

EXERCISE 4

1. Solve the inequality, $3x - 11 < 3$ where $x \in \{1, 2, 3, \dots, 10\}$. Also, represent its solution on a number line.

Solution:

Given inequality, $3x - 11 < 3$

$$3x < 3 + 11$$

$$3x < 14$$

$$\Rightarrow x < 14/3$$

But, $x \in \{1, 2, 3, \dots, 10\}$

Hence, the solution set is $\{1, 2, 3, 4\}$.

Representing the solution on a number line:



2. Solve $2(x - 3) < 1$, $x \in \{1, 2, 3, \dots, 10\}$

Solution:

Given inequality, $2(x - 3) < 1$

$$2x - 6 < 1$$

$$2x < 7$$

$$\Rightarrow x < 7/2$$

But, $x \in \{1, 2, 3, \dots, 10\}$

Hence, the solution set is $\{1, 2, 3\}$

3. Solve $5 - 4x > 2 - 3x$, $x \in \mathbb{W}$. Also represent its solution on the number line.

Solution:

Given inequality, $5 - 4x > 2 - 3x$

$$-4x + 3x > 2 - 5$$

$$-x > -3$$

On multiplying both sides by -1 , the inequality reverses

$$\Rightarrow x < 3$$

Since, $x \in \mathbb{W}$

The solution set is $\{0, 1, 2\}$

Representing the solution on a number line:



4. List the solution set of $30 - 4(2x - 1) < 30$, given that x is a positive integer.

Solution:

Given inequality, $30 - 4(2x - 1) < 30$

$$30 - 8x + 4 < 30$$

$$34 - 8x < 30$$

$$-8x < 30 - 34$$

$$-8x < -4 \quad [\text{On multiplying both sides by } -1, \text{ the inequality reverses}]$$

$$8x > 4$$

$$x > 4/8$$

$$\Rightarrow x > 1/2$$

As x is a positive integer

The solution set is $\{1, 2, 3, \dots\}$

5. Solve: $2(x - 2) < 3x - 2$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Solution:

Given inequation, $2(x - 2) < 3x - 2$

$$2x - 4 < 3x - 2$$

$$2x - 3x < -2 + 4$$

$$-x < 2$$

$$\Rightarrow x > -2$$

But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Hence, the solution set is $\{-1, 0, 1, 2, 3\}$.

6. If x is a negative integer, find the solution set of $2/3 + 1/3(x + 1) > 0$.

Solution:

Given inequation, $2/3 + 1/3(x + 1) > 0$.

$$2/3 + x/3 + 1/3 > 0$$

$$x/3 + 1 > 0$$

$$x/3 > -1$$

$$\Rightarrow x > -3$$

As x is a negative integer

The solution set is $\{-1, -2\}$.

7. Solve $x - 3(2 + x) > 2(3x - 1)$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$. Also represent its solution on the number line.

Solution:

Given inequation, $x - 3(2 + x) > 2(3x - 1)$

$$x - 6 - 3x > 6x - 2$$

$$-2x - 6 > 6x - 2$$

$$-6x - 2x > -2 + 6$$

$$-8x > 4$$

$$x < -4/8$$

$$\Rightarrow x < -1/2$$

But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Hence, the solution set is $\{-3, -2, -1\}$

8. Given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$ solve $x - 3 < 2x - 1$.

Solution:

Given inequation, $x - 3 < 2x - 1$

$$x - 2x < -1 + 3$$

$$-x < 2$$

$$\Rightarrow x > -2$$

But, $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$

Hence, the solution set is $\{1, 2, 3, 4, 5, 6, 7, 9\}$.

