin A)

$= (\sin^2 A + \cos^2 A)/(\sin A \cos A)$

Also we know that, $\sin^2 A + \cos^2 A = 1$

= 1/(sin A cos A)

= (1/sin A)(1/cos A)

= cosec A sec A
Then, Right Hand Side (RHS) = sec A cosec A
Therefore, LHS = RHS

(iv) sin θ(1 + tan θ) + cos θ (1 + cot θ) = sec θ + cosec θ
Solution:-
From the question first we consider Left Hand Side (LHS),
= sin θ(1 + tan θ) + cos θ (1 + cot θ)
We know that, tan θ = sin θ/cos θ, cot θ = cos θ/sin θ
= sin θ(1 + (sin θ/cos θ)) + cos θ (1 + (cos θ/sin θ))
= sin θ((cos θ + sin θ)/cos θ)) + cos θ ((sin θ + cos θ)/sin θ))
= cos θ + sin θ ((sin θ/cos θ) + (cos θ/sin θ))
= cos θ + sin θ ((1/sin θ)(1/cos θ))
= sec θ + cosec θ
Then, Right Hand Side (RHS) = sec θ + cosec θ
Therefore, LHS = RHS

(v) (1 + cot θ – cosec θ) (1 + tan θ + sec θ) = 2
Solution:-
From the question first we consider Left Hand Side (LHS),
= (1 + cot θ – cosec θ) (1 + tan θ + sec θ)
We know that,
cot θ = sin θ/cos θ, cosec θ = 1/cos θ, tan θ = cos θ/sin θ, sec θ = 1/sin θ
= (1 + (sin θ/cos θ) + (1/cos θ)) (1 + (cos θ/sin θ) – (1/sin θ))
Taking LCM we get,
= ((cos θ + sin θ + 1)/cos θ) ((sin θ + cos θ - 1)/sin θ)
= ((sin θ + cos θ)^2 – 1^2)/(sin θ cos θ)
= (1 + 2 sin θ cos θ - 1)/(sin θ cos θ)
= (2 sin θ cos θ)/(sin θ cos θ)
By simplification we get,
= 2
Then, Right Hand Side (RHS) = 2
Therefore, LHS = RHS

(vi) sin θ cot θ + sin θ cosec θ = 1 + cos θ
Solution:-
From the question first we consider Left Hand Side (LHS),

= $\sin \theta \cot \theta + \sin \theta \operatorname{cosec} \theta$
We know that, $\cot \theta = \cos \theta/\sin \theta$, $\operatorname{cosec} \theta = 1/\sin \theta$
= $\sin \theta (\cos \theta/\sin \theta) + \sin \theta (1/\sin \theta)$
= $\cos \theta + 1$
Then, Right Hand Side (RHS) = $1 + \cos \theta$
Therefore, LHS = RHS

(vii) sec A (1 – sin A) (sec A + tan A) = 1
Solution:-
From the question first we consider Left Hand Side (LHS),
= sec A (1 – sin A) (sec A + tan A)
We know that, sec A = 1/cos A, tan A = sin A/cos A
= (1/cos A) (1 – sin A) ((1/cos A) + (sin A/cos A))
= ((1 – sin A)/cos A) ((1 + sin A)/cos A)
By simplification we get,
= $(1 - \sin^2 A)/\cos^2 A$
= $\cos^2 A/\cos^2 A$
= 1
Then, Right Hand Side (RHS) = 1
Therefore, LHS = RHS

(viii) sec A (1 + sin A) (sec A - tan A) = 1
Solution:-
From the question first we consider Left Hand Side (LHS),
= sec A (1 + sin A) (sec A - tan A)
We know that, sec A = 1/cos A, tan A = sin A/cos A
= (1/cos A) (1 + sin A) ((1/cos A) - (sin A/cos A))
= ((1 + sin A)/cos A) ((1 - sin A)/cos A)
By simplification we get,
= $(1 - \sin^2 A)/\cos^2 A$
= $\cos^2 A/\cos^2 A$
= 1
Then, Right Hand Side (RHS) = 1
Therefore, LHS = RHS

(ix) cosec θ (1 + cos θ) (cosec θ – cot θ) = 1
Solution:-

From the question first we consider Left Hand Side (LHS),
= cosec θ (1 + cos θ) (cosec θ – cot θ)
We know that, cosec = 1/sin θ, cot θ = cos θ/sin θ
= (1/sin θ) (1 + cos θ) ((1/sin θ) – (cos θ/sin θ))
= ((1 + cos θ)/sin θ) ((1 – cos θ)/sin θ)
= ((1 – $\cos^2$ θ)/$\sin^2$ θ)
= $\sin^2$ θ/$\sin^2$ θ
= 1
Then, Right Hand Side (RHS) = 1
Therefore, LHS = RHS

(x) (sec A – 1)/(sec A + 1) = (1 – cos A)/(1 + cos A)
Solution:-
From the question first we consider Left Hand Side (LHS),
= (sec A – 1)/(sec A + 1)
We know that, sec A = 1/cos A
= ((1/cos A) – 1)/((1/cos A) + 1)
By simplification we get,
= (1 – cos A)/(1 + cos A)
Then, Right Hand Side (RHS) = (1 – cos A)/(1 + cos A)
Therefore, LHS = RHS

(xi) (1 + sin A)/(1 – sin A) = (cosec A + 1)/(cosec A - 1)
Solution:-
From the question first we consider Left Hand Side (LHS),
= (1 + sin A)/(1 – sin A)
Then consider Right Hand Side (RHS) = (cosec A + 1)/(cosec A - 1)
We know that, cosec A = 1/sin A
So,
= ((1/sin A) + 1)/((1/sin A) - 1)
= (1 + sin A)/(1 – sin A)
Therefore, LHS = RHS

(xii) cos A/(1 + sin A) = sec A – tan A
Solution:-
From the question first we consider Right Hand Side (RHS),
= sec A – tan A

We know that, sec A = 1/cos A, tan A = sin A/cos A
= (1/cos A) – (sin A/cos A)
= (1 – sin A)/cos A
= ((1 – sin A)/cos A) ((1 + sin A)/(1 + sin A))
By cross multiplication we get,
= (1 – $\sin^2$ A)/(cos A(1 + sin A))
= $\cos^2$ A/(cos A (1 + sin A))
= cos A/(1 + sin A)
Then, Left Hand Side (LHS) = cos A/(1 + sin A)
Therefore, LHS = RHS

(xiii) (tan θ + sec θ - 1)/(tan θ – sec + 1) = (1 + sin θ)/cos θ
Solution:-
From the question first we consider Left Hand Side (LHS),
= (tan θ + sec θ - 1)/(tan θ – sec + 1)
The above terms can be written as,
= (tan θ + sec θ – ($\sec^2$ θ – $\tan^2$ θ))/(1 + tan θ – sec θ)
= [tan θ + sec θ – {(sec θ + tan θ) (sec θ – tan θ)}]/(1 + tan θ – sec θ)
= [(tan θ + sec θ) {1 – (sec θ – tan θ)}]/(1 + tan θ – sec θ)
= [(tan θ + sec θ) (1 + tan θ – sec θ)]/(1 + tan θ – sec θ)
= [tan θ + sec θ]
= (1 + sin θ)/ cos θ
Then, Right Hand Side (RHS) = (1 + sin θ)/cos θ
Therefore, LHS = RHS

