

**EXERCISE 6.1****1. Find the remainder (without division) on dividing  $f(x)$  by  $(x - 2)$  where**

**(i)  $f(x) = 5x^2 - 7x + 4$**

**Solutions:-**

Let us assume  $x - 2 = 0$

Then,  $x = 2$

Given,  $f(x) = 5x^2 - 7x + 4$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(2) = (5 \times 2^2) - (7 \times 2) + 4$$

$$= (5 \times 4) - 14 + 4$$

$$= 20 - 14 + 4$$

$$= 24 - 14$$

$$= 10$$

Therefore, the remainder is 10.

**(ii)  $f(x) = 2x^3 - 7x^2 + 3$**

**Solution:-**

Let us assume  $x - 2 = 0$

Then,  $x = 2$

Given,  $f(x) = 2x^3 - 7x^2 + 3$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(2) = (2 \times 2^3) - (7 \times 2^2) + 3$$

$$= (2 \times 8) - (7 \times 4) + 3$$

$$= 16 - 28 + 3$$

$$= 19 - 28$$

$$= -9$$

Therefore, the remainder is -9.

**2. Using the remainder theorem, find the remainder on dividing  $f(x)$  by  $(x + 3)$  where**

**(i)  $f(x) = 2x^2 - 5x + 1$**

**Solution:-**

Let us assume  $x + 3 = 0$

Then,  $x = -3$

Given,  $f(x) = 2x^2 - 5x + 1$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-3) = (2 \times (-3)^2) - (5 \times (-3)) + 1$$

$$= (2 \times 9) - (-15) + 1$$

$$= 18 + 15 + 1$$
$$= 34$$

Therefore, the remainder is 34.

**(ii)  $f(x) = 3x^3 + 7x^2 - 5x + 1$**

**Solution:-**

Let us assume  $x + 3 = 0$

Then,  $x = -3$

Given,  $f(x) = 3x^3 + 7x^2 - 5x + 1$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-3) = (3 \times -3^3) + (7 \times -3^2) - (5 \times -3) + 1$$
$$= (3 \times -27) + (7 \times 9) - (-15) + 1$$
$$= -81 + 63 + 15 + 1$$
$$= -81 + 79$$
$$= -2$$

Therefore, the remainder is -2.

**3. Find the remainder (without division) on dividing  $f(x)$  by  $(2x + 1)$  where,**

**(i)  $f(x) = 4x^2 + 5x + 3$**

**Solution:-**

Let us assume  $2x + 1 = 0$

Then,  $2x = -1$

$$x = -\frac{1}{2}$$

Given,  $f(x) = 4x^2 + 5x + 3$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-\frac{1}{2}) = 4(-\frac{1}{2})^2 + 5(-\frac{1}{2}) + 3$$
$$= (4 \times \frac{1}{4}) + (-\frac{5}{2}) + 3$$
$$= 1 - \frac{5}{2} + 3$$
$$= 4 - \frac{5}{2}$$
$$= \frac{8 - 5}{2}$$
$$= \frac{3}{2} = 1\frac{1}{2}$$

Therefore, the remainder is  $1\frac{1}{2}$ .

**(ii)  $f(x) = 3x^3 - 7x^2 + 4x + 11$**

**Solution:-**

Let us assume  $2x + 1 = 0$

Then,  $2x = -1$

$$X = -\frac{1}{2}$$

Given,  $f(x) = 3x^3 - 7x^2 + 4x + 11$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(-\frac{1}{2}) &= (3 \times (-\frac{1}{2})^3) - (7 \times (-\frac{1}{2})^2) + (4 \times -\frac{1}{2}) + 11 \\&= 3 \times (-\frac{1}{8}) - (7 \times \frac{1}{4}) + (-2) + 11 \\&= -\frac{3}{8} - \frac{7}{4} - 2 + 11 \\&= -\frac{3}{8} - \frac{7}{4} + 9 \\&= \frac{-3 - 14 + 72}{8} \\&= \frac{55}{8} \\&= 6\frac{7}{8}\end{aligned}$$

**4. Using remainder theorem, find the value of  $k$  if on dividing  $2x^3 + 3x^2 - kx + 5$  by  $x - 2$  leaves a remainder 7.**

**Solution:-**

Let us assume,  $x - 2 = 0$

Then,  $x = 2$

Given,  $2x^3 + 3x^2 - kx + 5$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(2) &= (2 \times 2^3) + (3 \times 2^2) - (k \times 2) + 5 \\&= (2 \times 8) + (3 \times 4) - 2k + 5 \\&= 16 + 12 - 2k + 5 \\&= 33 - 2k\end{aligned}$$

From the question it is given that, remainder is 7.

$$\text{So, } 7 = 33 - 2k$$

$$2k = 33 - 7$$

$$2k = 26$$

$$K = \frac{26}{2}$$

$$K = 13$$

Therefore, the value of  $k$  is 13.

**5. Using remainder theorem, find the value of 'a' if the division of  $x^3 + 5x^2 - ax + 6$  by  $(x - 1)$  leaves the remainder  $2a$ .**

**Solution:-**

Let us assume  $x - 1 = 0$

Then,  $x = 1$

Given,  $f(x) = x^3 + 5x^2 - ax + 6$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 f(1) &= 1^3 + (5 \times 1^2) - (a \times 1) + 6 \\
 &= 1 + 5 - a + 6 \\
 &= 12 - a
 \end{aligned}$$

From the question it is given that, remainder is  $2a$

$$\text{So, } 2a = 12 - a$$

$$2a + a = 12$$

$$3a = 12$$

$$a = 12/3$$

$$a = 4$$

Therefore, the value of  $a$  is 4.

**6. (i) What number must be divided be subtracted from  $2x^2 - 5x$  so that the resulting polynomial leaves the remainder 2, when divided by  $2x + 1$ ?**

**Solution:-**

let us assume 'p' be subtracted from  $2x^2 - 5x$

So, dividing  $2x^2 - 5x$  by  $2x + 1$ ,

$$\begin{array}{r}
 2x + 1 \overline{) 2x^2 - 5x - p} \\
 \underline{2x^2 + x} \phantom{- p} \\
 -6x - p \\
 \underline{-6x - 3} \\
 + \phantom{+} \\
 \underline{\phantom{+} +} \\
 -p + 3
 \end{array}$$

Hence, remainder is  $3 - p$

From the question it is given that, remainder is 2.

$$3 - p = 2$$

$$p = 3 - 2$$

$$p = 1$$

Therefore, 1 is to be subtracted.

**(ii) What number must be added to  $2x^3 - 7x^2 + 2x$  so that the resulting polynomial leaves the remainder  $-2$  when divided by  $2x - 3$ ?**

**Solution:-**

let us assume 'p' be subtracted from  $2x^3 - 7x^2 + 2x$ ,

So, dividing it by  $2x - 3$ ,

$$\begin{array}{r}
 2x - 3 \mid 2x^3 - 7x^2 + 2x + p \quad (x^2 - 2x - 2) \\
 \underline{2x^3 - 3x^2} \phantom{+ 2x + p} \\
 -4x^2 + 2x \phantom{+ p} \\
 \underline{-4x^2 + 6x} \phantom{+ p} \\
 + \phantom{-} - \\
 \phantom{+} -4x + p \\
 \phantom{+} -4x + 6 \\
 \underline{\phantom{+} + \phantom{-}} \\
 \phantom{+} \phantom{-} P - 6
 \end{array}$$

Hence, remainder is  $p - 6$

From the question it is given that, remainder is  $-2$ .

$$P - 6 = -2$$

$$P = -2 + 6$$

$$P = 4$$

Therefore, 4 is to be added.

**7. (i) When divided by  $x - 3$  the polynomials  $x^3 - px^2 + x + 6$  and  $2x^3 - x^2 - (p + 3)x - 6$  leave the same remainder. Find the value of 'p'.**

**Solution:-**

From the question it is given that, by dividing  $x^3 - px^2 + x + 6$  and  $2x^3 - x^2 - (p + 3)x - 6$  by  $x - 3 = 0$ , then  $x = 3$ .