9. List the solution set of the inequation $\frac{1}{2} + 8x > 5x - \frac{3}{2}$, $x \in \mathbb{Z}$

Solution:

Given inequation, $\frac{1}{2} + 8x > 5x - \frac{3}{2}$

$$8x - 5x > -\frac{3}{2} - \frac{1}{2}$$

$$3x > -\frac{4}{2}$$

$$\Rightarrow x > -\frac{2}{3}$$

As $x \in \mathbb{Z}$

The solution set is $\{0, 1, 2, 3, 4, 5, \dots\}$

10. List the solution set of $(11 - 2x)/5 \geq (9 - 3x)/8 + 3/4$, $x \in \mathbb{N}$

Solution:

Given inequation, $(11 - 2x)/5 \geq (9 - 3x)/8 + \frac{3}{4}$

$$(11 - 2x)/5 \geq (9 - 3x + 6)/8$$

$$8(11 - 2x) \geq 5(15 - 3x)$$

$$88 - 16x \geq 75 - 15x$$

$$15x - 16x \geq 75 - 88$$

$$-x \geq -13$$

$$\Rightarrow x \leq 13$$

As $x \in \mathbb{N}$

Hence, the solution set is $\{1, 2, 3, 4, \dots, 13\}$.

11. Find the values of x , which satisfy the inequation : $-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$, $x \in \mathbb{N}$. Graph the solution set on the number line.

Solution:

$$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$$

Given inequation,

$$-2 \leq (3 - 4x)/6 \leq 11/6$$

$$-12 \leq 3 - 4x \leq 11$$

$$-12 - 3 \leq -4x \leq 11 - 3$$

$$-15 \leq -4x \leq 8$$

$$-15/4 \leq -x \leq 8/4$$

$$\Rightarrow 15/4 \geq x \geq -2$$

As $x \in \mathbb{N}$,

The solution set is $\{1, 2, 3\}$.

Representing the solution on a number line:



12. If $x \in \mathbb{W}$, find the solution set of $3/5 x - (2x - 1)/3 > 1$. Also graph the solution set on the number line, if possible.

Solution:

Given inequation, $3/5 x - (2x - 1)/3 > 1$

$$9/15 x - 5(2x - 1)/15 > 1 \quad [\text{Taking L.C.M}]$$

$$9x - 5(2x - 1) > 15 \quad [\text{Multiplying by 15 on both sides}]$$

$$9x - 10x + 5 > 15$$

$$-x > 15 - 5$$

$$-x > 10$$

$$\Rightarrow x < -10$$

But, $x \in \mathbb{W}$

Hence, the solution set is a null set.

Thus, it can't be represented on number line.

13. Solve:

(i) $x/2 + 5 \leq x/3 + 6$, where x is a positive odd integer.

(ii) $(2x + 3)/3 \geq (3x - 1)/4$, where x is positive even integer.

Solution:

(i) Given inequation, $x/2 + 5 \leq x/3 + 6$

$$(x + 10)/2 \leq (x + 18)/3 \quad [\text{Taking L.C.M on both sides}]$$

$$3(x + 10) \leq 2(x + 18) \quad [\text{On cross-multiplying}]$$

$$3x + 30 \leq 2x + 36$$

$$3x - 2x \leq 36 - 30$$

$$\Rightarrow x \leq 6$$

As x is a positive odd integer.

Hence, the solution set is $\{1, 3, 5\}$.

(ii) Given inequation, $(2x + 3)/3 \geq (3x - 1)/4$

$$4(2x + 3) \geq 3(3x - 1) \quad [\text{On cross-multiplying}]$$

$$8x + 12 \geq 9x - 3$$

$$-9x + 8x \geq -12 - 3$$

$$-x \geq -15$$

$$\Rightarrow x \leq 15$$

As x is positive even integer.

Hence, the solution set is $\{2, 4, 6, 8, 10, 12, 14\}$.

14. Given that $x \in \mathbb{I}$, solve the inequation and graph the solution on the number line:

$$3 \geq (x - 4)/2 + x/3 \geq 2$$

Solution:

Given inequation, $3 \geq (x - 4)/2 + x/3 \geq 2$

Now, let's take

$$3 \geq (x - 4)/2 + x/3, \text{ we have}$$

$$3 \geq (3x - 12 + 2x)/6 \quad [\text{Taking L.C.M}]$$

$$18 \geq 5x - 12$$

$$30 \geq 5x$$

$$\Rightarrow x \leq 6 \dots (i)$$

Next,

$$(x - 4)/2 + x/3 \geq 2$$

$$(3x - 12 + 2x)/6 \geq 2$$

$$5x - 12 \geq 12$$

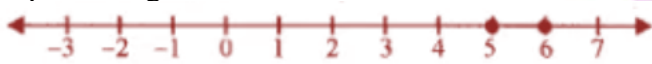
$$5x \geq 24$$

$$x \geq 24/5 \Rightarrow x \geq 4.8 \dots (ii)$$

Hence, from (i) and (ii) we have

$$\text{Solution of } x = \{5, 6\}$$

Representing the solution on a number line:



15. Solve: $1 \geq 15 - 7x > 2x - 27, x \in \mathbb{N}$

Solution:

Given inequation, $1 \geq 15 - 7x > 2x - 27,$

So, we have

$$1 \geq 15 - 7x \quad \text{and} \quad 15 - 7x > 2x - 27$$

$$7x \geq 15 - 1 \quad \text{and} \quad -2x - 7x > -27 - 15$$

$$7x \geq 14 \quad \text{and} \quad -9x > -42$$

$$x \geq 2 \quad \text{and} \quad -x > -42/9$$

$$x \geq 2 \quad \text{and} \quad x < 14/3$$

$$\Rightarrow 2 \leq x < 14/3$$

But as $x \in \mathbb{N}$

The solution set is $\{2, 3, 4\}$.

16. If $x \in \mathbb{Z}$, solve $2 + 4x < 2x - 5 \leq 3x$. Also represent its solution on the number line.

Solution

Given inequation, $2 + 4x < 2x - 5 \leq 3x$

So, we have

$$2 + 4x < 2x - 5 \quad \text{and} \quad 2x - 5 \leq 3x$$

$$4x - 2x < -5 - 2 \quad \text{and} \quad 2x - 3x \leq 5$$

$$2x < -7 \quad \text{and} \quad -x \leq 5$$

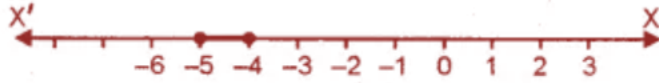
$$x < -7/2 \quad \text{and} \quad x \geq -5$$

$$\Rightarrow -5 \leq x < -7/2$$

As $x \in \mathbb{Z}$

The solution set is $\{-5, -4\}$.

Representing the solution on a number line:



17. Solve: $(4x - 10)/3 \leq (5x - 7)/2$, $x \in \mathbb{R}$ and represent the solution set on the number line.

Solution:

Given inequation, $(4x - 10)/3 \leq (5x - 7)/2$

$$2(4x - 10) \leq 3(5x - 7) \quad [\text{On cross-multiplying}]$$

$$8x - 20 \leq 15x - 21$$

$$8x - 15x \leq -21 + 20$$

$$-7x \leq -1$$

$$-x \leq -1/7$$

$$x \geq 1/7$$

As $x \in \mathbb{R}$

Hence, the solution set is $\{x: x \in \mathbb{R}, x \geq 1/7\}$

Representing the solution on a number line:



18. Solve $3x/5 - (2x - 1)/3 > 1$, $x \in \mathbb{R}$ and represent the solution set on the number line.

Solution:

Given inequation, $3x/5 - (2x - 1)/3 > 1$

$$(9x - 10x + 5)/15 > 1 \quad [\text{Taking L.C.M}]$$

$$-x + 5 > 15$$

$$-x > 15 - 5$$

$$-x > 10$$

$$x < -10$$

As $x \in \mathbb{R}$

Hence, the solution set is $\{x: x \in \mathbb{R}, x < -10\}$

Representing the solution on a number line:



19. Given that $x \in \mathbb{R}$, solve the following inequation and graph the solution on the number line: $-1 \leq 3 + 4x < 23$.