2. Prove the following identities:
(i) $\sin^2$ A $\cos^2$ B – $\cos^2$ A $\sin^2$ B = $\sin^2$ A – $\sin^2$ B
Solution:-
From the question first we consider Left Hand Side (LHS),
= $\sin^2$ A $\cos^2$ B – $\cos^2$ A $\sin^2$ B
We know that, $\cos^2$ B = (1 – $\sin^2$ B),
= $\sin^2$ A (1 – $\sin^2$ B) – (1 – $\sin^2$ A) $\sin^2$ B
= $\sin^2$ A – $\sin^2$ A $\sin^2$ B – $\sin^2$ B + $\sin^2$ A $\sin^2$ B
On simplification we get,
= $\sin^2$ A – $\sin^2$ B
Then, Right Hand Side (RHS) = $\sin^2$ A – $\sin^2$ B
Therefore, LHS = RHS

(ii). $(1 - \tan A)^2 + (1 + \tan A)^2 = 2 \sec^2 A$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (1 - \tan A)^2 + (1 + \tan A)^2$

Expanding the above terms we get,

$= 1 + \tan^2 A - 2 \tan A + 1 + \tan^2 A + 2 \tan A$

$= 2(1 + \tan^2 A)$

$= 2 \sec^2 A$

Then, Right Hand Side (RHS) = $2 \sec^2 A$

Therefore, LHS = RHS

(iii) $\text{cosec}^4 A - \text{cosec}^2 A = \cot^4 A + \cot^4 A$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= \text{cosec}^4 A - \text{cosec}^2 A$

By taking common we get,

$= \text{cosec}^2 A (\text{cosec}^2 A - 1)$

Now we consider Right Hand Side (RHS) = $\cot^4 A + \cot^4 A$

Again taking common we get,

$= \cot^2 A (\cot^2 A + 1)$

We know that, $\cot^2 A = \text{cosec}^2 A - 1$, $\cot^2 A + 1 = \text{cosec}^2 A$

So, $(\text{cosec}^2 A - 1) \text{cosec}^2 A$

Therefore, LHS = RHS

(iv) $\sec^2 A + \text{cosec}^2 A = \sec^2 A \text{ cosec}^2 A$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= \sec^2 A + \text{cosec}^2 A$

We know that, $\sec^2 A = 1/\cos^2 A$, $\text{cosec}^2 A = 1/\sin^2 A$

$= (1/\cos^2 A) + (1/\sin^2 A)$

$= (\sin^2 A + \cos^2 A)/(\cos^2 A \sin^2 A)$

$= 1/(\cos^2 A \sin^2)$

$= \sec^2 A \text{ cosec}^2 A$

Then, Right Hand Side (RHS) = $\sec^2 A \text{ cosec}^2 A$

Therefore, LHS = RHS

(v) $\cos^4 A - \sin^4 A = 2 \cos^2 A - 1$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= \cos^4 A - \sin^4 A$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - (1 - \cos^2 A))$

$= 2\cos^2 A - 1$

Then, Right Hand Side (RHS) $= 2\cos^2 A - 1$

Therefore, LHS = RHS

(vii) $(\sec A - \cos A)(\sec A + \cos A) = \sin^2 A + \tan^2 A$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (\sec A - \cos A)(\sec A + \cos A)$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$= (\sec^2 A - \cos^2 A)$

Also we know that, $\sec^2 A = 1 + \tan^2 A$, $\cos^2 A = 1 - \sin^2 A$

$= 1 + \tan^2 A - (1 - \sin^2 A)$

$= \tan^2 A + \sin^2 A$

Then, Right Hand Side (RHS) $= \tan^2 A + \sin^2 A$

Therefore, LHS = RHS

(viii) $(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (\cos A + \sin A)^2 + (\cos A - \sin A)^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$

$= \cos^2 A + \sin^2 A + 2\cos A \sin A + \cos^2 A + \sin^2 A - 2\cos A \sin A$

By simplification we get,

$= 2(\cos^2 A + \sin^2 A)$

$= 2$

Then, Right Hand Side (RHS) = 2

Therefore, LHS = RHS

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

Solution:-

From the question first we consider Left Hand Side (LHS),

= (cosec A – sin A) (sec A – cos A) (tan A + cot A)
We know that, cosec A = 1/sin A, sec A = 1/cos A, cot A = 1/tan A
= ((1/sin A) - sin A) ((1/cos A) – cos A) (tan A + (1/tan A))
= ((1 – $\sin^2$ A)/sin A) ((1 – $\cos^2$ A)/cos A) ((sin A/cos A) + (cos A/sin A))
= ($\cos^2$ A/sin A) ($\sin^2$ A/cos A) (($\sin^2$ A + $\cos^2$ A)/(sin A cos A))
By simplification we get,
= 1
Then, Right Hand Side (RHS) = 1
Therefore, LHS = RHS

(x) $\sec^2$ A cosec^2 A = $\tan^2$ A + $\cot^2$ A + 2
Solution:-
From the question first we consider Left Hand Side (LHS),
= $\sec^2$ A cosec^2 A
We know that, $\sec^2$ A = 1/$\cos^2$ A, cosec^2 A = 1/$\sin^2$ A
= 1/($\cos^2$ A $\sin^2$ A)
Now consider RHS = $\tan^2$ A + $\cot^2$ A + 2
= $\tan^2$ A + $\cot^2$ A + 2 $\tan^2$ A $\cot^2$ A
= $(\tan A + \cot A)^2$
= $((\sin A/\cos A) + (\cos A/\sin A))^2$
= (($\sin^2$ A + $\cos^2$ A)/(sin A cos A))
= 1/($\cos^2$ A $\sin^2$ A)
Therefore, LHS = RHS

(xii) (cos A/(1 – tan A)) + (sin A/(1 – cot A)) = sin A + cos A
Solution:-
From the question first we consider Left Hand Side (LHS),
= (cos A/(1 – tan A)) + (sin A/(1 – cot A))
= (cos A/(1 – (sin A/cos A))) + (sin A/(1 – (cos A/sin A)))
= (cos A/((cos A – sin A)/cos A)) + (sin A/((sin A – cos A)/sin A))
= ($\cos^2$A/(cos A – sin A)) + ($\sin^2$ A/(sin A – cos A))
= ($\cos^2$ A – $\sin^2$ A)/(cos A – sin A)
= sin A + cos A
Then, Right Hand Side (RHS) = sin A + cos A
Therefore, LHS = RHS

(xiii) (cosec A – sin A) (sec A – cos A) = 1/(tan A + cot A)

Solution:-

From the question first we consider Left Hand Side (LHS),

= (cosec A – sin A) (sec A – cos A)

We know that, cosec A = 1/sin A, sec A = 1/cos A

= ((1/sin A) – sin A) ((1/cos) – cos A)

= $((1 - \sin^2 A)/\sin A)((1 - \cos^2 A)/\cos A)$

= $(\cos^2 A/\sin A)(\sin^2 A/\cos A)$

= cos A sin A

Now consider the RHS = 1/(tan A + cot A)

= (1/((sin A/cos A)+(cos A/sin A))

= $1/((\sin^2 A + \cos^2 A)/(\sin A \cos A))$

= cos A sin A

Therefore, LHS = RHS

(xiv) $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

Solution:-

From the question first we consider Left Hand Side (LHS),

= $\sin^4 A + \cos^4 A$

= $1 - 2\sin^2 A \cos^2 A$

= $\sin^4 A + \cos^4 A + 2\sin^2 A \cos^2 A$

= $(\sin^2 A)^2 + (\cos^2 A)^2 + 2\sin^2 A \cos^2 A - 2\sin^2 A\cos^2 A$

[Adding and subtracting $2\sin^2 A \cos^2 A$]

= $(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A$

= $1 - 2\sin^2 A \cos^2 A$

Then, Right Hand Side (RHS) = $1 - 2\sin^2 A \cos^2 A$

Therefore, LHS = RHS

3. Prove the following identities:

(i) (sin A/(1 + cos A)) + ((1 + cos A)/sin A) = 2 cosec A

Solution:-

From the question first we consider Left Hand Side (LHS),

= (sin A/(1 + cos A)) + ((1 + cos A)/sin A)