Let us assume  $p(x) = x^3 - px^2 + x + 6$

Now, substitute the value of  $x$  in  $p(x)$ ,

$$\begin{aligned}
 p(3) &= 3^3 - (p \times 3^2) + 3 + 6 \\
 &= 27 - 9p + 9 \\
 &= 36 - 9p
 \end{aligned}$$

Then,  $q(x) = 2x^3 - x^2 - (p + 3)x - 6$

Now, substitute the value of  $x$  in  $q(x)$ ,

$$\begin{aligned}
 q(3) &= (2 \times 3^3) - (3)^2 - (p + 3) \times 3 - 6 \\
 &= (2 \times 27) - 9 - 3p - 9 - 6 \\
 &= 54 - 24 - 3p \\
 &= 30 - 3p
 \end{aligned}$$

Given, the remainder in each case is same,

$$\text{So, } 36 - 9p = 30 - 3p$$

$$36 - 30 = 9p - 3p$$

$$6 = 6p$$

$$p = 6/6$$

$$p = 1$$

Therefore, value of p is 1.

**(ii) Find 'a' if the two polynomials  $ax^3 + 3x^2 - 9$  and  $2x^3 + 4x + a$ , leaves the same remainder when divided by  $x + 3$ .**

**Solution:-**

Let us assume  $p(x) = ax^3 + 3x^2 - 9$  and  $q(x) = 2x^3 + 4x + a$

From the question it is given that, both  $p(x)$  and  $q(x)$  leaves the same remainder when divided by  $x + 3$ .

Let us assume that,  $x + 3 = 0$

$$\text{Then, } x = -3$$

Now, substitute the value of  $x$  in  $p(x)$  and in  $q(x)$ ,

$$\text{So, } p(-3) = q(-3)$$

$$a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$

$$-27a + 27 - 9 = -54 - 12 + a$$

$$-27a + 18 = -66 + a$$

$$-27a - a = -66 - 18$$

$$-28a = -84$$

$$a = 84/28$$

Therefore,  $a = 3$

**(iii) The polynomials  $ax^3 + 3x^2 - 3$  and  $2x^3 - 5x + a$  when divided by  $x - 4$  leave the remainder  $r_1$  and  $r_2$  respectively. If  $2r_1 = r_2$ , then find the value of  $a$ .**

**Solution:**

Let us assume  $p(x) = ax^3 + 3x^2 - 3$  and  $q(x) = 2x^3 - 5x + a$

From the question it is given that, both  $p(x)$  and  $q(x)$  leaves the remainder  $r_1$  and  $r_2$  respectively when divided by  $x - 4$ .

Also, given relation  $2r_1 = r_2$

Let us assume that,  $x - 4 = 0$

$$\text{Then, } x = 4$$

Now, substitute the value of  $x$  in  $p(x)$  and in  $q(x)$ ,

By factor theorem,  $r_1 = p(x)$  and  $r_2 = q(x)$

$$\text{So, } 2 \times p(4) = q(4)$$

$$2[a(4)^3 + 3(4)^2 - 3] = 2(4)^3 - 5(4) + a$$

$$2[64a + 48 - 3] = 128 - 20 + a$$

$$128a + 96 - 6 = 128 - 20 + a$$

$$128a + 90 = 108 + a$$

$$128a - a = 108 - 90$$

$$127a = 18$$

$$a = 18/127$$

Therefore, the value of  $a = 18/127$ .

**8. Using remainder theorem, find the remainders obtained when  $x^3 + (kx + 8)x + k$  is divided by  $x + 1$  and  $x - 2$ . Hence, find  $k$  if the sum of two remainders is 1.**

**Solution:**

Let us assume  $p(x) = x^3 + (kx + 8)x + k$

From the question it is given that, the sum of the remainders when  $p(x)$  is divided by  $(x + 1)$  and  $(x - 2)$  is 1.

Let us assume that,  $x + 1 = 0$

$$\text{Then, } x = -1$$

$$\text{Also, when } x - 2 = 0$$

$$\text{Then, } x = 2$$

Now, by remainder theorem we have

$$p(-1) + p(2) = 1$$

$$(-1)^3 + [k(-1) + 8](-1) + k + (2)^3 + [k(2) + 8](2) + k = 1$$

$$-1 + k - 8 + k + 8 + 4k + 16 + k = 1$$

$$7k + 15 = 1$$

$$7k = 1 - 15$$

$$k = -14/7$$

$$k = -2$$

Therefore,  $k = -2$ .

**9. By factor theorem, show that  $(x + 3)$  and  $(2x - 1)$  are factors of  $2x^2 + 5x - 3$ .**

**Solution:-**

Let us assume,  $x + 3 = 0$

$$\text{Then, } x = -3$$

$$\text{Given, } f(x) = 2x^2 + 5x - 3$$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-3) = (2 \times (-3)^2) + (5 \times -3) - 3$$

$$= (2 \times 9) + (-15) - 3$$

$$= 18 - 15 - 3$$

$$= 18 - 18$$

$$= 0$$

$$\text{Now, } 2x - 1 = 0$$

$$\text{Then, } 2x = 1$$

$$x = \frac{1}{2}$$

$$\text{Given, } f(x) = 2x^2 + 5x - 3$$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f\left(\frac{1}{2}\right) = \left(2 \times \left(\frac{1}{2}\right)^2\right) + \left(5 \times \frac{1}{2}\right) - 3$$

$$= \left(2 \times \left(\frac{1}{4}\right)\right) + \frac{5}{2} - 3$$

$$= \frac{1}{2} + \frac{5}{2} - 3$$

$$= \frac{(1 + 5)}{2} - 3$$

$$= \frac{6}{2} - 3$$

$$= 3 - 3$$

$$= 0$$

Hence, it is proved that,  $(x + 3)$  and  $(2x - 1)$  are factors of  $2x^2 + 5x - 3$ .

**10. Without actual division, prove that  $x^4 + 2x^3 - 2x^2 + 2x + 3$  is exactly divisible by  $x^2 + 2x - 3$ .**

**Solution:-**

$$\text{Consider } x^2 + 2x - 3$$

$$\text{By factor method, } x^2 + 3x - x - 3$$

$$= x(x + 3) - 1(x + 3)$$

$$= (x - 1)(x + 3)$$

$$\text{So, } f(x) = x^4 + 2x^3 - 2x^2 + 2x + 3$$

$$\text{Now take, } x + 3 = 0$$

$$x = -3$$

$$\text{Then, } f(-3) = (-3)^4 + 2 \times (-3)^3 - (2 \times (-3)^2) + (2 \times -3) + 3$$

$$= 81 - 54 - 18 - 6 - 3$$

$$= 0$$

Therefore,  $(x + 3)$  is a factor of  $f(x)$

$$\text{And also, take } x - 1 = 0$$

$$x = 1$$

$$\text{Then, } f(1) = 1^4 + 2(1)^3 - 2(1)^2 + 2(1) - 3$$

$$= 0$$

Therefore,  $(x - 1)$  is a factor of  $f(x)$

By comparing both results,  $p(x)$  is exactly divisible by  $x^2 + 2x - 3$ .

**11. Show that  $(x - 2)$  is a factor of  $3x^2 - x - 10$ . Hence factories  $3x^2 - x - 10$ .**

**Solution:-**



Let us assume  $x - 2 = 0$

Then,  $x = 2$

Given,  $f(x) = 3x^2 - x - 10$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(2) &= (3 \times 2^2) - 2 - 10 \\ &= (3 \times 4) - 2 - 10 \\ &= 12 - 2 - 10 \\ &= 12 - 12 \\ &= 0 \end{aligned}$$

Therefore,  $(x - 2)$  is a factor of  $f(x)$

Then, dividing  $(3x^2 - x - 10)$  by  $(x - 2)$ , we get

$$\begin{array}{r} x-2 \overline{) 3x^2 - x - 10} \phantom{+ 5} \\ \underline{3x^2 + x} \phantom{- 10} \\ 5x - 10 \\ \underline{5x - 10} \\ 0 \end{array}$$

Therefore,  $3x^2 - x - 10 = (x - 2)(3x + 5)$

**12. Using the factor theorem, show that  $(x - 2)$  is a factor of  $x^3 + x^2 - 4x - 4$ . Hence factorize the polynomial completely.**

**Solution:-**

Let us assume,  $x - 2 = 0$

Then,  $x = 2$

Given,  $f(x) = x^3 + x^2 - 4x - 4$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(2) &= (2)^3 + (2)^2 - 4(2) - 4 \\ &= 8 - 4 - 8 - 4 \\ &= 0 \end{aligned}$$