Solution:

Given inequation, $-1 \leq 3 + 4x < 23$

$$-1 - 3 \leq 4x < 23 - 3$$

$$\begin{aligned} -4 &\leq 4x < 20 \\ -4/4 &\leq x < 20/4 \\ -1 &\leq x < 5 \end{aligned}$$

Hence, the solution set is $\{-1 \leq x < 5; x \in \mathbb{R}\}$
Representing the solution on a number line:



20. Solve the following inequation and graph the solution on the number line.

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3}, x \in \mathbb{R}$$

Solution:

Given inequation,

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3}$$

$$-8/3 \leq (3x + 1)/3 < 10/3$$

$$-8 \leq 3x + 1 < 10 \quad [\text{Multiplying by 3}]$$

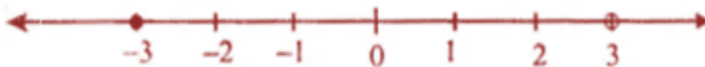
$$-8 - 1 \leq 3x < 10 - 1$$

$$-9 \leq 3x < 9$$

$$-3 \leq x < 3 \quad [\text{Dividing by 3}]$$

Thus, the solution set is $\{x: x \in \mathbb{R}, -3 \leq x < 3\}$

Representing the solution on a number line:



21. Solve the following inequation and represent the solution set on the number line:

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in \mathbb{R}$$

Solution:

Given in equation,

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in \mathbb{R}$$

$$-3 < -(3 + 4x)/6 \leq 5/6 \quad [\text{Taking L.C.M}]$$

$$-18 < -3 - 4x \leq 5 \quad [\text{Multiplying by 6}]$$

$$-18 + 3 < -4x \leq 5 + 3$$

$$-15 < -4x \leq 8$$

$$-15/4 < -x \leq 8/4$$

$$-2 \leq x < 15/4$$

Hence, the solution set is $\{x : x \in \mathbb{R}, -2 \leq x < 15/4\}$

Representing the solution on a number line:



22. Solving the following inequation, write the solution set and represent it on the number line

$$-3(x - 7) \geq 15 - 7x > \frac{x+1}{3}, x \in \mathbb{R}$$

Solution:

Given inequality, $-3(x - 7) \geq 15 - 7x > \frac{x+1}{3}$
 $-3x + 21 \geq 15 - 7x > (x + 1)/3$

So,

$$-3x + 21 \geq 15 - 7x$$

$$7x - 3x \geq 15 - 21$$

$$4x \geq -6$$

$$x \geq -6/4$$

$$x \geq -3/2$$

And,

$$15 - 7x > (x + 1)/3$$

$$3(15 - 7x) > x + 1$$

$$45 - 21x > x + 1$$

$$-21x - x > 1 - 45$$

$$-22x > -44$$

$$-x > -44/22$$

$$x < 2$$

Hence, the solution set is $\{x : x \in \mathbb{R}, -3/2 \leq x < 2\}$

Representing the solution on a number line:



23. Solve the following inequality, write down the solution set and represent it on the real number line:

$$-2 + 10x \leq 13x + 10 \leq 24 + 10x, x \in \mathbb{Z}$$

Solution:

Given inequality, $-2 + 10x \leq 13x + 10 \leq 24 + 10x$

So, we have

$$-2 + 10x \leq 13x + 10 \quad \text{and} \quad 13x + 10 \leq 24 + 10x$$

$$10x - 13x \leq 10 + 2 \quad \text{and} \quad 13x - 10x \leq 24 - 10$$

$$-3x \leq 12 \quad \text{and} \quad 3x \leq 14$$

$$x \geq -12/3 \quad \text{and} \quad x \leq 14/3$$

$$x \geq -4 \quad \text{and} \quad x \leq 14/3$$

So, $-4 \leq x \leq 14/3$

As $x \in \mathbb{Z}$

Thus, the solution set is $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:



24. Solve the inequality $2x - 5 \leq 5x + 4 < 11$, where $x \in \mathbb{I}$. Also represent the solution set on the number line.

Solution:

Given inequation, $2x - 5 \leq 5x + 4 < 11$

So, we have

$$2x - 5 \leq 5x + 4 \quad \text{and} \quad 5x + 4 < 11$$

$$2x - 5x \leq 4 + 5 \quad \text{and} \quad 5x < 11 - 4$$

$$-3x \leq 9 \quad \text{and} \quad 5x < 7$$

$$-x \leq 9/3 \quad \text{and} \quad x < 7/5$$

$$x \geq -3 \quad \text{and} \quad x < 7/5$$

$$-3 \leq x < 7/5$$

As $x \in I$

Thus, the solutions set is $\{-3, -2, -1, 0, 1\}$

Representing the solution on a number line:



25. If $x \in I$, A is the solution set of $2(x - 1) < 3x - 1$ and B is the solution set of $4x - 3 \leq 8 + x$, find $A \cap B$.