By cross multiplication we get,

= $((1 + \cos A)^2 + \sin^2 A)/(\sin A (1 + \cos))$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

Then,

= $(1 + 2\cos A + \cos^2 A + \sin^2 A)/(\sin A (1 + \cos A)$

= (2 + 2 cos A)/(sin A (1 + cos A))
Now taking common outside we get,
= (2(1 + cos A))/(sin A (1 + cos A))
= 2 cosec A
Then, Right Hand Side (RHS) = 2 cosec A
Therefore, LHS = RHS

(ii) (1 + cos A)/(1 – cos A) = (cosec A + cot A)^2
Solution:-
From the question first we consider Left Hand Side (LHS),
= (1 + cos A)/(1 – cos A)
Now, multiply and divide by (1 + cos A) we get,
= ((1 + cos A)/(1 – cos A)) ((1 + cos A)/(1 + cos A))
By cross multiplication we get,
= (1 + cos A)^2/(1 – cos^2 A)
We know that, sin^2 A + cos^2 A = 1
So, (1 + cos A)^2/sin^2 A
= ((1 + cos A)/sin A)^2
= ((1/sin A) + (cos A/sin A))^2
= (cosec A + cot A)^2
Then, Right Hand Side (RHS) = (cosec A + cot A)^2
Therefore, LHS = RHS

(iii) (cot A + tan B)/(cot B + tan A) = cot A tan B
Solution:-
From the question first we consider Left Hand Side (LHS),
= (cot A + tan B)/(cot B + tan A)
We know that, cot A = 1/tan A
= ((1/tan A) + (tan B))/((1/tan A) + tan A)
= ((1 + tan A tan B)/tan A)/((1 + tan A tan B)/tan B)
= ((1 + tan A tan B)/tan A) (tan B/(1 + tan A tan B))
= tan B/tan A
= (1/tan A) tan B
= cot A tan B
Then, Right Hand Side (RHS) = cot A tan B
Therefore, LHS = RHS

(iv) 1/(tan A + cot A) = sin A cos A
Solution:-
From the question first we consider Left Hand Side (LHS),
= 1/(tan A + cot A)
We know that, tan A = sin A/cos A, cot A = cos A/sin A
= 1/((sin A/cos A) + (cos A/sin A))
By cross multiplication we get,
= 1/(($\sin^2$ A + $\cos^2$ A)/(sin A cos A))
Also we know that, $\sin^2$ A + $\cos^2$ A = 1
= 1/(1/(sin A cos A))
= sin A cos A
Then, Right Hand Side (RHS) = sin A cos A
Therefore, LHS = RHS

(v) tan A – cot A = (1 – 2 $\cos^2$ A)/(sin A cos A)
Solution:-
From the question first we consider Left Hand Side (LHS),
= tan A – cot A
We know that, tan A = sin A/cos A, cot A = cos A/sin A
Then,
= (sin A/cos A) – (cos A /sin A)
= ($\sin^2$ A – $\cos^2$ A)/(sin A cos A)
We know that, $\sin^2$ A = 1 – $\cos^2$ A
= (1 – $\cos^2$ A – $\cos^2$ A)/(sin A cos A)
So,
= (1 – 2 $\cos^2$ A)/(sin A cos A)
Then, Right Hand Side (RHS) = (1 – 2 $\cos^2$ A)/(sin A cos A)
Therefore, LHS = RHS

(vi) ((1 + $\tan^2$ A) cot A)/cosec^2 A = tan A
Solution:-
From the question first we consider Left Hand Side (LHS),
= ((1 + $\tan^2$ A) cot A)/cosec^2 A
We know that, 1 + $\tan^2$ A = $\sec^2$
= ($\sec^2$ cot A)/cosec^2 A
Also we know that, $\sec^2$ A = 1/$\cos^2$ A, cot A = cos A/sin A
= ((1/$\cos^2$ A)(cos A/sin A))/(1/($\sin^2$ A))

= (1/(cos A sin A))/(1/$\sin^2$ A)
= sin A/cos A
= tan A
Then, Right Hand Side (RHS) = tan A
Therefore, LHS = RHS

(vii) cosec A + cot A = (1/(cosec A – cot A))
Solution:-
From the question first we consider Left Hand Side (LHS),
= cosec A + cot A
Now multiply and divide by cosec A – cot A we get,
= ((cosec A + cot A)/1) ((cosec A – cot A)/(cosec A – cot A))
By cross multiplication we get,
= (cosec^2 A – $\cot^2$ A)/(cosec A – cot A)
= (1 + $\cot^2$ A – $\cot^2$ A)/(cosec A – cot A)
= (1/(cosec A – cot A))
Then, Right Hand Side (RHS) = (1/(cosec A – cot A))
Therefore, LHS = RHS

(viii) ((cosec A)/(cosec A - 1)) + ((cosec A)/(cosec A + 1)) = 2 $\sec^2$ A
Solution:-
From the question first we consider Left Hand Side (LHS),
= ((cosec A)/(cosec A - 1)) + ((cosec A)/(cosec A + 1))
By cross multiplication we get,
= (cosec^2 A + cosec A + cosec^2 A – cosec A)/(cosec^2 A - 1)
We know that, cosec^2 A – 1 = $\cot^2$ A
= 2 cosec^2 A/$\cot^2$ A
Also we know that, cosec^2 A = 1/$\sin^2$ A
= (2/$\sin^2$ A)/($\cos^2$ A/$\sin^2$ A)
= 2/$\cos^2$ A
= 2 $\sec^2$ A
Then, Right Hand Side (RHS) = 2 $\sec^2$ A
Therefore, LHS = RHS

(ix) (1 + cos A)/(1 – cos A) = ($\tan^2$ A)/(sec A – 1)^2
Solution:-
From the question first we consider Left Hand Side (LHS),

= (1 + cos A)/(1 – cos A)
We know that, cos A = 1/sec A
Then,
= (1 + (1/sec A))/(1 – (1/sec))
= (sec A + 1)/(sec A - 1)
Now multiply and divide by (sec A - 1) we get,
= ((sec A + 1)/(sec A - 1)) ((sec A - 1)/(sec A - 1))
By cross multiplication we get,
= $(\sec^2 A - 1)/(\sec A - 1)^2$
= $\tan^2 A/(\sec A - 1)^2$
Then, Right Hand Side (RHS) = $\tan^2 A/(\sec A - 1)^2$
Therefore, LHS = RHS

(x) $(\cot A – \text{cosec } A)^2 = ((1 – \cos A)/(1 + \cos A))$
Solution:-
From the question first we consider Left Hand Side (LHS),
= $(\cot A – \text{cosec } A)^2$
We know that, cot A = cos A/sin A, cosec A = 1/sin A
= $((\cos A/\sin A) – (1/\sin A))^2$
= $((\cos A - 1)/\sin A)^2$
= $(\cos A - 1)^2/\sin^2 A$
= $(\cos A - 1)^2/1 – \cos^2 A$
= $(-(1 – \cos A)^2/((1 – \cos A) (1 + \cos A))$
= ((1 – cos A) (1 – cos A))/((1 – cos A)(1 + cos A))
= (1 – cos A)/(1 + cos A)
Then, Right Hand Side (RHS) = (1 – cos A)/(1 + cos A)
Therefore, LHS = RHS

