

## EXERCISE 7.1

1. An alloy consists of 27 ½ kg of copper and 2 ¾ kg of tin. Find the ratio by weight of tin to the alloy. Solution:

It is given that Copper =  $27 \frac{1}{2} \text{ kg} = 55/2 \text{ kg}$ Tin =  $2 \frac{3}{4} \text{ kg} = 11/4 \text{ kg}$ 

We know that Total alloy = 55/2 + 11/4 Taking LCM = (110 + 11)/ 4 = 121/4 kg

Here Ratio between tin and alloy = 11/4 kg: 121/4 kg So we get = 11: 121 = 1: 11

2. Find the compounded ratio of:
(i) 2: 3 and 4: 9
(ii) 4: 5, 5: 7 and 9: 11
(iii) (a - b): (a + b), (a + b)<sup>2</sup>: (a<sup>2</sup> + b<sup>2</sup>) and (a<sup>4</sup> - b<sup>4</sup>): (a<sup>2</sup> - b<sup>2</sup>)<sup>2</sup>
Solution:

(i) 2: 3 and 4: 9 We know that Compound ratio = 2/3 × 4/9 = 8/27 = 8: 27

(ii) 4: 5, 5: 7 and 9: 11 We know that Compound ratio = 4/5 × 5/7 × 9/11 = 36/77 = 36: 77

(iii) (a - b): (a + b),  $(a + b)^2$ :  $(a^2 + b^2)$  and  $(a^4 - b^4)$ :  $(a^2 - b^2)^2$ We know that Compound ratio =  $(a - b)/(a + b) \times (a + b)^2/(a^2 + b^2) \times (a^4 - b^4)/(a^2 - b^2)^2$ By further calculation =  $(a - b)/(a + b) \times [(a + b) (a + b)]/(a^2 + b^2) \times [(a^2 + b^2) (a + b) (a - b)]/[(a + b)^2 (a - b)^2]$ So we get = 1/1= 1: 1

3. Find the duplicate ratio of (i) 2: 3



(ii) √5: 7
(iii) 5a: 6b
Solution:

(i) 2: 3 We know that Duplicate ratio of 2:  $3 = 2^2$ :  $3^2 = 4$ : 9

(ii)  $\sqrt{5}$ : 7 We know that Duplicate ratio of  $\sqrt{5}$ : 7 =  $\sqrt{5^2}$ : 7<sup>2</sup> = 5: 49

(iii) 5a: 6b We know that Duplicate ratio of 5a:  $6b = (5a)^2$ :  $(6b)^2 = 25a^2$ :  $36b^2$ 

4. Find the triplicate ratio of
(i) 3: 4
(ii) <sup>1</sup>/<sub>2</sub>: 1/3
(iii) 1<sup>3</sup>: 2<sup>3</sup>
Solution:

(i) 3: 4 We know that Triplicate ratio of 3:  $4 = 3^3$ :  $4^3 = 27$ : 64

(ii)  $\frac{1}{2}$ : 1/3 We know that Triplicate ratio of  $\frac{1}{2}$ : 1/3 =  $(1/2)^3$ :  $(1/3)^3 = 1/8$ : 1/27 = 27: 8

(iii)  $1^3: 2^3$ We know that Triplicate ratio of  $1^3: 2^3 = (1^3)^3: (2^3)^3 = 1^3: 8^3 = 1: 512$ 

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5. Find the sub-duplicate ratio of
(i) 9: 16
(ii) <sup>1</sup>/<sub>4</sub>: 1/9
(iii) 9a<sup>2</sup>: 49b<sup>2</sup>
Solution:
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(i) 9: 16 We know that Sub-duplicate ratio of 9:  $16 = \sqrt{9}$ :  $\sqrt{16} = 3: 4$ 

(ii)  $\frac{1}{4}$ : 1/9 We know that Sub-duplicate ratio of  $\frac{1}{4}$ : 1/9 =  $\sqrt{1/4}$ :  $\sqrt{1/9}$ So we get =  $\frac{1}{2}$ : 1/3 = 3: 2



(iii)  $9a^2$ :  $49b^2$ We know that Sub-duplicate ratio of  $9a^2$ :  $49b^2 = \sqrt{9a^2}$ :  $\sqrt{49b^2} = 3a$ : 7b

6. Find the sub-triplicate ratio of
(i) 1: 216
(ii) 1/8: 1/125
(iii) 27a<sup>3</sup>: 64b<sup>3</sup>
Solution:

(i) 1: 216 We know that Sub-triplicate ratio of 1: 216 =  $\sqrt[3]{1: \sqrt[3]{216}}$ By further calculation =  $(1^3)^{1/3}$ :  $(6^3)^{1/3}$ = 1: 6

(ii) 1/8: 1/125 We know that Sub-triplicate ratio of 1/8:  $1/125 = (1/8)^{1/3}$ :  $(1/125)^{1/3}$ It can be written as =  $[(1/2)^3]^{1/3}$ :  $[(1/5)^3]^{1/3}$ So we get =  $\frac{1}{2}$ : 1/5 = 5: 2