Therefore, by factor theorem  $(x - 2)$  is a factor of  $x^3 + x^2 - 4x - 4$

Then, dividing  $f(x)$  by  $(x - 2)$ , we get

$$\begin{array}{r}
 x^2 + 3x + 2 \\
 x - 2 \overline{) x^3 + x^2 - 4x - 4} \\
 \underline{-} \\
 x^3 - 2x^2 \\
 \underline{-} \\
 3x^2 - 4x - 4 \\
 \underline{-} \\
 3x^2 - 6x \\
 \underline{-} \\
 2x - 4 \\
 \underline{-} \\
 2x - 4 \\
 \underline{-} \\
 0
 \end{array}$$

Therefore,  $x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$   
 $= (x - 2)(x^2 + 2x + x + 2)$   
 $= (x - 2)(x(x + 2) + 1(x + 2))$   
 $= (x - 2)(x + 2)(x + 1)$

**13. Show that  $2x + 7$  is a factor of  $2x^3 + 5x^2 - 11x - 14$ . Hence factorize the given expression completely, using the factor theorem.**

**Solution:-**

Let us assume  $2x + 7 = 0$

Then,  $2x = -7$

$$x = -7/2$$

Given,  $f(x) = 2x^3 + 5x^2 - 11x - 14$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 f(-7/2) &= 2(-7/2)^3 + 5(-7/2)^2 + 11(-7/2) - 14 \\
 &= 2(-343/8) + 5(49/4) + (-77/2) - 14 \\
 &= -343/4 + 245/4 - 77/2 - 14 \\
 &= (-343 + 245 + 154 - 56)/4 \\
 &= -399 + 399/4 \\
 &= 0
 \end{aligned}$$

Therefore,  $(2x + 7)$  is a factor of  $2x^3 + 5x^2 - 11x - 14$

Then, dividing  $f(x)$  by  $(2x + 1)$ , we get

$$\begin{array}{r}
 x^2 - x - 2 \\
 2x + 7 \overline{) 2x^3 + 5x^2 - 11x - 14} \\
 \underline{-} \\
 2x^3 + 7x^2 \\
 \underline{-} \\
 -2x^2 - 11x - 14 \\
 \underline{-} \\
 -2x^2 - 7x \\
 \underline{-} \\
 -4x - 14 \\
 \underline{-} \\
 -4x - 14 \\
 \underline{-} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } 2x^3 + 5x^2 - 11x - 14 &= (2x + 7)(x^2 - x - 2) \\
 &= (2x + 7)(x^2 - 2x + x - 2) \\
 &= (2x + 7)(x(x - 2) + 1(x - 2)) \\
 &= (x + 1)(x - 2)(2x + 7)
 \end{aligned}$$

14. Use factor theorem to factorize the following polynomials completely.

(i)  $x^3 + 2x^2 - 5x - 6$

**Solution:-**

Let us assume  $x = -1$ ,

Given,  $f(x) = x^3 + 2x^2 - 5x - 6$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 f(-1) &= (-1)^3 + 2(-1)^2 - 5(-1) - 6 \\
 &= -1 + 2(1) + 5 - 6 \\
 &= -1 + 2 + 5 - 6 \\
 &= -7 + 7 \\
 &= 0
 \end{aligned}$$

Then, dividing  $f(x)$  by  $(x + 1)$ , we get

$$\begin{array}{r}
 x^2 + x - 6 \\
 x + 1 \overline{) x^3 + 2x^2 - 5x - 6} \\
 \underline{x^3 + x^2} \phantom{- 5x - 6} \\
 x^2 - 5x - 6 \\
 \underline{x^2 + x} \phantom{- 6} \\
 -6x - 6 \\
 \underline{-6x - 6} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } x^3 + 2x^2 - 5x - 6 &= (x + 1)(x^2 + 3x - 2x - 6) \\
 &= (x + 1)(x(x + 3) - 2(x + 3)) \\
 &= (x + 1)(x - 2)(x + 3)
 \end{aligned}$$

(ii)  $x^3 - 13x - 12$

**Solution:-**

Let us assume  $x = -1$ ,

Given,  $f(x) = x^3 - 13x - 12$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-1) = (-1)^3 - 13(-1) - 12$$

$$= -1 + 13 - 12$$

$$= -13 + 13$$

$$= 0$$

Then, dividing  $f(x)$  by  $(x + 1)$ , we get

$$\begin{array}{r}
 x^2 \quad -x \quad -12 \\
 x + 1 \overline{) x^3 + 0x^2 - 13x - 12} \\
 \underline{-} \\
 x^3 \quad +x^2 \\
 \underline{-} \\
 \quad -x^2 \quad -13x \quad -12 \\
 \quad \underline{-} \\
 \quad \quad -x^2 \quad -x \\
 \quad \quad \underline{-} \\
 \quad \quad \quad -12x \quad -12 \\
 \quad \quad \quad \underline{-} \\
 \quad \quad \quad \quad -12x \quad -12 \\
 \quad \quad \quad \quad \underline{-} \\
 \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } x^3 - 13x - 12 &= (x + 1)(x^2 - x - 12) \\
 &= (x + 1)(x^2 - 4x + 3x - 12) \\
 &= (x + 1)(x(x - 4)) + 3(x - 4) \\
 &= (x + 1)(x + 3)(x - 4)
 \end{aligned}$$

**15. Use the remainder theorem to factorize the following expression.**

**(i)  $2x^3 + x^2 - 13x + 6$**

**Solution:-**

Let us assume  $x = 2$ ,

Then,  $f(x) = 2x^3 + x^2 - 13x + 6$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 f(2) &= (2 \times 2^3) + 2^2 - 13 \times 2 + 6 \\
 &= (2 \times 8) + 4 - 26 + 6 \\
 &= 16 + 4 - 26 + 6 \\
 &= 26 - 26 \\
 &= 0
 \end{aligned}$$

Then, dividing  $f(x)$  by  $(x - 2)$ , we get

$$\begin{array}{r}
 2x^2 + 5x - 3 \\
 x - 2 \overline{) 2x^3 + x^2 - 13x + 6} \\
 \underline{2x^3 - 4x^2} \phantom{+ 6} \\
 5x^2 - 13x + 6 \\
 \underline{5x^2 - 10x} \phantom{+ 6} \\
 -3x + 6 \\
 \underline{-3x + 6} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } 2x^3 + x^2 - 13x + 6 &= (x - 2)(2x^2 + 5x - 3) \\
 &= (x - 2)(2x^2 + 6x - x - 3) \\
 &= (x - 2)(2x(x + 3) - 1(x + 3)) \\
 &= (x - 2)(x + 3)(2x - 1)
 \end{aligned}$$

(ii)  $3x^3 + 2x^2 - 19x + 6$

**Solution:-**

Given,  $f(x) = 3x^3 + 2x^2 - 19x + 6$

Let us assume  $x = 1$

$$\begin{aligned}
 \text{Then, } f(1) &= 3(1)^3 + 2(1)^2 - (19 \times 1) + 6 \\
 &= 3 + 2 - 19 + 6 \\
 &= 11 - 19 \\
 &= -8
 \end{aligned}$$

So,  $-8 \neq 0$

Let us assume  $x = -1$

$$\begin{aligned}
 \text{Then, } f(-1) &= 3(-1)^3 + 2(-1)^2 - (19 \times (-1)) + 6 \\
 &= -3 + 2 + 19 + 6 \\
 &= -3 + 27 \\
 &= 24
 \end{aligned}$$

So,  $24 \neq 0$

Now, assume  $x = 2$

$$\begin{aligned}
 \text{Then, } f(2) &= 3(2)^3 + 2(2)^2 - (19 \times (2)) + 6 \\
 &= 24 + 8 - 38 + 6
 \end{aligned}$$

$$= 38 - 38$$

$$= 0$$

So,  $0 = 0$

Therefore,  $(x - 2)$  is a factor of  $f(x)$ .