4. Prove the following identities:
(i) $\sqrt{(\text{cosec}^2 q - 1)}$ = cos q cosec q
Solution:-
From the question first we consider Left Hand Side (LHS),
= $\sqrt{(\text{cosec}^2 q - 1)}$
We know that, $\text{cosec}^2 q – 1 = \cot^2 q$
= $\sqrt{(\cot^2 q)}$
Then,
= cot q

= cos q/sin q
= cos q (1/sin q)
Also we know that, 1/ sin q = cosec q
= cos q cosec q
Then, Right Hand Side (RHS) = cos q cosec q
Therefore, LHS = RHS

(ii) √((1 + sin q)/(1 – sin q)) + √((1 – sin q)/(1 + sin q)) = 2 sec q
Solution:-
From the question first we consider Left Hand Side (LHS),
= √((1 + sin q)/(1 – sin q)) + √((1 – sin q)/(1 + sin q))
Then,
= √(((1 + sin q)/(1 – sin q)) ((1 + sin q)/(1 + sin q))) + √(((1 – sin q)/(1 + sin q)) ((1 + sin q)/(1 + sin q)))
= $\sqrt{(1 + \sin q)^2/(1 - \sin^2 q)} + \sqrt{(1 - \sin q)^2/(1 - \sin^2 q)}$
We know that, $1 - \sin^2 q = \cos^2 q$
= $\sqrt{(1 + \sin q)^2/\cos^2 q} + \sqrt{(1 - \sin q)^2/\cos^2 q}$
= ((1 + sin q)/cos q) + ((1 – sin q)/cos q)
= 2/cos q
= 2 sec q
Then, Right Hand Side (RHS) = 2 sec q
Therefore, LHS = RHS

(iii) √((1 – cos A)/(1 + cos A)) = (sin A/(1 + cos A))
Solution:-
From the question first we consider Left Hand Side (LHS),
= √((1 – cos A)/(1 + cos A))
Then,
= √(((1 – cos A)/(1 + cos A)) ((1 + cos A)/(1 + cos A)))
= $\sqrt{(1 - \cos^2 A)/(1 + \cos A)^2}$
= $\sqrt{(\sin^2 A)/(1 + \cos A)^2}$
= sin A/(1 + cos A)
Then, Right Hand Side (RHS) = sin A/(1 + cos A)
Therefore, LHS = RHS

(iv) √((1 + cos A)/(1 - cos A)) = cosec A + cot A
Solution:-

From the question first we consider Left Hand Side (LHS),

$= \sqrt{((1 + \cos A)/(1 - \cos A))}$

Then,

$= \sqrt{(((1 + \cos A)/(1 - \cos A)) ((1 + \cos A)/(1 + \cos A)))}$

$= \sqrt{((1 + \cos A)^2/(1 - \cos^2 A))}$

We know that, $1 - \cos^2 A = \sin^2 A$

$= \sqrt{((1 + \cos^2 A)/(\sin^2 A))}$

$= \sqrt{((1 + \cos A)/(\sin A))^2}$

$= \sqrt{((1/\sin A) + (\cos A/\sin A))^2}$

$= \sqrt{(\text{cosec } A + \cot A)^2}$

$= \text{cosec } A + \cot A$

Then, Right Hand Side (RHS) = cosec A + cot A

Therefore, LHS = RHS

(v) $\sqrt{((\sec q - 1)/(\sec q + 1))} + \sqrt{((\sec q + 1)/(\sec q - 1))} = 2 \text{ cosec } q$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= \sqrt{((\sec q - 1)/(\sec q + 1))} + \sqrt{((\sec q + 1)/(\sec q - 1))}$

Then,

$= \sqrt{(((\sec q - 1)/(\sec q + 1)) ((\sec q - 1)/(\sec q - 1)))} + \sqrt{(((\sec q + 1)/(\sec q - 1)) ((\sec q - 1)/(\sec q - 1)))}$

By simplification we get,

$= \sqrt{((\sec q - 1)^2/(\sec^2 q - 1))} + \sqrt{((\sec q + 1)^2/(\sec^2 q - 1))}$

We know that, $\sec^2 q - 1 = \tan^2 q$

$= \sqrt{((\sec q - 1)^2/\tan^2 q)} + \sqrt{((\sec q + 1)^2/\tan^2 q)}$

$= ((\sec q - 1)/\tan q) + ((\sec q + 1)/\tan q)$

$= (\sec q - 1 + \sec q + 1)/\tan q$

$= (2 \sec q/\tan q)$

$= (2/\cos q)/(\sin q/\cos q)$

$= 2/\sin q$

$= 2 \text{ cosec } q$

Then, Right Hand Side (RHS) = 2 cosec q

Therefore, LHS = RHS

5. Prove the following identities:

(i) $(\sec \theta - \tan \theta)^2 = (1 - \sin \theta)/(1 + \sin \theta)$

Solution:-

From the question first we consider Left Hand Side (LHS),
$= (\sec \theta - \tan \theta)^2$
We know that, $\sec \theta = 1/\cos \theta$, $\tan \theta = \sin \theta/\cos \theta$
$= ((1/\cos \theta) - (\sin \theta/\cos \theta))^2$
By taking LCM we get,
$= ((1 - \sin \theta)/\cos \theta)^2$
$= (1 - \sin \theta)^2/\cos^2 \theta$
Also we know that, $\cos^2 \theta = 1 - \sin^2 \theta$
$= (1 - \sin \theta)^2/(1 - \sin^2 \theta)$
$= (1 - \sin \theta)^2/((1 - \sin \theta)(1 + \sin \theta))$
$= (1 - \sin \theta)/(1 + \sin \theta)$
Then, Right Hand Side $(1 - \sin \theta)/(1 + \sin \theta)$
Therefore, LHS = RHS

(ii) $(1/(\sin A + \cos A)) + (1/(\sin A - \cos A)) = 2 \sin A/(1 - 2 \cos^2 A)$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= (1/(\sin A + \cos A)) + (1/(\sin A - \cos A))$
By taking LCM we get,
$= (\sin A - \cos A + \sin A + \cos A)/(\sin^2 A - \cos^2 A)$
$= 2 \sin A/(1 - \cos^2 A - \cos^2 A)$
$= 2 \sin A/(1 - 2 \cos^2 A)$
Then, Right Hand Side $2 \sin A/(1 - 2 \cos^2 A)$
Therefore, LHS = RHS

(iii) $((\sin A + \cos A)/(\sin A - \cos A)) + ((\sin A - \cos A)/(\sin A + \cos A)) = 2 /(2 \sin^2 A - 1)$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= ((\sin A + \cos A)/(\sin A - \cos A)) + ((\sin A - \cos A)/(\sin A + \cos A))$
By taking LCM we get,
$= ((\sin A + \cos A)^2 + (\sin A - \cos A)^2)/((\sin A + \cos A)(\sin A - \cos A))$
Then,
$=(\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin \cos A)/(\sin^2 A - \cos^2 A)$
$= (2 \sin^2 A + 2 \cos^2 A + 2 \sin A \cos A - 2 \sin \cos A)/(\sin^2 A - \cos^2 A)$
$= (2 \sin^2 A + 2 \cos^2 A)/(\sin^2 A - \cos^2 A)$
$= 2(\sin^2 A + \cos^2 A)/(\sin^2 A - \cos^2 A)$
We know that, $\sin^2 A + \cos^2 A = 1$

= $2(1)/(\sin^2 A - \cos^2 A)$
= $2/(\sin^2 A - \cos^2 A)$
= $2/(\sin^2 A - (1 - \sin^2 A))$
= $2/(2 \sin^2 A - 1)$
Then, Right Hand Side $2/(2 \sin^2 A - 1)$
Therefore, LHS = RHS