(iii)  $27a^3$ :  $64b^3$ We know that Sub-triplicate ratio of  $27a^3$ :  $64b^3 = [(3a)^3]^{1/3}$ :  $[(4b)^3]^{1/3}$ So we get = 3a: 4b

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7. Find the reciprocal ratio of
(i) 4: 7
(ii) 3<sup>2</sup>: 4<sup>2</sup>
(iii) 1/9: 2
Solution:
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(i) 4: 7 We know that Reciprocal ratio of 4: 7 = 7: 4

(ii)  $3^2$ :  $4^2$ We know that Reciprocal ratio of  $3^2$ :  $4^2 = 4^2$ :  $3^2 = 16$ : 9

(iii) 1/9: 2 We know that Reciprocal ratio of 1/9: 2 = 2: 1/9 = 18: 1



#### 8. Arrange the following ratios in ascending order of magnitude: 2: 3, 17: 21, 11: 14 and 5: 7 Solution:

It is given that 2: 3, 17: 21, 11: 14 and 5: 7 We can write it in fractions as 2/3, 17/21, 11/14, 5/7 Here the LCM of 3, 21, 14 and 7 is 42 By converting the ratio as equivalent  $2/3 = (2 \times 14)/(3 \times 14) = 28/42$  $17/21 = (17 \times 2)/(21 \times 2) = 34/42$  $11/14 = (11 \times 3)/(14 \times 3) = 33/42$  $5/7 = (5 \times 6)/(7 \times 6) = 30/42$ Now writing it in ascending order 28/42, 30/42, 33/42, 34/42 By further simplification 2/3, 5/7, 11/14, 17/21 So we get 2: 3, 5: 7, 11: 14 and 17: 21

### 9. (i) If A: B = 2: 3, B: C = 4: 5 and C: D = 6: 7, find A: D. (ii) If x: y = 2: 3 and y: z = 4: 7, find x: y: z. Solution:

(i) It is given that A: B = 2: 3, B: C = 4: 5 and C: D = 6: 7 We can write it as A/B = 2/3, B/C = 4/5, C/D = 6/7 By multiplication A/B × B/C × C/D =  $2/3 \times 4/5 \times 6/7$ So we get A/D = 16/35A: D = 16: 35

(ii) We know that the LCM of y terms 3 and 4 is 12 Now making equals of y as 12  $x/y = 2/3 = (2 \times 4)/(3 \times 4) = 8/12 = 8: 12$  $y/z = 4/7 \times 3/3 = 12/21 = 12: 21$ So x: y: z = 8: 12: 21

### 10. (i) If A: B = 1/4: 1/5 and B: C = 1/7: 1/6, find A: B: C. (ii) If 3A = 4B = 6C, find A: B: C Solution:

(i) We know that A:  $B = 1/4 \times 5/1 = 5/4$ B:  $C = 1/7 \times 6/1 = 6/7$ Here the LCM of B terms 4 and 6 is 12 Now making terms of B as 12



 $A/B = (5 \times 3)/(4 \times 3) = 15/12 = 15:12$   $B/C = (6 \times 2)/(7 \times 2) = 12/14 = 12:14$ So A: B: C = 15:12:14

(ii) It is given that 3A = 4BWe can write it as A/B = 4/3 A: B = 4: 3Similarly 4B = 6CWe can write it as B/C = 6/4 = 3/2B: C = 3: 2 So we get A: B: C = 4: 3: 2

11. (i) If 3x + 5y/ 3x - 5y = 7/3, find x: y. (ii) If a: b = 3: 11, find (15a - 3b): (9a + 5b). Solution:

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(i) 3x + 5y/3x - 5y = 7/3
By cross multiplication
9x + 15y = 21x - 35y
By further simplification
21x - 9x = 15y + 35y
12x = 50y
So we get
x/y = 50/12 = 25/6
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Therefore, x: y = 25: 6

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(ii) It is given that
a: b = 3: 11
a/b = 3/11
It is given that
(15a - 3b)/(9a + 5b)
Now dividing both numerator and denominator by b
= [15a/b - 3b/b]/[9a/b + 5b/b]
By further calculation
= [15a/b - 3]/[9a/b + 5]
Substituting the value of a/b
= [15 \times 3/11 - 3]/[9 \times 3/11 + 5]
So we get
= [45/11 - 3]/[27/11 + 5]
Taking LCM
= [(45 - 33)/11]/[(27 + 55)/11]
= 12/11/82/11
We can write it as
= 12/11 \times 11/82
= 12/82
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= 6/41

Hence, (15a - 3b): (9a + 5b) = 6: 41.