Solution:

Given inequations,

$$2(x - 1) < 3x - 1 \quad \text{and} \quad 4x - 3 \leq 8 + x \text{ for } x \in I$$

Solving for both, we have

$$2x - 3x < 2 - 1 \quad \text{and} \quad 4x - x \leq 8 + 3$$

$$-x < 1 \quad \text{and} \quad 3x \leq 11$$

$$x > -1 \quad \text{and} \quad x \leq 11/3$$

Hence,

$$\text{Solution set A} = \{0, 1, 2, 3, \dots\}$$

$$\text{Solution set B} = \{3, 2, 1, 0, -1, \dots\}$$

$$\text{Thus, } A \cap B = \{0, 1, 2, 3\}$$

26. If P is the solution set of $-3x + 4 < 2x - 3$, $x \in N$ and Q is the solution set of $4x - 5 < 12$, $x \in W$, find

(i) $P \cap Q$

(ii) $Q - P$.

Solution:

Given inequations,

$$-3x + 4 < 2x - 3 \text{ where } x \in N \text{ and } 4x - 5 < 12 \text{ where } x \in W$$

So, solving

$$-3x + 4 < 2x - 3 \text{ where } x \in N$$

$$-3x - 2x < -3 - 4$$

$$-5x < -7$$

$$x > 7/5$$

Hence, the solution set P is $\{2, 3, 4, 5, \dots\}$

And, solving

$$4x - 5 < 12 \text{ where } x \in W$$

$$4x < 12 + 5$$

$$4x < 17$$

$$x < 17/4$$

Hence, the solution set Q is $\{0, 1, 2, 3, 4\}$

Therefore,

$$(i) P \cap Q = \{2, 3, 4\}$$

$$(ii) Q - P = \{0, 1\}$$

27. A = $\{x : 11x - 5 > 7x + 3, x \in R\}$ and

B = $\{x : 18x - 9 \geq 15 + 12x, x \in R\}$

Find the range of set $A \cap B$ and represent it on a number line

Solution:

Given, A = $\{x : 11x - 5 > 7x + 3, x \in R\}$ and B = $\{x : 18x - 9 \geq 15 + 12x, x \in R\}$

Solving for A,

$$11x - 5 > 7x + 3$$

$$11x - 7x > 3 + 5$$

$$4x > 8$$

$$x > 2$$

Hence, A = $\{x : x > 2, x \in R\}$

Next, solving for B

$$18x - 9 \geq 15 + 12x$$

$$18x - 12x \geq 15 + 9$$

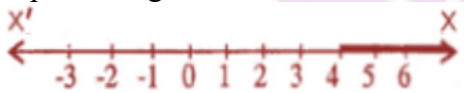
$$6x \geq 24$$

$$x \geq 4$$

Hence, B = $\{x : x \geq 4, x \in R\}$

Thus, $A \cap B = x \geq 4$

Representing the solution on a number line:



28. Given: P $\{x : 5 < 2x - 1 \leq 11, x \in R\}$

Q $\{x : -1 \leq 3 + 4x < 23, x \in I\}$ where

R = (real numbers), I = (integers)

Represent P and Q on number line. Write down the elements of $P \cap Q$.

Solution:

Given, P $\{x : 5 < 2x - 1 \leq 11, x \in R\}$ and Q $\{x : -1 \leq 3 + 4x < 23, x \in I\}$

Solving for P,

$$5 < 2x - 1 \leq 11$$

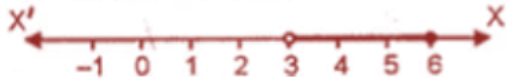
$$5 + 1 < 2x \leq 11 + 1$$

$$6 < 2x \leq 12$$

$$3 < x \leq 6$$

Hence, $P = \{x : 3 < x \leq 6, x \in \mathbb{R}\}$

Representing the solution on a number line:



Next, solving for Q

$$-1 \leq 3 + 4x < 23$$

$$-1 - 3 \leq 4x < 23 - 3$$

$$-4 \leq 4x < 20$$

$$-1 \leq x < 5$$

Hence, solution $Q = \{-1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:



Therefore, $P \cap Q = \{4\}$

29. If $x \in \mathbb{I}$, find the smallest value of x which satisfies the inequation $2x + \frac{5}{2} > \frac{5x}{3} + 2$

Solution:

Given inequation, $2x + \frac{5}{2} > \frac{5x}{3} + 2$

$$(4x + 5)/2 > (5x + 6)/3 \quad [\text{Taking L.C.M}]$$

$$3(4x + 5) > 2(5x + 6) \quad [\text{On cross-multiplication}]$$

$$12x + 15 > 10x + 12$$

$$12x - 10x > 12 - 15$$

$$2x > -3$$

$$x > -3/2$$

Hence, for $x \in \mathbb{I}$ the smallest value of x is -1 .

30. Given $20 - 5x < 5(x + 8)$, find the smallest value of x , when

(i) $x \in \mathbb{I}$

(ii) $x \in \mathbb{W}$

(iii) $x \in \mathbb{N}$.

Solution:

Given inequation, $20 - 5x < 5(x + 8)$

$$20 - 5x < 5x + 40$$

$$-5x - 5x < 40 - 20$$

$$-10x < 20$$

$$-x < 20/10$$

$$x > -2$$

Thus,

(i) For $x \in \mathbb{I}$, the smallest value = -1

(ii) For $x \in \mathbb{W}$, the smallest value = 0

(iii) For $x \in \mathbb{N}$, the smallest value = 1

31. Solve the following inequation and represent the solution set on the number line:

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$

Solution:

Given inequation,

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$

So, we have

$$4x - 19 < \frac{3x}{5} - 2 \quad \text{and} \quad \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$$

$$4x - \frac{3x}{5} < 19 - 2 \quad \text{and} \quad \frac{3x}{5} - x \leq 2 - \frac{2}{5}$$

$$\frac{(20x - 3x)}{5} < 17 \quad \text{and} \quad \frac{(3x - 5x)}{5} \leq \frac{(10 - 2)}{5}$$

$$17x < 85 \quad \text{and} \quad -2x \leq 8 \quad \text{[Multiplying by 5]}$$

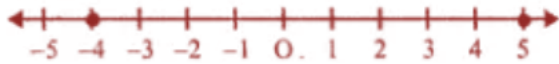
$$x < 5 \quad \text{and} \quad -x \leq 4$$

$$x < 5 \quad \text{and} \quad x \geq 4$$

$$-4 \leq x < 5, x \in R$$

Hence, the solution set is $\{x : -4 \leq x < 5, x \in R\}$

Representing the solution on a number line:



32. Solve the given inequation and graph the solution on the number line:

$$2y - 3 < y + 1 \leq 4y + 7; y \in R.$$

Solution:

Given inequation, $2y - 3 < y + 1 \leq 4y + 7$

So, we have

$$2y - 3 < y + 1 \quad \text{and} \quad y + 1 \leq 4y + 7$$

$$2y - y < 1 + 3 \quad \text{and} \quad y - 4y \leq 7 - 1$$

$$y < 4 \quad \text{and} \quad -3y \leq 6$$

$$y < 4 \quad \text{and} \quad -y \leq 2 \Rightarrow y \geq -2$$

$$\text{Thus, } -2 \leq y < 4$$

The solution set is $\{y : -2 \leq y < 4, y \in R\}$

Representing the solution on a number line:



33. Solve the inequation and represent the solution set on the number line.

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \text{ Where } x \in I$$

Solution:

Given inequation,

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \text{ Where } x \in I$$

So, we have

$$\begin{aligned}
 -3 + x &\leq 8x/3 + 2 & \text{and} & & 8x/3 + 2 &\leq 14/3 + 2x \\
 x - 8x/3 &\leq 2 + 3 & \text{and} & & 8x/3 - 2x &\leq 14/3 - 2 \\
 (3x - 8x)/3 &\leq 5 & \text{and} & & (8x - 6x)/3 &\leq (14 - 6)/3 \\
 -5x/3 &\leq 5 & \text{and} & & 2x &\leq 8 \\
 -5x &\leq 15 & \text{and} & & x &\leq 8/2 \\
 -x &\leq 3 & \text{and} & & x &\leq 4 \\
 x &\geq -3 & \text{and} & & x &\leq 4 \\
 \Rightarrow & -3 \leq x \leq 4
 \end{aligned}$$

[Taking L.C.M]

Thus, the solution set is $\{-3, -2, -1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:



34. Find the greatest integer which is such that if 7 is added to its double, the resulting number becomes greater than three times the integer.