(iv) $\tan^2 A - \tan^2 B = (\sin^2 A - \sin^2 B)/(\cos^2 A \cos^2 B)$
Solution:-
From the question first we consider Left Hand Side (LHS),
= $\tan^2 A - \tan^2 B$
We know that, $\tan^2 \theta = \sin^2 \theta / \cos^2 \theta$,
= $(\sin^2 A \cos^2 B - \cos^2 A \sin^2 B)/(\cos^2 A \cos^2 B)$
Then,
= $(((1 - \cos^2 A) \cos^2 B) - (\cos^2 A \sin^2 B))/(\cos^2 A \cos^2 B)$
= $(\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 A \cos^2 B)/(\cos^2 A \cos^2 B)$
By simplification we get,
= $(\cos^2 B - \cos^2 A)/(\cos^2 A \cos^2 B)$
= $((1 - \sin^2 B) - (1 - \sin^2 A))/(\cos^2 A \cos^2 B)$
= $(\sin^2 A - \sin^2 B)/(\cos^2 A \cos^2 B)$
Then, Right Hand Side $(\sin^2 A - \sin^2 B)/(\cos^2 A \cos^2 B)$
Therefore, LHS = RHS

(v) $(\cos A/(1 - \tan A)) + (\sin^2 A/(\sin A - \cos A)) = \cos A + \sin A$
Solution:-
From the question first we consider Left Hand Side (LHS),
= $(\cos A/(1 - \tan A)) + (\sin^2 A/(\sin A - \cos A))$
We know that, $\tan A = \sin A/\cos A$
= $(\cos A/(1 - (\sin A/\cos A))) + (\sin^2 A/(\sin A - \cos A))$
Then,
= $(\cos A/((\cos A - \sin A)/\cos A)) + (\sin^2 A/(\sin A - \cos A))$
= $(\cos^2 A/(\cos A - \sin A)) - (\sin^2 A/(\cos A - \sin A))$
= $(\cos^2 A - \sin^2 A)/(\cos A - \sin A)$
= $((\cos A + \sin A)(\cos A - \sin A))/(\cos A - \sin A)$
By simplification we get,
= $\cos A + \sin A$
Then, Right Hand Side $\cos A + \sin A$

Therefore, LHS = RHS

(vi) $(1 + \tan^2 A) + (1 + (1/\tan^2 A)) = (1/(\sin^2 A - \sin^4 A))$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= (1 + \tan^2 A) + (1 + (1/\tan^2 A))$
We know that, $\tan^2 A = \sin^2 A/\cos^2 A$
$= (1 + (\sin^2 A/\cos^2 A)) + (1 + (1/(\sin^2 A/\cos^2 A)))$
By taking LCM we get,
$= ((\cos^2 A + \sin^2 A)/\cos^2 A) + ((\cos^2 A + \sin^2 A)/\sin^2 A)$
Also we know that, $\cos^2 A + \sin^2 A = 1$
So,
$= (1/(1 - \sin^2 A)) + (1/\sin^2 A)$
$= (\sin^2 A + 1 - \sin^2 A)/(\sin^2 A(1 - \sin^2 A))$
$= 1/(\sin^2 A - \sin^4 A)$
Then, Right Hand Side $= 1/(\sin^2 A - \sin^4 A)$
Therefore, LHS = RHS

(vii) $((\cos^3 A + \sin^3 A)/(\cos A + \sin A)) + ((\cos^3 A - \sin^3 A)/(\cos A - \sin A)) = 2$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= ((\cos^3 A + \sin^3 A)/(\cos A + \sin A)) + ((\cos^3 A - \sin^3 A)/(\cos A - \sin A))$
By taking LCM we get,
$= ((\cos^3 A + \sin^3 A)(\cos A - \sin A) + (\cos^3 A - \sin^3 A)(\cos A + \sin A))/(\cos^2 A - \sin^2 A)$
$= 2(\cos^4 A - \sin^4 A)/(\cos^2 A - \sin^2 A)$
$= (2(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A))/(\cos^2 A - \sin^2 A)$
$= 2(\cos^2 A + \sin^2 A)$
We know that, $\cos^2 A + \sin^2 A = 1$
$= 2$
Then, Right Hand Side = 2
Therefore, LHS = RHS

(viii) $(\tan \theta + (1/\cos \theta))^2 + (\tan \theta - (1/\cos \theta))^2 = 2((1 + \sin^2 \theta)/(1 - \sin^2 \theta))$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= (\tan \theta + (1/\cos \theta))^2 + (\tan \theta - (1/\cos \theta))^2$
We know that, $\tan \theta = \sin \theta/\cos \theta$

= ((sin θ/cos θ) + (1/cos θ))^2 + ((sin θ/cos θ) – (1/cos θ))^2
= ((sin θ + 1)/cos θ)^2 + ((sin θ - 1)/cos θ)^2
= ((sin θ + 1)^2/cos^2 θ) + ((sin θ - 1)/cos^2 θ)
= ((sin θ + 1)^2 + (sin θ - 1)^2)/cos^2 θ
Also we know that, (a + b)^2 = a^2 + 2ab + b^2
= (sin^2 θ + 1 + 2 sin θ + sin^2 θ + 1 – 2 sin θ)/(1 – sin^2 θ)
= 2(1 + sin^2 θ)/(1 – sin^2 θ)
Then, Right Hand Side = 2(1 + sin^2 θ)/(1 – sin^2 θ)
Therefore, LHS = RHS

(ix) ((sin A – sin B)/(cos A + cos B)) + ((cos A – cos B)/(sin A + sin B)) = 0
Solution:-
From the question first we consider Left Hand Side (LHS),
= ((sin A – sin B)/(cos A + cos B)) + ((cos A – cos B)/(sin A + sin B))
By taking LCM we get,
= (((sin A + sin B)(sin A – sin B)) + ((cos A + cos B)(cos A – cos B)))/((cos A + cos B)(sin A – sin B))
By simplification we get,
= ((sin^2 A – sin^2 B) + (cos^2 A – cos^2 B))/((cos A + cos B)(sin A – sin B))
= ((sin^2 A + cos^2 A) – (sin^2 B + cos^2 B))/((cos A + cos B)(sin A – sin B))
We know that, sin^2 A + cos^2 A = 1
= (1 - 1)/ ((cos A + cos B)(sin A – sin B))
= 0/((cos A + cos B)(sin A – sin B))
= 0
Then, Right Hand Side = 0
Therefore, LHS = RHS

(x) (1/(cos A + sin A - 1)) + (1/(cos A + sin A + 1)) = cosec A + sec A
Solution:-
From the question first we consider Left Hand Side (LHS),
= (1/(cos A + sin A - 1)) + (1/(cos A + sin A + 1))
By taking LCM we get,
= (cos A + sin A + 1 + cos A + sin A - 1)/((cos A + sin A)^2 - 1)
We know that, (a + b)^2 = a^2 + 2ab + b^2
= (2(cos A + sin A))/(cos^2 A + sin^2 A + 2 cos A sin A - 1)
= (cos A + sin A)/(cos A sin A)
= (cos A/(cos A sin A)) + (sin A/(cos A sin A))

= (1/sin A) + (1/cos A)
= cosec A + sec A
Then, Right Hand Side = cosec A + sec A
Therefore, LHS = RHS

(xi) (cot A + cosec A - 1)/(cot A – cosec A + 1) = (cos A + 1)/sin A
Solution:-
From the question first we consider Left Hand Side (LHS),
= (cot A + cosec A - 1)/(cot A – cosec A + 1)
We know that, $\text{cosec}^2 A - \cot^2 A = 1$
= (cot A + cosec – ($\text{cosec}^2 A - \cot^2 A$))/(cot A – cosec A + 1)
Also we know that, $(a^2 - b^2) = (a + b)(a - b)$
= [cot A + cosec A – ((cosec A – cot A)(cosec A + cot A))]/(cot A – cosec A + 1)
= (cot A + cosec A [1 – cosec A + cot A])/(cot A – cosec A + 1)
= cot A + cosec A
= (cos A/sin A) + (1/sin A)
= (1 + cos A)/sin A
Then, Right Hand Side = (1 + cos A)/sin A
Therefore, LHS = RHS