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12. (i) If (4x^2 + xy): (3xy - y^2) = 12: 5, find (x + 2y): (2x + y).
(ii) If y (3x - y): x (4x + y) = 5: 12. Find (x^2 + y^2): (x + y)^2.
Solution:
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(i) (4x^2 + xy): (3xy - y^2) = 12: 5
We can write it as
(4x^2 + xy)/(3xy - y^2) = 12/5
By cross multiplication
20x^2 + 5xy = 36xy - 12y^2
20x^2 + 5xy - 36xy + 12y^2 = 0
20x^2 - 31xy + 12y^2 = 0
Now divide the entire equation by y^2
20x^2/y^2 - 31xy/y^2 + 12y^2/y^2 = 0
So we get
20 (x/y)^2 - 31 (x/y) + 12 = 0
20 (x/y)^2 - 15(x/y) - 16 (x/y) + 12 = 0
Taking common terms
5 (x/y) [4 (x/y) - 3] - 4 [4 (x/y) - 3] = 0
[4 (x/y) - 3] [5 (x/y) - 4] = 0
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Here 4 (x/y) - 3 = 04 (x/y) = 3So we get  $x/y = \frac{3}{4}$ 

Similarly 5 (x/y) - 4 = 05 (x/y) = 4So we get x/y = 4/5

Now dividing by y (x + 2y)/(2x + y) = (x/y + 2)/(2x/y + 1)

(a) If x/y = 3/4, then = (x/y + 2)/(2 x/y + 1)Substituting the values =  $(3/4 + 2)/(2 \times 3/4 + 1)$ By further calculation = 11/4/(3/2 + 1)= 11/4/5/2=  $11/4 \times 2/5$ = 11/10So we get (x + 2y): (2x + y) = 11: 10

(b) If x/y = 4/5 then (x + 2y)/(2x + y) = [x/y + 2]/[2 x/y + 1] Substituting the value of x/y



 $= [4/5 + 2]/[2 \times 4/5 + 1]$ So we get = 14/5/[8/5+1]= 14/5/13/5 $= 14/5 \times 5/13$ = 14/13We get (x + 2y)/(2x + y) = 11/10 or 14/13(x + 2y): (2x + y) = 11: 10 or 14: 13(ii) y (3x - y): x (4x + y) = 5: 12It can be written as  $(3xy - y^2)/(4x^2 + xy) = 5/12$ By cross multiplication  $36xy - 12y^2 = 20x^2 + 5xy$  $20x^2 + 5xy - 36xy + 12y^2 = 0$  $20x^2 - 31xy + 12y^2 = 0$ Divide the entire equation by  $y^2$  $20x^2/y^2 - 31 xy/y^2 + 12y^2/y^2 = 0$  $20(x^2/y^2) - 31(xy/y^2) + 12 = 0$ We can write it as  $20(x^2/y^2) - 15(x/y) - 16(x/y) + 12 = 0$ Taking common terms 5 (x/y) [4 (x/y) - 3] - 4 [4 (x/y) - 3] = 0[4 (x/y) - 3] [5 (x/y) - 4] = 0Here 4(x/y) - 3 = 0So we get

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So we get 4 (x/y) = 3x/y = 3/4

#### Similarly

5 (x/y) - 4 = 0So we get 5 (x/y) = 4x/y = 4/5

(a) x/y = 3/4We know that  $(x^2 + y^2): (x + y)^2 = (x^2 + y^2)/(x + y)^2$ Dividing both numerator and denominator by  $y^2 = (x^2/y^2 + y^2/y^2)/[1/y^2 (x + y)^2]$  $= (x^2/y^2 + 1)(x/y + 1)^2$ Substituting the value of x/y $= [(3/4)^2 + 1]/[3/4 + 1]^2$ By further calculation  $= (9/16 + 1)/(7/4)^2$ So we get



= 25/16/49/16 $= 25/16 \times 16/49$ = 25/49So we get  $(x^{2} + y^{2}): (x + y)^{2} = 25:49$ (b) x/y = 4/5We know that  $(x^{2} + y^{2}): (x + y)^{2} = (x^{2} + y^{2})/(x + y)^{2}$ Dividing both numerator and denominator by  $y^2$  $= (x^2/y^2 + y^2/y^2) / [1/y^2 (x + y)^2]$  $=(x^{2}/y^{2}+1)(x/y+1)^{2}$ Substituting the value of x/y  $= [(4/5)^2 + 1]/[4/5 + 1]^2$ By further calculation  $=(16/25+1)/(9/5)^{2}$ So we get = 41/25/ 81/25  $=41/25 \times 25/81$ =41/81So we get  $(x^{2} + y^{2}): (x + y)^{2} = 41:81$ 

13. (i) If (x - 9): (3x + 6) is the duplicate ratio of 4: 9, find the value of x. (ii) If (3x + 1): (5x + 3) is the triplicate ratio of 3: 4, find the value of x. (iii) If (x + 2y): (2x - y) is equal to the duplicate ratio of 3: 2, find x: y. Solution:

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(i) (x-9)/(3x+6) = (4/9)^2
So we get
(x-9)/(3x+6) = 16/81
By cross multiplication
81x - 729 = 48x + 96
81x - 48x = 96 + 729
So we get
33x = 825
x = 825/33 = 25
(ii) (3x + 1)/(5x + 3) = 3^3/4^3
So we get
(3x + 1)/(5x + 3) = 27/64
By cross multiplication
64 (3x + 1) = 27 (5x + 3)
192x + 64 = 135x + 81
192x - 135x = 81 - 64
57x = 17
So we get
x = 17/57
(iii) (x + 2y)/(2x - y) = 3^2/2^2
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So we get (x + 2y)/(2x - y) = 9/4By cross multiplication 9(2x - y) = 4(x + 2y) 18x - 9y = 4x + 8y 18x = 4x = 8y + 9ySo we get 14x = 17y x/y = 17/14x: y = 17: 14

14. (i) Find two numbers in the ratio of 8: 7 such that when each is decreased by 12 ½, they are in the ratio 11: 9.