$$f(x) = 3x^3 + 2x^2 - 19x + 6$$

$$= 3x^3 - 6x^2 + 8x^2 - 16x - 3x + 6$$

$$= 3x^2(x - 2) + 8x(x - 2) - 3(x - 2)$$

$$= (x - 2)(3x^2 + 8x - 3)$$

$$= (x - 2)(3x^2 + 9x - x - 3)$$

$$= (x - 2)(3x(x + 3) - 1(x + 3))$$

$$= (x - 2)(x + 3)(3x - 1)$$

(iii)  $2x^3 + 3x^2 - 9x - 10$

**Solution:-**

Given,  $f(x) = 2x^3 + 3x^2 - 9x - 10$

Let us assume,  $x = -1$

$$= 2(-1)^3 + 3(-1)^2 - 9(-1) - 10$$

$$= -2 + 3 + 9 - 10$$

$$= 12 - 12$$

$$= 0$$

Therefore,  $(x + 1)$  is the factor of  $2x^3 + 3x^2 - 9x - 10$

Then, dividing  $f(x)$  by  $(x + 1)$ , we get

$$\begin{array}{r}
 2x^2 + x - 10 \\
 x + 1 \overline{) 2x^3 + 3x^2 - 9x - 10} \\
 \underline{2x^3 + 2x^2} \phantom{-9x - 10} \\
 \phantom{2x^3} x^2 - 9x - 10 \\
 \phantom{2x^3} \underline{x^2 + x} \phantom{-10} \\
 \phantom{2x^3} \phantom{x^2} -10x - 10 \\
 \phantom{2x^3} \phantom{x^2} \underline{-10x - 10} \\
 \phantom{2x^3} \phantom{x^2} \phantom{-10x} 0
 \end{array}$$

Therefore,  $2x^3 + 3x^2 - 9x - 10 = 2x^2 + 5x - 4x - 10$

$$= x(2x + 5) - 2(2x + 5) - (2x + 5)(x - 2)$$

Hence the factors are  $(x + 1)(x - 2)(2x + 5)$

**(iv)  $x^3 + 10x^2 - 37x + 26$**

**Solution:-**

Given,  $f(x) = x^3 + 10x^2 - 37x + 26$

Let us assume,  $x = 1$

Then,  $f(1) = 1^3 + 10(1)^2 - 37(1) + 26$

$$= 1 + 10 - 37 + 26$$

$$= 37 - 37$$

$$= 0$$

Therefore,  $x - 1$  is a factor of  $x^3 + 10x^2 - 37x + 26$

Then, dividing  $f(x)$  by  $(x - 1)$ , we get

$$\begin{array}{r}
 x^2 + 11x - 26 \\
 x - 1 \overline{) x^3 + 10x^2 - 37x + 26} \\
 \underline{-} \\
 x^3 \quad -x^2 \\
 \hline
 11x^2 \quad -37x \quad +26 \\
 \underline{-} \\
 11x^2 \quad -11x \\
 \hline
 -26x \quad +26 \\
 \underline{-} \\
 -26x \quad +26 \\
 \hline
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } x^3 + 10x^2 - 37x + 26 &= (x - 1)(x^2 + 11x - 26) \\
 &= (x - 1)(x^2 + 13x - 2x - 26) \\
 &= (x - 1)(x(x + 13) - 2(x + 13)) \\
 &= (x - 1)((x - 2)(x + 13))
 \end{aligned}$$

**16. If  $(2x + 1)$  is a factor of  $6x^3 + 5x^2 + ax - 2$  find the value of  $a$ .**

**Solution:-**

Let us assume  $2x + 1 = 0$

Then,  $2x = -1$

$$x = -\frac{1}{2}$$



Given,  $f(x) = 6x^3 + 5x^2 + ax - 2$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f\left(-\frac{1}{2}\right) &= 6\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) - 2 \\&= 6\left(-\frac{1}{8}\right) + 5\left(\frac{1}{4}\right) - \frac{1}{2}a - 2 \\&= -\frac{3}{4} + \frac{5}{4} - \frac{a}{2} - 2 \\&= \frac{-3 + 4 - 2a - 8}{4} \\&= \frac{-6 - 2a}{4}\end{aligned}$$

From the question,  $(2x + 1)$  is a factor of  $6x^3 + 5x^2 + ax - 2$

Then, remainder is 0.

So,  $\frac{-6 - 2a}{4} = 0$

$$-6 - 2a = 4 \times 0$$

$$-6 - 2a = 0$$

$$-2a = 6$$

$$a = -\frac{6}{2}$$

$$a = -3$$

Therefore, the value of  $a$  is  $-3$ .

**17. If  $(3x - 2)$  is a factor of  $3x^3 - kx^2 + 21x - 10$ , find the value of  $k$ .**

**Solution:-**

Let us assume  $3x - 2 = 0$

Then,  $3x = 2$

$$x = \frac{2}{3}$$

Given,  $f(x) = 3x^3 - kx^2 + 21x - 10$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f\left(\frac{2}{3}\right) &= 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21\left(\frac{2}{3}\right) - 10 \\&= 3\left(\frac{8}{27}\right) - k\left(\frac{4}{9}\right) + 14 - 10 \\&= \frac{8}{9} - \frac{4k}{9} + 14 - 10 \\&= \frac{8}{9} - \frac{4k}{9} + 4 \\&= \frac{8 - 4k + 36}{9} \\&= \frac{44 - 4k}{9}\end{aligned}$$

From the question,  $(3x - 2)$  is a factor of  $3x^3 - kx^2 + 21x - 10$

Then, remainder is 0

So,  $\frac{44 - 4k}{9} = 0$

$$44 - 4k = 0 \times 9$$

$$44 = 4k$$

$$k = \frac{44}{4}$$

$$k = 11$$

18. If  $(x - 2)$  is a factor of  $2x^3 - x^2 + px - 2$ , then (i) find the value of  $p$ . (ii) with this value of  $p$ , factorize the above expression completely.

**Solution:-**

Let us assume  $x - 2 = 0$

Then,  $x = 2$

Given,  $f(x) = 2x^3 - x^2 + px - 2$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(2) &= (2 \times 2^3) - 2^2 + (p \times 2) - 2 \\ &= (2 \times 8) - 4 + 2p - 2 \\ &= 16 - 4 + 2p - 2 \\ &= 16 - 6 + 2p \\ &= 10 + 2p \end{aligned}$$

From the question,  $(x - 2)$  is a factor of  $2x^3 - x^2 + px - 2$

Then, remainder is 0.

$$10 + 2p = 0$$

$$2p = -10$$

$$P = -10/2$$

$$P = -5$$

So,  $(x - 2)$  is a factor of  $2x^3 - x^2 + 5x - 2$

$$\begin{array}{r} \phantom{x-2} \quad \quad \quad 2x^2 \quad +3x \quad +11 \\ x-2 \quad \Big) \quad 2x^3 \quad -x^2 \quad +5x \quad -2 \\ \underline{\phantom{x-2} \quad \quad \quad 2x^3 \quad -4x^2} \phantom{+5x} \phantom{-2} \\ \phantom{x-2} \quad \quad \quad \phantom{2x^3} \quad 3x^2 \quad +5x \quad -2 \\ \underline{\phantom{x-2} \quad \quad \quad \phantom{2x^3} \quad 3x^2 \quad -6x} \phantom{-2} \\ \phantom{x-2} \quad \quad \quad \phantom{2x^3} \quad \phantom{3x^2} \quad 11x \quad -2 \\ \underline{\phantom{x-2} \quad \quad \quad \phantom{2x^3} \quad \phantom{3x^2} \quad 11x \quad -22} \\ \phantom{x-2} \quad \quad \quad \phantom{2x^3} \quad \phantom{3x^2} \quad \phantom{11x} \quad 20 \end{array}$$

$$\begin{aligned} \text{Therefore, } 2x^3 - x^2 + 5x - 2 &= (x - 2)(2x^2 + 3x + 1) \\ &= (x - 2)(2x^2 + 2x + x + 1) \\ &= (x - 2)(2x(x + 1) + 1(x + 1)) \end{aligned}$$

$$= (x + 1)(x - 2)(2x + 1)$$

**19. What number should be subtracted from  $2x^3 - 5x^2 + 5x$  so that the resulting polynomial has  $2x - 3$  as a factor?**

**Solution:-**

Let us assume the number to be subtracted from  $2x^3 - 5x^2 + 5x$  be  $p$ .

$$\text{Then, } f(x) = 2x^3 - 5x^2 + 5x - p$$

$$\text{Given, } 2x - 3 = 0$$

$$x = 3/2$$

$$f(3/2) = 0$$

$$\text{So, } f(3/2) = 2(3/2)^3 - 5(3/2)^2 + 5(3/2) - p = 0$$

$$2(27/8) - 5(9/4) + 15/2 - p = 0$$

$$27/4 - 45/4 + 15/2 - p = 0$$

[multiply by 4 for all numerator]

$$27 - 45 + 30 - 4p = 0$$

$$57 - 45 - 4p = 0$$

$$12 - 4p = 0$$

$$P = 12/4$$

$$P = 3$$

Therefore, 3 is the number should be subtracted from  $2x^3 - 5x^2 + 5x$ .