Solution:

Let's consider the greatest integer to be x

Then according to the given condition, we have

$$2x + 7 > 3x$$

$$2x - 3x > -7$$

$$-x > -7$$

$$x < 7, \quad x \in \mathbb{R}$$

Hence, the greatest integer value is 6.

35. One-third of a bamboo pole is buried in mud, one-sixth of it is in water and the part above the water is greater than or equal to 3 metres. Find the length of the shortest pole.

Solution:

Let's assume the length of the shortest pole = x metre

Now,

Length of the pole which is buried in mud = $x/3$

Length of the pole which is in the water = $x/6$

Then according to the given condition, we have

$$x - [x/3 + x/6] \geq 3$$

$$x - [(2x + x)/6] \geq 3$$

$$x - 3x/6 \geq 3$$

$$x - x/2 \geq 3$$

$$x/2 \geq 3$$

$$x \geq 6 \quad [\text{Multiplying by 6}]$$

Therefore, the length of the shortest pole is 6 metres.

CHAPTER TEST

1. Solve the inequality: $5x - 2 \leq 3(3 - x)$ where $x \in \{-2, -1, 0, 1, 2, 3, 4\}$. Also represent its solution on the number line.

Solution:

Given inequality, $5x - 2 \leq 3(3 - x)$

$$5x - 2 \leq 9 - 3x$$

$$5x + 3x \leq 9 + 2$$

$$8x \leq 11$$

$$x \leq 11/8$$

As $x \in \{-2, -1, 0, 1, 2, 3, 4\}$

The solution set is $\{-2, -1, 0, 1\}$

Representing the solution on a number line:



2. Solve the inequality: $6x - 5 < 3x + 4$, $x \in I$

Solution:

Given inequality, $6x - 5 < 3x + 4$

$$6x - 3x < 4 + 5$$

$$3x < 9$$

$$x < 9/3$$

$$x < 3$$

As $x \in I$

The solution set is $\{2, 1, 0, -1, -2, \dots\}$

3. Find the solution set of the inequality $x + 5 \leq 2x + 3$; $x \in R$

Graph the solution set on the number line.

Solution:

Given inequality, $x + 5 \leq 2x + 3$

$$x - 2x \leq 3 - 5$$

$$-x \leq -2$$

$$x \geq 2$$

As $x \in R$

Thus, the solution set is $\{2, 3, 4, 5, \dots\}$

Representing the solution on a number line:



4. If $x \in R$ (real numbers) and $-1 < 3 - 2x \leq 7$, find solution set and present it on a number line.

Solution:

Given inequation, $-1 < 3 - 2x \leq 7$

$$-1 - 3 < -2x \leq 7 - 3$$

$$-4 < -2x \leq 4$$

$$-4/2 < -x \leq 4/2$$

$$-2 < -x \leq 2$$

Thus, $-2 \leq x < 2$

The solution set is $\{x : x \in \mathbb{R}, -2 \leq x < 2\}$

Representing the solution on a number line:



5. Solve the inequation:

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}, x \in \mathbb{R}$$

Solution:

Given inequation,

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}, x \in \mathbb{R}$$

$$(5x + 1)/7 - 4(5x + 14)/35 \leq 8/5 + (3x - 1)/7$$

$$[5(5x + 1) - 4(5x + 14)]/35 \leq [56 + 5(3x - 1)]/35 \quad \text{[Taking L.C.M]}$$

$$(25x + 5 - 20x - 56) \leq 56 + 15x - 5$$

$$5x - 51 \leq 51 + 15x$$

$$5x - 15x \leq 51 + 51$$

$$-10x \leq 102$$

$$-x \leq 102/10$$

$$x \geq -51/5$$

Hence, the solution set is $\{x : x \in \mathbb{R}, x \geq -51/5\}$

6. Find the range of values of a, which satisfy $7 \leq -4x + 2 < 12$, $x \in \mathbb{R}$. Graph these values of a on the real number line.