(ii) The income of a man is increased in the ratio of 10: 11. If the increase in his income is Rs 600 per month, find his new income. Solution:

(i) Ratio = 8: 7 Consider the numbers as 8x and 7x Using the condition [8x - 25/2]/[7x - 25/2] = 11/9Taking LCM [(16x - 25)/2]/[(14x - 25)/2] = 11/9By further calculation  $[(16x - 25) \times 2]/[2(14x - 25)] = 11/9$ By cross multiplication 154x - 275 = 144x - 225154x - 144x = 275 - 22510x = 50x = 50/10 = 5

So the numbers are  $8x = 8 \times 5 = 40$  $7x = 7 \times 5 = 35$ 

(ii) Consider the present income = 10xIncreased income = 11xSo the increase per month = 11x - 10x = xHere x = Rs 600 New income =  $11x = 11 \times 600 = \text{Rs} 6600$ 

15. (i) A woman reduces her weight in the ratio 7: 5. What does her weight become if originally it was 91 kg.

(ii) A school collected Rs 2100 for charity. It was decided to divide the money between an orphanage and a blind school in the ratio of 3: 4. How much money did each receive? Solution:

(i) Ratio of original and reduced weight of woman = 7: 5 Consider original weight = 7x



Reduced weight = 5xHere original weight = 91 kgSo the reduced weight =  $(91 \times 5x)/7x = 65 \text{ kg}$ 

(ii) Amount collected for charity = Rs 2100 Here the ratio between orphanage and a blind school = 3: 4 Sum of ratios = 3 + 4 = 7

We know that Orphanage schools share =  $2100 \times 3/7$  = Rs 900 Blind schools share =  $2100 \times 4/7$  = Rs 1200

16. (i) The sides of a triangle are in the ratio 7: 5: 3 and its perimeter is 30 cm. Find the lengths of sides.(ii) If the angles of a triangle are in the ratio 2: 3: 4, find the angles.Solution:

(i) It is given that Perimeter of triangle = 30 cm Ratio among sides = 7: 5: 3 Here the sum of ratios = 7 + 5 + 3 = 15

We know that Length of first side =  $30 \times 7/15 = 14$  cm Length of second side =  $30 \times 5/15 = 10$  cm Length of third side =  $30 \times 3/15 = 6$  cm

Therefore, the sides are 14 cm, 10 cm and 6 cm.

(ii) We know that Sum of all the angles of a triangle =  $180^{\circ}$ Here the ratio among angles = 2: 3: 4 Sum of ratios = 2 + 3 + 4 = 9

So we get First angle =  $180 \times 2/9 = 40^{\circ}$ Second angle =  $180 \times 3/9 = 60^{\circ}$ Third angle =  $180 \times 4/9 = 80^{\circ}$ 

Hence, the angles are  $40^{\circ}$ ,  $60^{\circ}$  and  $80^{\circ}$ .

17. Three numbers are in the ratio 1/2: 1/3: <sup>1</sup>/<sub>4</sub>. If the sum of their squares is 244, find the numbers. Solution:

It is given that Ratio of three numbers = 1/2: 1/3: 1/4 = (6: 4: 3)/ 12 = 6: 4: 3

Consider first number = 6xSecond number = 4x



Third number = 3xSo based on the condition  $(6x)^2 + (4x)^2 + (3x)^2 = 244$  $36x^2 + 16x^2 + 9x^2 = 244$ So we get  $61x^2 = 244$  $x^2 = 244/61 = 4 = 2^2$ x = 2

Here First number =  $6x = 6 \times 2 = 12$ Second number =  $4x = 4 \times 2 = 8$ Third number =  $3x = 3 \times 2 = 6$ 

18. (i) A certain sum was divided among A, B and C in the ratio 7: 5: 4. If B got Rs 500 more than C, find the total sum divided.

(ii) In a business, A invests Rs 50000 for 6 months, B Rs 60000 for 4 months and C Rs 80000 for 5 months. If they together earn Rs 18800 find the share of each. Solution:

(i) It is given that Ratio between A, B and C = 7: 5: 4 Consider A share = 7xB share = 5xC share = 4xSo the total sum = 7x + 5x + 4x = 16x

Based on the condition 5x - 4x = 500 x = 500So the total sum =  $16x = 16 \times 500 = \text{Rs } 8000$ 

(ii) 6 months investment of A = Rs 50000 1 month investment of A =  $50000 \times 6 = \text{Rs} 300000$ 

4 months investment of B = Rs 600001 month investment of  $B = 60000 \times 4 = Rs 240000$ 

5 months investment of C = Rs 800001 month investment of C =  $80000 \times 5 = \text{Rs} 400000$ 

Here the ratio between their investments = 300000: 240000: 400000= 30: 24: 40Sum of ratio = 30 = 24 + 40 = 94Total earnings = Rs 18800

So we get A share = 30/94 × 18800 = Rs 6000 B share = 24/94 × 18800 = Rs 4800 C share = 40/94 = 18800 = Rs 8000



19. (i) In a mixture of 45 litres, the ratio of milk to water is 13: 2. How much water must be added to this mixture to make the ratio of milk to water as 3: 1?