**20. (i) Find the value of the constants  $a$  and  $b$ , if  $(x - 2)$  and  $(x + 3)$  are both factors of the expression  $x^3 + ax^2 + bx - 12$ .**

**Solution:-**

Let us assume  $x - 2 = 0$

$$\text{Then, } x = 2$$

$$\text{Given, } f(x) = x^3 + ax^2 + bx - 12$$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(2) = 2^3 + a(2)^2 + b(2) - 12$$

$$= 8 + 4a + 2b - 12$$

$$= 4a + 2b - 4$$

From the question,  $(x - 2)$  is a factor of  $x^3 + ax^2 + bx - 12$ .

$$\text{So, } 4a + 2b - 4 = 0$$

$$4a + 2b = 4$$

By dividing both the side by 2 we get,

$$2a + b = 2$$

... [equation (i)]

Now, assume  $x + 3 = 0$

$$\text{Then, } x = -3$$

Given,  $f(x) = x^3 + ax^2 + bx - 12$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(-3) &= (-3)^3 + a(-3)^2 + b(-3) - 12 \\ &= -27 + 9a - 3b - 12 \\ &= 9a - 3b - 39\end{aligned}$$

From the question,  $(x - 3)$  is a factor of  $x^3 + ax^2 + bx - 12$ .

So,  $9a - 3b - 39 = 0$

$$9a - 3b = 39$$

By dividing both the side by 3 we get,

$$3a - b = 13 \quad \dots \text{ [equation (ii)]}$$

Now, adding both equation (i) and equation (ii) we get,

$$(2a + b) + (3a - b) = 2 + 13$$

$$2a + 3a + b - b = 15$$

$$5a = 15$$

$$a = 15/5$$

$$a = 3$$

Consider the equation (i) to find out 'b'.

$$2a + b = 2$$

$$2(3) + b = 2$$

$$6 + b = 2$$

$$b = 2 - 6$$

$$b = -4$$

**(ii) If  $(x + 2)$  and  $(x + 3)$  are factors of  $x^3 + ax + b$ , Find the values of  $a$  and  $b$ .**

**Solution:-**

Let us assume  $x + 2 = 0$

Then,  $x = -2$

Given,  $f(x) = x^3 + ax + b$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(-2) &= (-2)^3 + a(-2) + b \\ &= -8 - 2a + b\end{aligned}$$

From the question,  $(x + 2)$  is a factor of  $x^3 + ax + b$ .

Therefore, remainder is 0.

$$f(x) = 0$$

$$-8 - 2a + b = 0$$

$$2a - b = -8$$

... [equation (i)]

Let us assume  $x + 3 = 0$

Then,  $x = -3$

Given,  $f(x) = x^3 + ax + b$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(-2) &= (-3)^3 + a(-3) + b \\ &= -27 - 3a + b \end{aligned}$$

From the question,  $(x + 3)$  is a factor of  $x^3 + ax + b$ .

Therefore, remainder is 0.

$$f(x) = 0$$

$$-27 - 3a + b = 0$$

$$3a - b = -27$$

... [equation (i)]

Now, subtracting both equation (i) and equation (ii) we get,

$$(2a - b) - (3a - b) = -8 - (-27)$$

$$2a - 3a - b + b = -8 + 27$$

$$-a = 19$$

$$a = -19$$

Consider the equation (i) to find out 'b'.

$$2a - b = -8$$

$$2(-19) - b = -8$$

$$-38 - b = -8$$

$$b = -38 + 8$$

$$b = -30$$

**21. If  $(x + 2)$  and  $(x - 3)$  are factors of  $x^3 + ax + b$ , find the values of  $a$  and  $b$ . With these values of  $a$  and  $b$ , factorize the given expression.**

**Solution:-**

Let us assume  $x + 2 = 0$

Then,  $x = -2$

Given,  $f(x) = x^3 + ax + b$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(-2) &= (-2)^3 + a(-2) + b \\ &= -8 - 2a + b \end{aligned}$$

From the question,  $(x + 2)$  is a factor of  $x^3 + ax + b$ .

Therefore, remainder is 0.

$$f(x) = 0$$

$$-8 - 2a + b = 0$$

$$2a - b = -8$$

... [equation (i)]

Now, assume  $x - 3 = 0$

Then,  $x = 3$

Given,  $f(x) = x^3 + ax + b$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(3) &= (3)^3 + a(3) + b \\ &= 27 + 3a + b\end{aligned}$$

From the question,  $(x - 3)$  is a factor of  $x^3 + ax + b$ .

Therefore, remainder is 0.

$$f(x) = 0$$

$$27 + 3a + b = 0$$

$$3a + b = -27$$

... [equation (ii)]

Now, adding both equation (i) and equation (ii) we get,

$$(2a - b) + (3a + b) = -8 - 27$$

$$2a - b + 3a + b = -35$$

$$5a = -35$$

$$a = -35/5$$

$$a = -7$$

Consider the equation (i) to find out 'b'.

$$2a - b = -8$$

$$2(-7) - b = -8$$

$$-14 - b = -8$$

$$b = -14 + 8$$

$$b = -6$$

Therefore, value of  $a = -7$  and  $b = -6$ .

Then,  $f(x) = x^3 - 7x - 6$

$$(x + 2)(x - 3)$$

$$= x(x - 3) + 2(x - 3)$$

$$= x^2 - 3x + 2x - 6$$

$$= x^2 - x - 6$$

Dividing  $f(x)$  by  $x^2 - x - 6$  we get,

$$\begin{array}{r}
 x^2 - x - 6 \quad \left. \begin{array}{l} x + 1 \\ \hline x^3 + 0x^2 - 7x - 6 \\ \hline x^3 - x^2 - 6x \\ \hline x^2 - x - 6 \\ \hline x^2 - x - 6 \\ \hline 0 \end{array} \right\}
 \end{array}$$

Therefore,  $x^3 - 7x - 6 = (x + 1)(x + 2)(x - 3)$

**22.  $(x - 2)$  is a factor of the expression  $x^3 + ax^2 + bx + 6$ . When this expression is divided by  $(x - 3)$ , it leaves the remainder 3. Find the values of a and b.**

**Solution:-**

From the question it is given that,  $(x - 2)$  is a factor of the expression  $x^3 + ax^2 + bx + 6$

Then,  $f(x) = x^3 + ax^2 + bx + 6$  ... [equation (i)]

Let assume  $x - 2 = 0$

Then,  $x = 2$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(2) = 2^3 + a(2)^2 + 2b + 6$$

$$= 8 + 4a + 2b + 6$$

$$= 14 + 4a + 2b$$

By dividing the numbers by 2 we get,

$$= 7 + 2a + b$$

From the question,  $(x - 2)$  is a factor of the expression  $x^3 + ax^2 + bx + 6$ .

So, remainder is 0.

$$f(x) = 0$$

$$7 + 2a + b = 0$$

$$2a + b = -7 \quad \dots \text{ [equation (ii)]}$$

Now, expression is divided by  $(x - 3)$ , it leaves the remainder 3.





Then,  $x = 2$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(2) &= 2(2)^3 + a(2)^2 + 2b - 14 \\ &= 16 + 4a + 2b - 14 \\ &= 2 + 4a + 2b \end{aligned}$$

By dividing the numbers by 2 we get,

$$= 1 + 2a + b$$

From the question,  $(x - 2)$  is a factor of the expression  $2x^3 + ax^2 + bx - 14$ .