(xii) (sec A - 1)/(sec A + 1) = ($\sin^2$ A)/$(1 + \cos A)^2$
Solution:-
From the question first we consider Left Hand Side (LHS),
= (sec A - 1)/(sec A + 1)
We know that, sec A = 1/cos A
= ((1/cos A) - 1)/((1/cos) + 1)
= (1 – cos A)/(1 + cos A)
Then,
= ((1 – cos A)/(1 + cos A)) × ((1 + cos A)/(1 + cos A))
By simplification we get,
= $(1 - \cos^2 A)/(1 + \cos A)^2$
Also we know that, $1 - \cos^2 A = \sin^2 A$
= $\sin^2 A/(1 + \cos A)^2$
Then, Right Hand Side = $\sin^2 A/(1 + \cos A)^2$
Therefore, LHS = RHS

6. Prove the following identities:

(i) $(1 + \cot A)^2 + (1 - \cot A)^2 = 2 \operatorname{cosec}^2 A$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (1 + \cot A)^2 + (1 - \cot A)^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$= 1 + \cot^2 A + 2 \cot A + 1 + \cot^2 A - 2 \cot A$

$= 2 + 2 \cot^2 A$

Taking common terms outside we get,

$= 2 (1 + \cot^2 A)$

Also we know that, $1 + \cot^2 A = \operatorname{cosec}^2 A$

$= 2 \operatorname{cosec}^2 A$

Then, Right Hand Side $= 2 \operatorname{cosec}^2 A$

Therefore, LHS = RHS

(ii) $(\operatorname{cosec} \theta/(\tan \theta + \cot \theta)) = \cos \theta$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (\operatorname{cosec} \theta/(\tan \theta + \cot \theta))$

We know that, $\operatorname{cosec} \theta = 1/\sin \theta$, $\tan \theta = \sin \theta/\cos \theta$, $\cot \theta = \cos \theta/\sin \theta$

Then,

$= (1/\sin \theta)/((\sin \theta/\cos \theta) + (\cos \theta/\sin \theta))$

Taking LCM in the denominator we get,

$= (1/\sin \theta)/((\sin^2 \theta + \cos^2 \theta)/(\cos \theta \sin \theta))$

Also we know that, $\sin^2 \theta + \cos^2 \theta = 1$

$= (1/\sin \theta)/(1/\cos \theta \sin \theta)$

$= (1/\sin \theta) \times ((\cos \theta \sin \theta)/1)$

$= \cos \theta$

Then, Right Hand Side $= \cos \theta$

Therefore, LHS = RHS

(iii) $(1 + \tan^2 \theta) \sin \theta \cos \theta = \tan \theta$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (1 + \tan^2 \theta) \sin \theta \cos \theta$

We know that, $\tan^2 \theta = \sin^2 \theta/\cos^2 \theta$

$= (1 + (\sin^2 \theta/\cos^2 \theta)) \sin \theta \cos \theta$

Taking LCM we get,

$= ((\cos^2 \theta + \sin^2 \theta)/\cos^2 \theta) \sin \theta \cos \theta$

Also we know that, $\sin^2 \theta + \cos^2 \theta = 1$

$= (1/\cos^2 \theta) \sin \theta \cos \theta$

$= \sin \theta/\cos \theta$

$= \tan \theta$

Then, Right Hand Side = $\tan \theta$

Therefore, LHS = RHS

(iv) $((1 + \sin \theta)/(\text{cosec } \theta - \cot \theta)) - ((1 - \sin \theta)/(\text{cosec } \theta + \cot \theta)) = 2 (1 + \cot \theta)$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= ((1 + \sin \theta)/(\text{cosec } \theta - \cot \theta)) - ((1 - \sin \theta)/(\text{cosec } \theta + \cot \theta))$

By taking LCM we get,

$= [((1 + \sin \theta)(\text{cosec } \theta + \cot \theta)) - ((1 - \sin \theta)(\text{cosec } \theta - \cot \theta))]/(\text{cosec}^2 \theta - \cot^2 \theta)$

We know that, $1 + \cot^2 \theta = \text{cosec}^2 \theta$

$= (\text{cosec } \theta + \cot \theta + 1 + \cos \theta - \text{cosec } \theta + \cot \theta + 1 - \cos \theta)/(1 + \cot^2 \theta - \cot^2 \theta)$

By simplification we get,

$= 2 + 2 \cot \theta$

Taking common terms outside we get,

$= 2(1 + \cot \theta)$

Then, Right Hand Side = $2(1 + \cot \theta)$

Therefore, LHS = RHS

(v) $(1 + \cot A + \tan A) (\sin A - \cos A) = (\sec A/\text{cosec}^2 A) - (\text{cosec } A/\sec^2 A)$

Solution:-

From the question first we consider Left Hand Side (LHS),

$= (1 + \cot A + \tan A) (\sin A - \cos A)$

We know that, $\cot A = \cos A/\sin A$, $\tan A = \sin A/\cos A$

$= [1 + (\cos A/\sin A) + (\sin A/\cos A)] (\sin A - \cos A)$

By taking LCM we get,

$= [(\sin A \cos A + \cos^2 A + \sin^2 A)/(\sin A \cos A)] (\sin A - \cos A)$

Also we know that, $(a^3 - b^3) = (a - b) (a^2 + b^2 + ab)$

So,

$= (\sin^3 A - \cos^3 A)/(\sin A \cos A)$

$= (\sin^3 A/(\sin A \cos A)) - (\cos^3 A/(\sin A \cos A))$

$= (\sin^2 A/\cos A) - (\cos^2 A/\sin A)$

$= ((1/\cos A) \times \sin^2 A) - ((1/\sin A) \times \cos^2 A)$

= sec A $\sin^2$ A – cosec A $\cos^2$ A
= (sec A/cosec^2 A) – (cosec A/$\sec^2$ A)
Then, Right Hand Side = (sec A/cosec^2 A) – (cosec A/$\sec^2$ A)
Therefore, LHS = RHS

(vi) $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$
The above terms can be written as,
$= 2[(\sin^2 \theta)^3 + (\cos^2 \theta)^3] - 3(\sin^4 \theta + \cos^4 \theta) + 1$
We know that, $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$
$= 2[(\sin^2 \theta + \cos^2 \theta)((\sin^2 \theta) + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta)] - 3(\sin^4 \theta + \cos^4 \theta) + 1$
$= 2[(\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta] - 3(\sin^4 \theta + \cos^4 \theta) + 1$
$= 2\sin^4 \theta + 2\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta - 3\sin^4 \theta - 3\cos^4 \theta + 1$
$= -\sin^4 \theta - \cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + 1$
$= -(\sin^4 \theta + \cos^4 \theta + 2\sin^2 \theta \cos^2 \theta) + 1$
$= -(\sin^2 \theta + \cos^2 \theta)^2 + 1$
Also we know that, $\sin^2 \theta + \cos^2 \theta = 1$
$= -1 + 1$
$= 0$
Then, Right Hand Side = 0
Therefore, LHS = RHS