(ii) The ratio of the number of boys to the numbers of girls in a school of 560 pupils is 5: 3. If 10 new boys are admitted, find how many new girls may be admitted so that the ratio of the number of boys to the number of girls may change to 3: 2.

Solution:

(i) It is given that Mixture of milk to water = 45 litres Ratio of milk to water = 13: 2 Sum of ratio = 13 + 2 = 15Here the quantity of milk =  $(45 \times 13)/15 = 39$  litres Quantity of water =  $45 \times 2/15 = 6$  litres

Consider x litre of water to be added, then water = (6 + x) litres Here the new ratio = 3: 1 39: (6 + x) = 3: 1 We can write it as 39/(6 + x) = 3/1By cross multiplication 39 = 18 + 3x3x = 39 - 18 = 21x = 21/3 = 7 litres

Hence, 7 litres of water is to be added to the mixture.

(ii) It is given that Ratio between boys and girls = 5: 3 Number of pupils = 560So the sum of ratios = 5 + 3 = 8

We know that Number of boys =  $5/8 \times 560 = 350$ Number of girls =  $3/8 \times 560 = 210$ Number of new boys admitted = 10So the total number of boys = 350 + 10 = 360

Consider x as the number of girls admitted Total number of girls = 210 + xBased on the condition 360: 210 + x = 3: 2We can write it as 360/210 + x = 3/2By cross multiplication 630 + 3x = 7203x = 720 - 630 = 90So we get x = 90/3 = 30

Hence, 30 new girls are to be admitted.



20. (i) The monthly pocket money of Ravi and Sanjeev are in the ratio 5: 7. Their expenditures are in the ratio 3: 5. If each saves Rs 80 per month, find their monthly pocket money.
(ii) In class X of a school, the ratio of the number of boys to that of the girls is 4: 3. If there were 20 more boys and 12 less girls, then the ratio would have been 2: 1. How many students were there in the class? Solution:

(i) Consider the monthly pocket money of Ravi and Sanjeev as 5x and 7x Their expenditure is 3y and 5y respectively.  $5x - 3y = 80 \dots (1)$  $7x - 5y = 80 \dots (2)$ Now multiply equation (1) by 7 and (2) by 5 Subtracting both the equations 35x - 21y = 56035x - 25y = 400So we get 4y = 160y = 40In equation (1)  $5x = 80 + 3 \times 40 = 200$ x = 40Here the monthly pocket money of Ravi =  $5 \times 40 = 200$ (ii) Consider x as the number of students in class Ratio of boys and girls = 4: 3Number of boys = 4x/7Number of girls = 3x/7Based on the problem (4x/7 + 20): (3x/7 - 12) = 2:1We can write it as (4x + 140)/7: (3x - 84)/7 = 2:1So we get  $(4x + 140)/7 \times 7/(3x - 84) = 2/1$ (4x + 140)/(3x - 84) = 2/16x - 168 = 4x + 1406x - 4x = 140 + 1682x = 308x = 308/2 = 154

Therefore, 154 students were there in the class.

21. In an examination, the ratio of passes to failures was 4: 1. If 30 less had appeared and 20 less passed, the ratio of passes to failures would have been 5: 1. How many students appeared for the examination. Solution:

Consider number of passes = 4xNumber of failures = xTotal number of students appeared = 4x + x = 5x



In case 2 Number of students appeared = 5x - 30Number of passes = 4x - 20So the number of failures = (5x - 30) - (4x - 20)By further calculation = 5x - 30 - 4x + 20= x - 10

Based on the condition (4x - 20)/(x - 10) = 5/1By cross multiplication 5x - 50 = 4x - 205x - 4x = -20 + 50x = 30

Number of students appeared =  $5x = 5 \times 30 = 150$ 



### **EXERCISE 7.2**

1. Find the value of x in the following proportions: (i) 10: 35 = x: 42(ii) 3: x = 24: 2 (iii) 2.5: 1.5 = x: 3(iv) x: 50 :: 3: 2 Solution: (i) 10: 35 = x: 42 We can write it as  $35 \times x = 10 \times 42$ So we get  $x = (10 \times 42)/35$  $x = 2 \times 6$ x = 12 (ii) 3: x = 24: 2We can write it as  $x \times 24 = 3 \times 2$ So we get  $x = (3 \times 2)/24$  $X = \frac{1}{4}$ (iii) 2.5: 1.5 = x: 3We can write it as  $1.5 \times x = 2.5 \times 3$ So we get  $x = (2.5 \times 3)/1.5$ x = 5.0(iv) x: 50 :: 3: 2 We can write it as  $x \times 2 = 50 \times 3$ So we get  $x = (50 \times 3)/2$ x = 75 2. Find the fourth proportional to (i) 3, 12, 15 (ii) 1/3, 1/4, 1/5

(ii) 1/3, 1/4, 1/5 (iii) 1.5, 2.5, 4.5 (iv) 9.6 kg, 7.2 kg, 28.8 kg Solution:

(i) 3, 12, 15 Consider x as the fourth proportional to 3, 12 and 15 3: 12 :: 15: x We can write it as  $3 \times x = 12 \times 15$ 