So, remainder is 0.

$$f(x) = 0$$

$$1 + 2a + b = 0$$

$$2a + b = -1 \quad \dots \text{ [equation (ii)]}$$

Now, expression is divided by  $(x - 3)$ , it leaves the remainder 52.

$$\begin{array}{r} \phantom{x-3} \quad 2x^2 + x(a+6) \\ x-3 \quad \overline{) 2x^3 + ax^2 + bx - 14} \\ \underline{-} \phantom{x-3} \quad 2x^3 \phantom{+ ax^2} - 6x^2 \phantom{+ bx} - 14 \\ \phantom{x-3} \quad \phantom{2x^3} \quad x^2(a+6) \phantom{+ bx} - 14 \\ \underline{-} \phantom{x-3} \quad \phantom{2x^3} \quad x^2(a+6) + x(-3a-18) \\ \phantom{x-3} \quad \phantom{2x^3} \quad \phantom{x^2(a+6)} \phantom{+ x(-3a-18)} - 14 \\ \underline{-} \phantom{x-3} \quad \phantom{2x^3} \quad \phantom{x^2(a+6)} \phantom{+ x(-3a-18)} \phantom{- 14} \\ \phantom{x-3} \quad \phantom{2x^3} \quad \phantom{x^2(a+6)} \phantom{+ x(-3a-18)} \phantom{- 14} \quad x(3a+b+18) + -9a-3b-54 \\ \underline{-} \phantom{x-3} \quad \phantom{2x^3} \quad \phantom{x^2(a+6)} \phantom{+ x(-3a-18)} \phantom{- 14} \phantom{x(3a+b+18)} \phantom{+ -9a-3b-54} \\ \phantom{x-3} \quad \phantom{2x^3} \quad \phantom{x^2(a+6)} \phantom{+ x(-3a-18)} \phantom{- 14} \phantom{x(3a+b+18)} \phantom{+ -9a-3b-54} \quad 9a + 3b + 40 \end{array}$$

So, remainder =  $9a + 3b + 40 = 52$

$$9a + 3b = 52 - 40$$

$$9a + 3b = 12$$

By dividing the numbers by 3 we get,

$$= 3a + b = 4 \quad \dots \text{ [equation (iii)]}$$

Now, subtracting equation (iii) from equation (ii) we get,

$$(3a + b) - (2a + b) = 4 - (-1)$$

$$3a - 2a + b - b = 4 + 1$$

$$a = 5$$

$$a = 5$$

Consider the equation (ii) to find out 'b'.

$$2a + b = -1$$

$$2(5) + b = -1$$

$$10 + b = -1$$

$$b = -1 - 10$$

$$b = -11$$

**24. If  $ax^3 + 3x^2 + bx - 3$  has a factor  $(2x + 3)$  and leaves remainder  $-3$  when divided by  $(x + 2)$ , find the values of  $a$  and  $b$ . With these values of  $a$  and  $b$ , factorize the given expression.**

**Solution:-**

Let us assume,  $2x + 3 = 0$

Then,  $2x = -3$

$$x = -3/2$$

Given,  $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-3/2) = a(-3/2)^3 + 3(-3/2)^2 + b(-3/2) - 3$$

$$= a(-27/8) + 3(9/4) - 3b/2 - 3$$

$$= -27a/8 + 27/4 - 3b/2 - 3$$

From the question it is given that,  $ax^3 + 3x^2 + bx - 3$  has a factor  $(2x + 3)$ .

So, remainder is 0.

$$-27a/8 + 27/4 - 3b/2 - 3 = 0$$

$$-27a + 54 - 12b - 24 = 0$$

$$-27a - 12b = -30$$

By dividing the numbers by  $-3$  we get,

$$9a + 4b = 10$$

[equation (i)]

Now, let us assume  $x + 2 = 0$

Then,  $x = -2$

Given,  $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$$

$$= -8a + 12 - 2b - 3$$

$$= -8a - 2b + 9$$

Leaves the remainder  $-3$

$$\text{So, } -8a - 2b + 9 = -3$$

$$-8a - 2b = -3 - 9$$

$$-8a - 2b = -12$$

By dividing both sides by -2 we get,

$$4a + b = 6$$

[equation (ii)]

By multiplying equation (ii) by 4,

$$16a + 4b = 24$$

Now, subtracting equation (ii) from equation (i) we get,

$$(16a + 4b) - (9a + 4b) = 24 - 10$$

$$16a - 9a + 4b - 4b = 14$$

$$7a = 14$$

$$a = 14/7$$

$$a = 2$$

Consider the equation (i) to find out 'b'.

$$9a + 4b = 10$$

$$9(2) + 4b = 10$$

$$18 + 4b = 10$$

$$4b = 10 - 18$$

$$4b = -8$$

$$b = -8/4$$

$$b = -2$$

$$\begin{aligned} \text{Therefore, } f(x) &= ax^3 + 3x^2 + bx - 3 \\ &= 2x^3 + 3x^2 - 2x - 3 \end{aligned}$$

Given,  $2x + 3$  is a factor of  $f(x)$

So, divide  $f(x)$  by  $2x + 3$

$$\begin{array}{r} x^2 - 1 \\ 2x + 3 \overline{) 2x^3 + 3x^2 - 2x - 3} \\ \underline{2x^3 + 3x^2} \phantom{- 2x - 3} \\ 0 - 2x - 3 \\ \phantom{0} \underline{-2x - 3} \\ \phantom{0} 0 \end{array}$$

$$\begin{aligned} \text{Therefore, } 2x^3 + 3x^2 - 2x - 3 &= (2x + 3)(x^2 - 1) \\ &= (2x + 3)(x + 1)(x - 1) \end{aligned}$$

**25. Given  $f(x) = ax^2 + bx + 2$  and  $g(x) = bx^2 + ax + 1$ . If  $x - 2$  is a factor of  $f(x)$  but leaves the remainder  $-15$  when it divides  $g(x)$ , find the values of  $a$  and  $b$ . With these values**

of  $a$  and  $b$ , factorize the expression.  $f(x) + g(x) + 4x^2 + 7x$ .

**Solution:-**

From the question it is given that,  $f(x) = ax^2 + bx + 2$  and  $g(x) = bx^2 + ax + 1$  and  $x - 2$  is a factor of  $f(x)$ ,

So,  $x = 2$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(2) = 0$$

$$a(2)^2 + b(2) + 2 = 0$$

$$4a + 2b + 2 = 0$$

By dividing both sides by 2 we get,

$$2a + b + 1 = 0 \quad \dots \text{[equation (i)]}$$

Given,  $g(x)$  divide by  $(x - 2)$ , leaves remainder  $-15$

$$g(x) = bx^2 + ax + 1$$

So,  $g(2) = -15$

$$b(2)^2 + 2a + 1 = -15$$

$$4b + 2a + 1 + 15 = 0$$

$$4b + 2a + 16 = 0$$

By dividing both sides by 2 we get,

$$2b + a + 8 = 0 \quad \dots \text{[equation (ii)]}$$

Now, subtracting equation (ii) from equation (i) multiplied by 2,

$$(4a + 2b + 2) - (a + 2b + 8) = 0 - 0$$

$$4a - a + 2b - 2b + 2 - 8 = 0$$

$$3a - 6 = 0$$

$$3a = 6$$

$$a = 6/3$$

$$a = 2$$

Consider the equation (i) to find out 'b'.

$$2a + b + 1 = 0$$

$$2(2) + b = -1$$

$$4 + b = -1$$

$$b = -1 - 4$$

$$b = -5$$

Now,  $f(x) = ax^2 + bx + 2 = 2x^2 - 5x + 2$

$g(x) = bx^2 + ax + 1 = -5x^2 + 2x + 1$

then,  $f(x) + g(x) + 4x^2 + 7x$

$$= 2x^2 - 5x + 2 - 5x^2 + 2x + 1 + 4x^2 + 7x$$

$$= x^2 + 4x + 3$$

$$\begin{aligned} &= x^2 + 3x + x + 3 \\ &= x(x + 3) + 1(x + 3) \\ &= (x + 1)(x + 3) \end{aligned}$$



## CHAPTER TEST

1. Find the remainder when  $2x^3 - 3x^2 + 4x + 7$  is divided by

(i)  $x - 2$

(ii)  $x + 3$

(iii)  $2x + 1$

**Solution:-**

From the question it is given that,  $f(x) = 2x^3 - 3x^2 + 4x + 7$

(i) Consider  $x - 2$

let us assume  $x - 2 = 0$

Then,  $x = 2$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(2) &= 2(2)^3 - 3(2)^2 + 4(2) + 7 \\&= 16 - 12 + 8 + 7 \\&= 31 - 12 \\&= 19\end{aligned}$$

Therefore, the remainder is 19

(ii) consider  $x + 3$

let us assume  $x + 3 = 0$

Then,  $x = -3$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(-3) &= 2(-3)^3 - 3(-3)^2 + 4(-3) + 7 \\&= 2(-27) - 3(9) - 12 + 7 \\&= -54 - 27 - 12 + 7 \\&= -93 + 7 \\&= -86\end{aligned}$$

Therefore, remainder is -86.