Solution:

$$7 < -4x + 2 < 12$$

$$7 < -4x + 2 \text{ and } -4x + 2 < 12$$

7. If $x \in \mathbb{R}$, solve $2x - 3 \geq x + (1 - x)/3 > 2x/5$

Solution:

Given inequation, $2x - 3 \geq x + (1 - x)/3 > 2x/5$

So, we have

$$2x - 3 \geq x + (1 - x)/3 \quad \text{and} \quad x + (1 - x)/3 > 2x/5$$

$$2x - 3 \geq (3x + 1 - x)/3 \quad \text{and} \quad (3x + 1 - x)/3 > 2x/5$$

$$3(2x - 3) \geq 2x + 1 \quad \text{and} \quad (2x + 1) \times 5 > 2x \times 3$$

$$6x - 9 \geq 2x + 1 \quad \text{and} \quad 10x + 5 > 6x$$

[On taking L.C.M]

[Upon cross multiplication]

$$\begin{aligned}
 6x - 2x &\geq 1 + 9 && \text{and} && 10x - 6x > -5 \\
 4x &\geq 10 && \text{and} && 4x > -5 \\
 x &\geq 10/4 && \text{and} && x > -5/4 \\
 x &\geq 5/2
 \end{aligned}$$

As $x \in \mathbb{R}$

Thus, the solution set is $\{x: x \in \mathbb{R}, x \geq 5/2\}$

Representing the solution on a number line:



7. If $x \in \mathbb{R}$, solve $2x - 3 \geq x + (1 - x)/3 > 2x/5$. Also represent the solution on the number line.

Solution:

Given inequation, $2x - 3 \geq x + (1 - x)/3 > 2x/5$

So, we have

$$\begin{aligned}
 2x - 3 &\geq x + (1 - x)/3 && \text{and} && x + (1 - x)/3 > 2x/5 \\
 2x - 3 &\geq (3x + 1 - x)/3 && \text{and} && (3x + 1 - x)/3 > 2x/5 && \text{[On taking L.C.M]} \\
 3(2x - 3) &\geq 2x + 1 && \text{and} && 5 \times (2x + 1) > 3 \times 2x \\
 6x - 9 &\geq 2x + 1 && \text{and} && 10x + 5 > 6x \\
 6x - 2x &\geq 1 + 9 && \text{and} && 10x - 6x > -5 \\
 4x &\geq 10 && \text{and} && 4x > -5 \\
 x &\geq 10/4 && \text{and} && x > -5/4 \\
 x &\geq 5/2
 \end{aligned}$$

As $x \in \mathbb{R}$

The solution set = $\{x: x \in \mathbb{R}, x \geq 5/2\}$

Representing the solution on a number line:



8. Find positive integers which are such that if 6 is subtracted from five times the integer then the resulting number cannot be greater than four times the integer.

Solution:

Let's consider the positive integer be x

Then according to the problem, we have

$$5x - 6 < 4x$$

$$5x - 4x < 6$$

$$\Rightarrow x < 6$$

Hence, the solution set = $\{x : x < 6\}$

$$= \{1, 2, 3, 4, 5, 6\}$$

9. Find three smallest consecutive natural numbers such that the difference between one-third of the largest and one-fifth of the smallest is at least 3.

Solution:

Let's consider the first least natural number as x

Then, second number = $x + 1$

And third number = $x + 2$

So, according the conditions given in the problem, we have

$$\frac{1}{3} \times (x + 2) - \frac{x}{5} \geq 3$$

$$5x + 10 - 3x \geq 3 \times 15$$

[Multiplying by 15 the L.C.M of 3 and 5]

$$2x \geq 45 - 10$$

$$2x \geq 35$$

$$x \geq \frac{35}{2}$$

$$x \geq 17.5$$

As x is a natural least number

Thus, first least natural number = 18

Second number = $18 + 1 = 19$

And, third number = $18 + 2 = 20$

Hence, the least natural numbers are 18, 19 and 20