So we get  $x = (12 \times 15)/3$ x = 60(ii) 1/3, 1/4, 1/5 Consider x as the fourth proportional to 1/3, 1/4 and 1/51/3: 1/4:: 1/5: x We can write it as  $1/3 \times x = 1/4 \times 1/5$ So we get  $x = 1/4 \times 1/5 \times 3/1$ x = 3/20(iii) 1.5, 2.5, 4.5 Consider x as the fourth proportional to 1,5, 2.5 and 4.5 1.5: 2.5 :: 4.5: x We can write it as  $1.5 \times x = 2.5 \times 4.5$ So we get  $x = (2.5 \times 4.5)/1.5$ x = 7.5(iv) 9.6 kg, 7.2 kg, 28.8 kg Consider x as the fourth proportional to 9.6, 7.2 and 28.8 9.6: 7.2 :: 28.8: x We can write it as  $9.6 \times x = 7.2 \times 28.8$ So we get  $x = (7.2 \times 28.8)/9.6$ x = 21.63. Find the third proportional to (i) 5, 10 (ii) 0.24, 0.6 (iii) Rs. 3, Rs. 12 (iv) 5 <sup>1</sup>/<sub>4</sub> and 7. Solution: (i) Consider x as the third proportional to 5, 10 5: 10 :: 10: x It can be written as  $5 \times x = 10 \times 10$  $x = (10 \times 10)/5 = 20$ Hence, the third proportional to 5, 10 is 20.

(ii) Consider x as the third proportional to 0.24, 0.6 0.24: 0.6 :: 0.6: x It can be written as  $0.24 \times x = 0.6 \times 0.6$ 



 $x = (0.6 \times 0.6) / 0.24 = 1.5$ 

Hence, the third proportional to 0.24, 0.6 is 1.5.

(iii) Consider x as the third proportional to Rs. 3 and Rs. 12 3: 12 :: 12: x It can be written as  $3 \times x = 12 \times 12$  $x = (12 \times 12)/3 = 48$ 

Hence, the third proportional to Rs. 3 and Rs. 12 is Rs. 48

(iv) Consider x as the third proportional to 5 <sup>1</sup>/<sub>4</sub> and 7 5 <sup>1</sup>/<sub>4</sub>: 7 :: 7: x It can be written as  $21/4 \times x = 7 \times 7$  $x = (7 \times 7 \times 4)/21 = 28/3 = 9 1/3$ 

Hence, the third proportional to  $5\frac{1}{4}$  and 7 is  $9\frac{1}{3}$ .

4. Find the mean proportion of:
(i) 5 and 80
(ii) 1/12 and 1/75
(iii) 8.1 and 2.5
(iv) (a - b) and (a<sup>3</sup> - a<sup>2</sup>b), a > b
Solution:

(i) Consider x as the mean proportion of 5 and 80 5: x :: x: 80 It can be written as  $x^2 = 5 \times 80 = 400$  $x = \sqrt{400} = 20$ 

Therefore, mean proportion of 5 and 80 is 20.

(ii) Consider x as the mean proportion of 1/12 and 1/75 1/12: x :: x: 1/75 It can be written as  $x^2 = 1/12 \times 1/75 = 1/900$  $x = \sqrt{1/900} = 1/30$ 

Therefore, mean proportion of 1/12 and 1/75 is 1/30.

(iii) Consider x as the mean proportion of 8.1 and 2.5 8.1: x :: x: 2.5 It can be written as  $x^2 = 8.1 \times 2.5 = 20.25$  $x = \sqrt{20.25} = 4.5$ 

Therefore, mean proportion of 8.1 and 2.5 is 4.5.



(iv) Consider x as the mean proportion of (a - b) and  $(a^3 - a^2b)$ , a > b (a - b): x ::  $(a^3 - a^2b)$ It can be written as  $x^2 = (a - b) (a^3 - a^2b)$ So we get  $x^2 = (a - b) a^2 (a - b)$   $x^2 = a^2 (a - b)^2$ Here x = a (a - b)

Therefore, mean proportion of (a - b) and  $(a^3 - a^2b)$ , a > b is a (a - b).

# 5. If a, 12, 16 and b are in continued proportion find a and b. Solution:

It is given that a, 12, 16 and b are in continued proportion a/12 = 12/16 = 16/b

We know that a/12 = 12/16By cross multiplication 16a = 144a = 144/16 = 9

Similarly 12/16 = 16/bBy cross multiplication  $12b = 16 \times 16 = 256$  $b = 256/12 = 64/3 = 21 \ 1/3$ 

Therefore, a = 9 and b = 64/3 or 21 1/3.

## 6. What number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion? Solution:

Consider x to be added to 5, 11, 19 and 37 to make them in proportion 5 + x: 11 + x :: 19 + x: 37 + xIt can be written as (5 + x) (37 + x) = (11 + x) (19 + x)By further calculation  $185 + 5x + 37x + x^2 = 209 + 11x + 19x + x^2$   $185 + 42x + x^2 = 209 + 30x + x^2$ So we get  $42x - 30x + x^2 - x^2 = 209 - 185$  12x = 24x = 2

Hence, the least number to be added is 2.