(vii) $\sin^8 \theta - \cos^8 \theta = (\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= \sin^8 \theta - \cos^8 \theta$
The above terms can be written as,
$= (\sin^4 \theta)^2 - (\cos^4 \theta)^2$
We know that, $(a^2 - b^2) = (a + b)(a - b)$
$= (\sin^4 \theta - \cos^4 \theta)(\sin^4 \theta + \cos^4 \theta)$
$= ((\sin^2 \theta)^2 - (\cos^2 \theta)^2)(\sin^4 \theta + \cos^4 \theta)$
$= (\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta)$
$= (\sin^2 \theta - \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta)$
$= (\sin^2 \theta - \cos^2 \theta)((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2\sin^2 \theta \cos^2 \theta - 2\sin^2 \theta \cos^2 \theta)$
$= (\sin^2 \theta - \cos^2 \theta)((\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta)$

$= (\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$
Then, Right Hand Side $= (\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$
Therefore, LHS = RHS

(viii) $\sec^4 A - \sec^2 A = (\sin^2 A/\cos^4 A)$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= \sec^4 A - \sec^2 A$
We know that, $\sec A = 1/\cos A$
$= (1/\cos^4 A) - (1/\cos^2 A)$
$= (1 - \cos^2 A)/\cos^4 A$
Also we know that, $1 - \cos^2 A = \sin^2 A$
$= \sin^2 A/\cos^4 A$
Then, Right Hand Side $= \sin^2 A/\cos^4 A$
Therefore, LHS = RHS

(ix) $(\tan^2 \theta/(\tan^2 \theta - 1)) + (\text{cosec}^2 \theta/(\sec^2 \theta - \text{cosec}^2 \theta)) = (1/(\sin^2 \theta - \cos^2 \theta))$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= (\tan^2 \theta/(\tan^2 \theta - 1)) + (\text{cosec}^2 \theta/(\sec^2 \theta - \text{cosec}^2 \theta))$
We know that, $\tan^2 \theta = \sin^2 \theta/\cos^2 \theta$, $\text{cosec}^2 \theta = 1/\sin^2 \theta$, $\sec^2 \theta = 1/\cos^2 \theta$
$= ((\sin^2 \theta/\cos^2 \theta)/((\sin^2 \theta/\cos^2 \theta) - 1)) + ((1/\sin^2 \theta)/((1/\cos^2 \theta) - (1/\sin^2 \theta)))$
$= (\sin^2 \theta/(\sin^2 \theta - \cos^2 \theta)) + ((1/\sin^2 \theta)/((\sin^2 \theta - \cos^2 \theta)/(\cos^2 \theta \sin^2 \theta)))$
$= (\sin^2 \theta/(\sin^2 \theta - \cos^2 \theta)) + (\cos^2 \theta/(\sin^2 \theta - \cos^2 \theta))$
$= (\sin^2 \theta + \cos^2 \theta)/(\sin^2 \theta - \cos^2 \theta)$
$= 1/(\sin^2 \theta - \cos^2 \theta)$
Then, Right Hand Side $= 1/(\sin^2 \theta - \cos^2 \theta)$
Therefore, LHS = RHS

(x) $(\sec^2 \theta - \sin^2 \theta)/(\tan^2 \theta) = \text{cosec}^2 \theta - \cos^2 \theta$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= (\sec^2 \theta - \sin^2 \theta)/(\tan^2 \theta)$
We know that, $\sec^2 \theta = 1/\cos^2 \theta$, $\tan^2 \theta = \sin^2 \theta/\cos^2 \theta$
$= ((1/\cos^2 \theta) - \sin^2 \theta)/(\sin^2 \theta/\cos^2 \theta)$
$= ((1 - \sin^2 \theta \cos^2 \theta)/\cos^2 \theta)/(\sin^2 \theta/\cos^2 \theta)$
$= (1 - \sin^2 \theta \cos^2 \theta)/\sin^2 \theta$

$= (1/\sin^2 \theta) - ((\sin^2 \theta \cos^2 \theta)/\sin^2 \theta)$
$= \text{cosec}^2 \theta - \cos^2 \theta$
Then, Right Hand Side $= \text{cosec}^2 \theta - \cos^2 \theta$
Therefore, LHS = RHS

(xi) $((\cos^3 \theta + \sin^3 \theta)/(\cos \theta + \sin \theta)) + ((\cos^3 \theta - \sin^3 \theta)/(\cos \theta - \sin \theta)) = 2$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= ((\cos^3 \theta + \sin^3 \theta)/(\cos \theta + \sin \theta)) + ((\cos^3 \theta - \sin^3 \theta)/(\cos \theta - \sin \theta))$
By taking LCM we get,
$= [((\cos^3 \theta + \sin^3 \theta)(\cos \theta - \sin \theta)) + ((\cos^3 \theta - \sin^3 \theta)(\cos \theta + \sin \theta))]/((\cos \theta + \sin \theta)(\cos \theta - \sin \theta))$
Then,
$= (\cos^4 \theta - \cos^3 \theta \sin \theta + \sin^3 \theta \cos \theta - \sin^4 \theta + \cos^4 \theta + \cos^3 \theta \sin \theta - \sin^3 \theta \cos \theta - \sin^4 \theta)/(\cos^2 \theta - \sin^2 \theta)$
By simplification we get,
$= (2 \cos^4 \theta - 2 \sin^4 \theta)/(\cos^2 \theta - \sin^2 \theta)$
Taking common outside,
$= 2(\cos^4 \theta - \sin^4 \theta)/(\cos^2 \theta - \sin^2 \theta)$
$= (2(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta))/(\cos^2 \theta - \sin^2 \theta)$
$= 2(\cos^2 \theta + \sin^2 \theta)$
We know that, $\sin^2 \theta + \cos^2 \theta = 1$
$= 2$
Then, Right Hand Side = 2
Therefore, LHS = RHS

(xii) $(\tan \theta + \sin \theta)/(\tan \theta - \sin \theta) = (\sec \theta + 1)/(\sec \theta - 1)$
Solution:-
From the question first we consider Left Hand Side (LHS),
$= (\tan \theta + \sin \theta)/(\tan \theta - \sin \theta)$
We know that, $\tan \theta = \sin \theta/\cos \theta$
$= ((\sin \theta/\cos \theta) + \sin \theta)/((\sin \theta/\cos \theta) - \sin \theta)$
$= (\sin \theta + \sin \theta \cos \theta)/(\sin \theta - \sin \theta \cos \theta)$
$= (\sin \theta(1 + \cos \theta))/(\sin \theta(1 - \cos \theta))$
$= (1 + \cos \theta)/(1 - \cos \theta)$
Also we know that, $\cos \theta = 1/\sec \theta$
$= (1 + (1/\cos \theta))/(1 - (1/\sec \theta))$

= ((sec θ + 1)/sec θ)/((sec θ - 1)sec θ)
= (sec θ + 1)/(sec θ - 1)
Then, Right Hand Side = (sec θ + 1)/(sec θ - 1)
Therefore, LHS = RHS

7. If m = a sec A + b tan A and n = a tan A + b sec A, prove that $m^2 – n^2 = a^2 – b^2$.
Solution:-
From the question it is given that,
m = a sec A + b tan A
n = a tan A + b sec A
We have to prove that, $m^2 – n^2 = a^2 – b^2$
Then,
$m^2 – n^2 = (a \sec A + b \tan A)^2 – (a \tan A + b \sec A)^2$
We know that, $(a + b)^2 = a^2 + b^2 + 2ab$
$= a^2\sec^2 A + b^2 \tan^2 A + 2a \sec A\, b \tan A – (a^2 \tan^2 A + b^2 \sec^2 A + 2ab \sec A \tan A)$
$= \sec^2 A(a^2 – b^2) + \tan^2 A(b^2 – a^2)$
$= (a^2 – b^2)[\sec^2 A – \tan^2 A]$
Also we know that, $\sec^2 A – \tan^2 A = 1$
$= (a^2 – b^2)$
Hence it is proved that, $m^2 – n^2 = a^2 – b^2$