(iii) consider  $2x + 1$

Let us assume,  $2x + 1 = 0$

Then,  $2x = -1$

$$x = -\frac{1}{2}$$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 \\&= 2\left(-\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) + 4\left(-\frac{1}{2}\right) + 7 \\&= -\frac{1}{4} - \frac{3}{4} - 2 + 7 \\&= -1 - 2 + 7 \\&= 4\end{aligned}$$

Therefore, remainder is 4.

**2. When  $2x^3 - 9x^2 + 10x - p$  is divided by  $(x + 1)$ , the remainder is  $-24$ . Find the value of  $p$ .**

**Solution:-**

Let us assume  $x + 1 = 0$

Then,  $x = -1$

Given,  $f(x) = 2x^3 - 9x^2 + 10x - p$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(-1) &= 2(-1)^3 - 9(-1)^2 + 10(-1) - p \\ &= -2 - 9 - 10 + p \\ &= -21 + p \end{aligned}$$

From the question it is given that, the remainder is  $-24$ ,

$$\text{So, } -21 + p = -24$$

$$p = -24 + 21$$

$$p = -3$$

$$\begin{aligned} \text{So, } f(x) &= 2x^3 - 9x^2 + 10x - (-3) \\ &= 2x^3 - 9x^2 + 10x + 3 \end{aligned}$$

Therefore, the value of  $p$  is 3.

**3. If  $(2x - 3)$  is a factor of  $6x^2 + x + a$ , find the value of  $a$ . With this value of  $a$ , factorise the given expression.**

**Solution:-**

Let us assume  $2x - 3 = 0$

Then,  $2x = 3$

$$x = 3/2$$

Given,  $f(x) = 6x^2 + x + a$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned} f(3/2) &= 6(3/2)^2 + (3/2) + a \\ &= 6(9/4) + (3/2) + a \\ &= 3(9/2) + (3/2) + a \\ &= 27/2 + 3/2 + a \\ &= 30/2 + a \\ &= 15 + a \end{aligned}$$

From the question,  $(2x - 3)$  is a factor of  $6x^2 + x + a$ .

So, remainder is 0.

$$\text{Then, } 15 + a = 0$$

$$a = -15$$

Therefore,  $f(x) = 6x^2 + x - 15$

Dividing  $f(x)$  by  $2x - 3$  we get,

$$\begin{array}{r}
 3x + 5 \\
 2x - 3 \overline{) 6x^2 + x - 15} \\
 \underline{6x^2 - 9x} \phantom{-15} \\
 10x - 15 \\
 \underline{10x - 15} \\
 0
 \end{array}$$

Therefore,  $6x^2 + x - 15 = (2x - 3)(3x + 5)$

**4. When  $3x^2 - 5x + p$  is divided by  $(x - 2)$ , the remainder is 3. Find the value of  $p$ . Also factorize the polynomial  $3x^2 - 5x + p - 3$ .**

**Solution:-**

Let us assume  $x - 2 = 0$

Then,  $x = 2$

Given,  $f(x) = 3x^2 - 5x + p$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 \text{So, } f(2) &= 3(2)^2 - 5(2) + p \\
 &= 3(4) - 10 + p \\
 &= 12 - 10 + p \\
 &= 2 + p
 \end{aligned}$$

From the question it is given that, remainder is 3.

So,  $2 + p = 3$

$$p = 3 - 2$$

$$p = 1$$

Therefore,  $f(x) = 3x^2 - 5x + 1$

Consider the polynomial,  $3x^2 - 5x + p - 3$

Now, substitute the value of  $p$  in polynomial,

$$\begin{aligned}
 &= 3x^2 - 5x + 1 - 3 \\
 &= 3x^2 - 5x - 2
 \end{aligned}$$

Now, by factorizing the polynomial  $3x^2 - 5x - 2$ ,

Dividing  $3x^2 - 5x - 2$  by  $x - 2$  we get,



$$\begin{array}{r}
 3x + 1 \\
 x - 2 \overline{) 3x^2 - 5x - 2} \\
 \underline{-} \\
 3x^2 - 6x \\
 \underline{-} \\
 x - 2 \\
 \underline{-} \\
 x - 2 \\
 \underline{-} \\
 0
 \end{array}$$

Therefore,  $3x^2 - 5x - 2 = (x - 2)(3x + 1)$

**5. Prove that  $(5x + 4)$  is a factor of  $5x^3 + 4x^2 - 5x - 4$ . Hence factorize the given polynomial completely.**

**Solution:-**

Let us assume  $(5x + 4) = 0$

Then,  $5x = -4$

$$x = -4/5$$

Given,  $f(x) = 5x^3 + 4x^2 - 5x - 4$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 \text{So, } f(-4/5) &= 5(-4/5)^3 + 4(-4/5)^2 - 5(-4/5) - 4 \\
 &= 5(-64/125) + 4(16/25) + 4 - 4 \\
 &= -64/25 + 64/25 \\
 &= (-64 + 64)/25 \\
 &= 0/25 \\
 &= 0
 \end{aligned}$$

Hence,  $(5x + 4)$  is a factor of  $5x^3 + 4x^2 - 5x - 4$ .

So, dividing  $5x^3 + 4x^2 - 5x - 4$  by  $5x + 4$  we get,

$$\begin{array}{r}
 x^2 - 1 \\
 5x + 4 \overline{) 5x^3 + 4x^2 - 5x - 4} \\
 \underline{-} \\
 5x^3 + 4x^2 \\
 \underline{-} \\
 0 - 5x - 4 \\
 \underline{-} \\
 -5x - 4x \\
 \underline{-} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } 5x^3 + 4x^2 - 5x - 4 &= (5x + 4)(x^2 - 1) \\
 &= (5x + 4)(x^2 - 1^2) \\
 &= (5x + 4)(x + 1)(x - 1)
 \end{aligned}$$

**6. Use factor theorem to factorize the following polynomials completely:**

**(i)  $4x^3 + 4x^2 - 9x - 9$**

**Solution:-**

Let us assume  $x = -1$ ,

Given,  $f(x) = 4x^3 + 4x^2 - 9x - 9$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 f(-1) &= 4(-1)^3 + 4(-1)^2 - 9(-1) - 9 \\
 &= -4 + 4 + 9 - 9 \\
 &= 0
 \end{aligned}$$

Therefore,  $x + 1$  is the factor of  $4x^3 + 4x^2 - 9x - 9$ .

Now, dividing  $4x^3 + 4x^2 - 9x - 9$  by  $x + 1$  we get,

$$\begin{array}{r}
 \phantom{x+1} \quad 4x^2 \quad -9 \\
 x+1 \overline{) 4x^3 + 4x^2 - 9x - 9} \\
 \underline{4x^3 + 4x^2} \phantom{-9x - 9} \\
 \phantom{4x^3 + 4x^2} 0 \quad -9x \quad -9 \\
 \phantom{4x^3 + 4x^2} \underline{-9x \quad -9} \\
 \phantom{4x^3 + 4x^2} \phantom{-9x} 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } 4x^3 + 4x^2 - 9x - 9 &= (x + 1)(4x^2 - 9) \\
 &= (x + 1)((2x)^2 - (3)^2) \\
 &= (x + 1)(2x + 3)(2x - 3)
 \end{aligned}$$

**(ii)  $x^3 - 19x - 30$**

**Solution:-**

Let us assume  $x = -2$ ,

Given,  $f(x) = x^3 - 19x - 30$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}
 f(-2) &= (-2)^3 - 19(-2) - 30 \\
 &= -8 + 38 - 30
 \end{aligned}$$

$$= -38 + 38$$

$$= 0$$

Therefore,  $x + 2$  is the factor of  $x^3 - 19x - 30$ .