7. What number should be subtracted from each of the numbers 23, 30, 57 and 78 so that the remainders are in proportion? Solution:

Consider x be subtracted from each term 23 - x, 30 - x, 57 - x and 78 - x are proportional It can be written as 23 - x: 30 - x :: 57 - x: 78 - x (23 - x)/(30 - x) = (57 - x)/(78 - x)By cross multiplication (23 - x)(78 - x) = (30 - x)(57 - x)By further calculation  $1794 - 23x - 78x + x^2 = 1710 - 30x - 57x + x^2$   $x^2 - 101x + 1794 - x^2 + 87x - 1710 = 0$ So we get - 14x + 84 = 0 14x = 84x = 84/14 = 6

Therefore, 6 is the number to be subtracted from each of the numbers.

## 8. If k + 3, k + 2, 3k - 7 and 2k - 3 are in proportion, find k. Solution:

It is given that k + 3, k + 2, 3k - 7 and 2k - 3 are in proportion We can write it as (k + 3) (2k - 3) = (k + 2) (3k - 7)By further calculation  $2k^2 - 3k + 6k - 9 = 3k^2 - 7k + 6k - 14$   $3k^2 - 7k + 6k - 14 - 2k^2 + 3k - 6k + 9 = 0$   $k^2 - 4k - 5 = 0$   $k^2 - 5k + k - 5 = 0$  k(k - 5) + 1(k - 5) = 0 k(k - 5) + 1(k - 5) = 0So, k + 1 = 0 or k - 5 = 0 k = -1 or k = 5Therefore, the value of k is -1, 5.

## 9. If x + 5 is the mean proportion between x + 2 and x + 9, find the value of x. Solution:

It is given that x + 5 is the mean proportion between x + 2 and x + 9We can write it as  $(x + 5)^2 = (x + 2) (x + 9)$ By further calculation  $x^2 + 10x + 25 = x^2 + 11x + 18$  $x^2 + 10x - x^2 - 11x = 18 - 25$ 



So we get -x = -7x = 7

Hence, the value of x is 7.

10. What number must be added to each of the numbers 16, 26 and 40 so that the resulting numbers may be in continued proportion? Solution:

Consider x be added to each number 16 + x, 26 + x and 40 + x are in continued proportion It can be written as (16 + x)/(26 + x) = (26 + x)/(40 + x)By cross multiplication (16 + x) (40 + x) = (26 + x) (26 + x)On further calculation  $640 + 16x + 40x + x^2 = 676 + 26x + 26x + x^2$   $640 + 56x + x^2 = 676 + 52x + x^2$   $56x + x^2 - 52x - x^2 = 676 - 640$ So we get 4x = 36x = 36/4 = 9

Hence, 9 is the number to be added to each of the numbers.

11. Find two numbers such that the mean proportional between them is 28 and the third proportional to them is 224. Solution:

Consider a and b as the two numbers It is given that 28 is the mean proportional a: 28 :: 28: b We get  $ab = 28^2 = 784$ Here a = 784/b ..... (1)

We know that 224 is the third proportional a: b :: b: 224 So we get  $b^2 = 224a \dots (2)$ 

Now by substituting the value of a in equation (2)  $b^2 = 224 \times 784/b$ So we get  $b^3 = 224 \times 784$   $b^3 = 175616 = 56^3$ b = 56

By substituting the value of b in equation (1)



a = 784/56 = 14

Therefore, 14 and 56 are the two numbers.

12. If b is the mean proportional between a and c, prove that a, c,  $a^2 + b^2$  and  $b^2 + c^2$  are proportional. Solution:

It is given that b is the mean proportional between a and c We can write it as  $b^2 = a \times c$  $b^2 = ac \dots (1)$ 

We know that a, c,  $a^2 + b^2$  and  $b^2 + c^2$  are in proportion It can be written as  $a/c = (a^2 + b^2)/(b^2 + c^2)$ By cross multiplication  $a (b^2 + c^2) = c (a^2 + b^2)$ Using equation (1)  $a (ac + c^2) = c (a^2 + ac)$ So we get  $ac (a + c) = a^2c + ac^2$ Here ac (a + c) = ac (a + c) which is true.

Therefore, it is proved.

13. If b is the mean proportional between a and c, prove that (ab + bc) is the mean proportional between  $(a^2 + b^2)$  and  $(b^2 + c^2)$ . Solution:

It is given that b is the mean proportional between a and c  $b^2 = ac \dots (1)$ 

Here (ab + bc) is the mean proportional between  $(a^2 + b^2)$  and  $(b^2 + c^2)$  $(ab + bc)^2 = (a^2 + b^2) (b^2 + c^2)$ 

```
Consider LHS = (ab + bc)^2
Expanding using formula
= a^2b^2 + b^2c^2 + 2ab^2c
Using equation (1)
= a^2 (ac) + ac (c)^2 + 2a. ac. c
= a^3c + ac^3 + 2a^2c^2
Taking ac as common
= ac (a^2 + c^2 + 2ac)
= ac (a + c)^2
```

 $RHS = (a^2 + b^2) (b^2 + c^2)$ Using equation (1)



= (a<sup>2</sup> + ac) (ac + c<sup>2</sup>)Taking common terms out = a (a + c) c (a + c)= ac (a + c)<sup>2</sup>Hence, LHS = RHS.