8. If x = r sin A cos B, y = r sin A sin B and z = r cos A, prove that $x^2 + y^2 + z^2 = r^2$.
Solution:-
From the question it is given that,
x = r sin A cos B
y = r sin A sin B
z = r cos A
We have to prove that, $x^2 + y^2 + z^2 = r^2$
First we consider Left Hand Side (LHS),
$= x^2 + y^2 + z^2$
$= (r \sin A \cos B)^2 + (r \sin A \sin B)^2 + (r \cos A)^2$
$= r^2 \sin^2 A \cos^2 B + r^2 \sin^2 A \sin^2 B + r^2 \cos^2 A$
Taking common terms outside we get,
$= r^2 \sin^2 A (\cos^2 B + \sin^2 B) + r^2 \cos^2 A$
$= r^2 (\sin^2 A + \cos^2 A)$
We know that, $\sin^2 A + \cos^2 A = 1$
$= r^2$

Then, Right Hand Side = r^2
Therefore, LHS = RHS
Hence it is proved that, $x^2 + y^2 + z^2 = r^2$

9. If sin A + cos A = m and sec A + cosec A = n, prove that n (m^2 - 1) = 2m
Solution:-
From the question it is given that,
sin A + cos A = m
sec A + cosec A = n
We have to prove that, n (m^2 - 1) = 2m
First we consider Left Hand Side (LHS),
= n(m^2 - 1)
= (sec A + cosec A)((sin A + cos A)^2 - 1)
We know that, sec A = 1/cos A, cosec A = 1/sin A
= ((1/cos A) + (1/sin A))[$\sin^2$ A + $\cos^2$ A + 2 sin A cos A - 1]
= ((cos A + sin A)/(sin A cos A)) (1 + 2 sin A cos A - 1)
= ((cos A + sin A)/(sin A cos A))(2 sin A cos A)
= 2(sin A + cos A)
= 2m
Then, Right Hand Side = 2m
Therefore, LHS = RHS
Hence it is proved that, n (m^2 - 1) = 2m

10. If x = a cos θ, y = b cot θ, prove that (a^2/x^2) – (b^2/y^2) = 1.
Solution:-
From the question it is given that,
x = a cos θ
y = b cot θ
We have to prove that, (a^2/x^2) – (b^2/y^2) = 1
First we consider Left Hand Side (LHS),
= (a^2/x^2) – (b^2/y^2)
= ($a^2/a^2\cos^2$ θ) – (b^2/b^2 $\cot^2$ θ)
= (1/$\cos^2$ θ) – (1/$\cot^2$ θ)
= $\sec^2$ θ – $\tan^2$ θ
We know that, 1 + $\tan^2$ θ = $\sec^2$ θ
= 1
Then, Right Hand Side = 1

Therefore, LHS = RHS

Hence it is proved that, $(a^2/x^2) - (b^2/y^2) = 1$.

11. If sec θ + tan θ = m, sec θ – tan θ = n, prove that mn = 1.

Solution:-

From the question it is given that,

$\sec\theta + \tan\theta = m$

$\sec\theta - \tan\theta = n$

We have to prove that, mn = 1

First we consider Left Hand Side (LHS),

= mn

$= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$= \sec^2\theta - \tan^2\theta$

Also we know that, $1 + \tan^2\theta = \sec^2\theta$

= 1

Then, Right Hand Side = 1

Therefore, LHS = RHS

Hence it is proved that, mn = 1.

12. If x = a sec θ + b tan θ and y = a tan θ + b sec θ, prove that $x^2 - y^2 = a^2 - b^2$

Solution:-

From the question it is given that,

$x = a\sec\theta + b\tan\theta$

$y = a\tan\theta + b\sec\theta$

We have to prove that, $x^2 - y^2 = a^2 - b^2$

First we consider Left Hand Side (LHS),

$= x^2 - y^2$

$= (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - (a^2\tan^2\theta + b^2\sec^2\theta + 2ab\sec\theta\tan\theta)$

Then,

$= \sec^2\theta(a^2 - b^2) + \tan^2\theta(b^2 - a^2)$

$= (a^2 - b^2)[\sec^2\theta - \tan^2\theta]$

Also we know that, $\sec^2\theta - \tan^2\theta = 1$

$= a^2 - b^2$

Then, Right Hand Side $= a^2 - b^2$

Therefore, LHS = RHS

Hence it is proved that, $x^2 – y^2 = a^2 – b^2$.

13. If tan A + sin A = m and tan A – sin A = n, prove that $(m^2 – n^2)^2 = 16mn$

Solution:-

From the question it is given that,

tan A + sin A = m

tan A – sin A = n

We have to prove that, $(m^2 – n^2)^2 = 16mn$

First we consider Left Hand Side (LHS),

$= (m^2 – n^2)^2$

$= [(\tan A + \sin A)^2 – (\tan A - \sin A)^2]^2$

$=[\{(\tan A + \sin A) – (\tan A – \sin A)\}\{(\tan A + \sin A) + (\tan A – \sin A)\}]^2$

By simplification we get,

$= [(2 \sin A)(2 \tan A)]^2$

$= [4 \sin A \tan A]^2$

$= 16\sin^2 A \tan^2 A$

Then, Right Hand Side = 16mn

$= 16(\tan^2 A – \sin^2 A)$

$= 16((\sin^2 A/\cos^2 A) – \sin^2 A)$

$= 16 \sin^2 A ((1 – \cos^2 A)/\cos^2 A)$

$= 16 \sin^2 A (\sin^2 A/\cos^2 A)$

$= 16 \sin^2 A \tan^2 A$

Therefore, LHS = RHS

Hence it is proved that, $(m^2 – n^2)^2 = 16mn$.

14. If sin A + cos A = √2, prove that sin A cos A = ½

Solution:-

From the question it is given that, sin A + cos A = √2

We have to prove that, sin A cos A = ½

We know that, $(\sin A + \cos A)^2 = \sin^2 A + \cos^2 A + 2 \sin A \cos A$

So, 2 = 1 + 2 sin A cos A

2 sin A cos A = 1

sin A cos A = ½

15. If $a \sin^2 \theta + b \cos^2 \theta = c$ and $p \sin^2 \theta + q \cos^2 \theta = r$, prove that (b - c)(r - p) = (c - a)(q - r).

Solution:-

From the question it is given that,

$a \sin^2 \theta + b \cos^2 \theta = c$

$p \sin^2 \theta + q \cos^2 \theta = r$

We have to prove that, (b - c)(r - p) = (c - a)(q - r)

Consider, LHS = (b - c)(r - p)

$= (b - a \sin^2 \theta + b \cos^2 \theta)(p \sin^2 \theta + q \cos^2 \theta - p)$

$= [b(1 - \cos^2 \theta) - a \sin^2 \theta] [p(\sin^2 \theta - 1) + q \cos^2 \theta]$

$= [(b - a) \sin^2 \theta][(q - p) \cos^2 \theta]$

$= (b - a)(q - p)\sin^2 \theta \cos^2 \theta$

Now consider, RHS = (c - a) (q – r)

$= (a \sin^2 \theta + b \cos^2 \theta - a) (q - p \sin^2 \theta - q \cos^2 \theta)$

$= [(b - a) \cos^2 \theta][(q - p) \sin^2 \theta]$

$= (b - a)(q - p) \sin^2 \theta \cos^2 \theta$

Therefore, LHS = RHS

Hence it is proved that, (b - c)(r - p) = (c - a)(q - r).