Now, dividing  $x^3 - 19x - 30$  by  $x + 2$  we get,

$$\begin{array}{r}
 x^2 - 2x - 15 \\
 x + 2 \overline{) x^3 + 0x^2 - 19x - 30} \\
 \underline{-} \\
 x^3 + 2x^2 \\
 \underline{-} \\
 -2x^2 - 19x - 30 \\
 \underline{-} \\
 -2x^2 - 4x \\
 \underline{-} \\
 -15x - 30 \\
 \underline{-} \\
 -15x - 30 \\
 \underline{-} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } x^3 - 19x - 30 &= (x + 2)(x^2 - 2x - 15) \\
 &= (x + 2)(x^2 - 5x + 3x - 15) \\
 &= (x + 2)(x - 5)(x + 3)
 \end{aligned}$$

**7. If  $x^3 - 2x^2 + px + q$  has a factor  $(x + 2)$  and leaves a remainder 9, when divided by  $(x + 1)$ , find the values of  $p$  and  $q$ . With these values of  $p$  and  $q$ , factorize the given polynomial completely.**

**Solution:-**

From the question it is given that,  $(x + 2)$  is a factor of the expression  $x^3 - 2x^2 + px + q$

$$\text{Then, } f(x) = x^3 - 2x^2 + px + q$$

$$\text{Let assume } x + 2 = 0$$

$$\text{Then, } x = -2$$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-2) = (-2)^3 - 2(-2)^2 + p(-2) + q$$

$$= -8 - 8 - 2p + q$$

$$= -16 - 2p + q$$

$$2p - q = -16$$

... [equation (i)]

Now, consider  $(x + 1)$

Then,  $f(x) = x^3 - 2x^2 + px + q$

Let assume  $x + 1 = 0$

Then,  $x = -1$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$f(-1) = (-1)^3 - 2(-1)^2 + p(-1) + q$$

$$= -1 - 2 - p + q$$

$$= -3 - p + q$$

Given, remainder is 9

So,  $-3 - p + q = 9$

$$-p + q = 9 + 3$$

$$-p + q = 12$$

... [equation (ii)]

Now, adding equation (i) and equation (ii) we get,

$$(2p - q) + (-p + q) = -16 + 12$$

$$2p - q - p + q = -4$$

$$p = -4$$

Consider the equation (ii) to find out 'b'.

$$-p + q = 12$$

$$-(-4) + q = 12$$

$$4 + q = 12$$

$$q = 12 - 4$$

$$q = 8$$

Therefore, by substituting the value of  $p$  and  $q$   $f(x) = x^3 - 2x^2 - 4x + 8$

Dividing  $f(x)$  by  $(x + 2)$  we get,

$$\begin{array}{r}
 x^2 - 4x + 4 \\
 x + 2 \overline{) x^3 - 2x^2 - 4x + 8} \\
 \underline{-} \\
 x^3 + 2x^2 \\
 \underline{-} \\
 -4x^2 - 4x + 8 \\
 \underline{-} \\
 -4x^2 - 8x \\
 \underline{-} \\
 4x + 8 \\
 \underline{-} \\
 4x + 8 \\
 \underline{-} \\
 0
 \end{array}$$

$$\begin{aligned}x^3 - 2x^2 - 4x + 8 &= (x + 2)(x^2 - 4x + 4) \\ &= (x + 2)(x^2 - 2 \times x(-2) + 2^2) \\ &= (x + 2)(x - 2)^2\end{aligned}$$

**8. If  $(x + 3)$  and  $(x - 4)$  are factors of  $x^3 + ax^2 - bx + 24$ , find the values of  $a$  and  $b$ : With these values of  $a$  and  $b$ , factorize the given expression.**

**Solution:-**

Let us assume  $x + 3 = 0$

Then,  $x = -3$

Given,  $f(x) = x^3 + ax^2 - bx + 24$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(-3) &= (-3)^3 + a(-3)^2 - b(-3) + 24 \\ &= -27 + 9a + 3b + 24 \\ &= 9a + 3b - 3\end{aligned}$$

Dividing all terms by 3 we get,

$$= 3a + b - 1$$

From the question,  $(x + 3)$  is a factor of  $x^3 + ax^2 - bx + 24$ .

Therefore, remainder is 0.

$$f(x) = 0$$

$$3a + b - 1 = 0$$

$$3a + b = 1$$

... [equation (i)]

Now, assume  $x - 4 = 0$

Then,  $x = 4$

Given,  $f(x) = x^3 + ax^2 - bx + 24$

Now, substitute the value of  $x$  in  $f(x)$ ,

$$\begin{aligned}f(4) &= 4^3 + a(4)^2 - b(4) + 24 \\ &= 64 + 16a - 4b + 24 \\ &= 88 + 16a - 4b\end{aligned}$$

Dividing all terms by 4 we get,

$$= 22 + 4a - b$$

From the question,  $(x - 4)$  is a factor of  $x^3 + ax^2 - bx + 24$ .

Therefore, remainder is 0.

$$f(x) = 0$$

$$22 + 4a - b = 0$$

$$4a - b = -22$$

... [equation (ii)]

Now, adding both equation (i) and equation (ii) we get,

$$(3a + b) + (4a - b) = 1 - 22$$

$$3a + b + 4a - b = -21$$

$$7a = -21$$

$$a = -21/7$$

$$a = -3$$

Consider the equation (i) to find out 'b'.

$$3a + b = 1$$

$$3(-3) + b = 1$$

$$-9 + b = 1$$

$$b = 1 + 9$$

$$b = 10$$

Therefore, value of  $a = -3$  and  $b = 10$ .

Then, by substituting the value of  $a$  and  $b$   $f(x) = x^3 - 3x^2 - 10x + 24$

$$(x + 3)(x - 4)$$

$$= x(x - 4) + 3(x - 4)$$

$$= x^2 - 4x + 3x - 12$$

$$= x^2 - x - 12$$

Dividing  $f(x)$  by  $x^2 - x - 12$  we get,

$$\begin{array}{r}
 x^2 - x - 12 \quad \overline{) \quad x^3 - 3x^2 - 10x + 24} \\
 \underline{x^3 - x^2 - 12x} \phantom{+ 24} \\
 -2x^2 + 2x + 24 \\
 \underline{-2x^2 + 2x + 24} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Therefore, } x^3 - 3x^2 - 10x + 24 &= (x^2 - x - 12)(x - 2) \\
 &= (x + 3)(x - 4)(x - 2)
 \end{aligned}$$

9. If  $(2x + 1)$  is a factor of both the expressions  $2x^2 - 5x + p$  and  $2x^2 + 5x + q$ , find the value of  $p$  and  $q$ . Hence find the other factors of both the polynomials.

**Solution:-**

Let us assume  $2x + 1 = 0$

Then,  $2x = -1$

$$x = -\frac{1}{2}$$







$$2a = 10$$

$$a = 10/2$$

$$a = 5$$

To find out the value of b, substitute the value of a in equation (i) we get,

$$a + b = 13$$

$$5 + b = 13$$

$$b = 13 - 5$$

$$b = 8$$

Therefore, value of a = 5 and b = 8

**11. When a polynomial  $f(x)$  is divided by  $(x - 1)$ , the remainder is 5 and when it is, divided by  $(x - 2)$ , the remainder is 7. Find the remainder when it is divided by  $(x - 1)(x - 2)$ .**

**Solution:-**

From the question it is given that,

Polynomial  $f(x)$  is divided by  $(x - 1)$ ,

Remainder = 5

Let us assume  $x - 1 = 0$

$$x = 1$$

$$f(1) = 5$$

and the divided by  $(x - 2)$ , remainder = 7

let us assume  $x - 2 = 0$

$$x = 2$$

Therefore,  $f(2) = 7$

So,  $f(x) = (x - 1)(x - 2)q(x) + ax + b$

Where,  $q(x)$  is the quotient and  $ax + b$  is remainder,

Now put  $x = 1$ , we get,

$$f(1) = (1 - 1)(1 - 2)q(1) + (a \times 1) + b$$

$$a + b = 5 \quad \dots \text{ [equation (i)]}$$

$$x = 2,$$

$$f(2) = (2 - 1)(2 - 2)q(2) + (a \times 2) + b$$

$$2a + b = 7 \quad \dots \text{ [equation (ii)]}$$

Now subtracting equation (i) from equation (ii) we get,

$$(2a + b) - (a + b) = 7 - 5$$

$$2a + b - a - b = 2$$

$$a = 2$$

To find out the value of b, substitute the value of a in equation (i) we get,

$$a + b = 5$$

$$2 + b = 5$$

$$b = 5 - 2$$

$$b = 3$$

Therefore, the remainder =  $ax + b = 2x + 3$