14. If y is mean proportional between x and z, prove that  $xyz (x + y + z)^3 = (xy + yz + zx)^3$ . Solution:

It is given that y is mean proportional between x and z We can write it as  $y^2 = xz$  ..... (1)

Consider LHS = xyz  $(x + y + z)^3$ It can be written as = xz. y  $(x + y + z)^3$ Using equation (1) =  $y^2 y (x + y + z)^3$ =  $y^3 (x + y + z)^3$ So we get =  $[y (x + y + z)]^3$ By further calculation =  $(xy + y^2 + yz)^3$ Using equation (1) =  $(xy + yz + zx)^3$ = RHS

Hence, it is proved.

15. If a + c = mb and 1/b + 1/d = m/c, prove that a, b, c and d are in proportion. Solution:

It is given that a + c = mb and 1/b + 1/d = m/c

a + c = mbDividing the equation by b a/b + c/d = m ......(1)

1/b + 1/d = m/cMultiplying the equation by c c/b + c/d = m ..... (2)

Using equation (1) and (2) a/b + c/b = c/b + c/dSo we get a/b = c/d



Therefore, it is proved that a, b, c and d are in proportion.

16. If 
$$\mathbf{x}/\mathbf{a} = \mathbf{y}/\mathbf{b} = \mathbf{z}/\mathbf{c}$$
, prove that  
(i)  $\frac{\mathbf{x}^{3}}{\mathbf{a}^{2}} + \frac{\mathbf{y}^{3}}{\mathbf{b}^{2}} + \frac{\mathbf{z}^{3}}{\mathbf{c}^{2}} = \frac{(\mathbf{x} + \mathbf{y} + \mathbf{z})^{3}}{(\mathbf{a} + \mathbf{b} + \mathbf{c})^{2}}$   
(ii)  $[\frac{\mathbf{a}^{2}\mathbf{x}^{2} + \mathbf{b}^{2}\mathbf{y}^{2} + \mathbf{c}^{2}\mathbf{z}^{2}}{\mathbf{a}^{3}\mathbf{x} + \mathbf{b}^{3}\mathbf{y} + \mathbf{c}^{3}\mathbf{z}}]^{3} = \frac{\mathbf{x}\mathbf{y}\mathbf{z}}{\mathbf{a}\mathbf{b}\mathbf{c}}$   
(iii)  $\frac{\mathbf{a}\mathbf{x} - \mathbf{b}\mathbf{y}}{(\mathbf{a} + \mathbf{b})(\mathbf{x} - \mathbf{y})} + \frac{\mathbf{b}\mathbf{y} - \mathbf{c}\mathbf{z}}{(\mathbf{b} + \mathbf{c})(\mathbf{y} - \mathbf{z})} + \frac{\mathbf{c}\mathbf{z} - \mathbf{a}\mathbf{x}}{(\mathbf{c} + \mathbf{a})(\mathbf{z} - \mathbf{x})} = 3$   
Solution:

It is given that x/a = y/b = z/cWe can write it as x = ak, y = bk and z = ck



$$(i)LHS = \frac{x^3}{a^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2}$$

 $It\ can\ be\ written\ as$ 

$$= \frac{a^3k^3}{a^2} + \frac{b^3k^3}{b^2} + \frac{c^3k^3}{c^2}$$

So we get

$$=ak^3+bk^3+ck^3$$

 $Taking\ common\ terms$ 

$$=k^3(a+b+c)$$

$$RHS = \frac{(x+y+z)^3}{(a+b+c)^2}$$

 $It \ can \ be \ written \ as$ 

$$=\frac{(ak+bk+ck)^3}{(a+b+c)^2}$$

So we get

$$= \frac{k^3(a+b+c)^3}{(a+b+c)^2} \\ = k^3(a+b+c)$$

Therefore, LHS = RHS.



$$(ii)LHS = [\frac{a^2x^2 + b^2y^2 + c^2z^2}{a^3x + b^3y + c^3z}]^3$$

 $It\ can\ be\ written\ as$ 

$$= [\frac{a^2a^2k^2 + b^2b^2k^2 + c^2c^2k^2}{a^3.ak + b^3.bk + c^3.ck}]^3$$

 $By\ further\ calculation$ 

$$= \left[\frac{a^4k^2 + b^4k^2 + c^4k^2}{a^4k + b^4k + c^4k}\right]^3$$

So we get

$$= \left[\frac{k^2(a^4 + b^4 + c^4)}{k(a^4 + b^4 + c^4)}\right]^3$$
$$= k^3$$

$$RHS = \frac{xyz}{abc}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{ak.bk.ck}{abc}$$
$$= k^{3}$$

Therefore, LHS = RHS.

$$(iii)LHS = \frac{ax - by}{(a+b)(x-y)} + \frac{by - cz}{(b+c)(y-z)} + \frac{cz - ax}{(c+a)(z-x)}$$

It can be written as

$$=\frac{ax-by}{(a+b)(ak-bk)}+\frac{by-cz}{(b+c)(bk-ck)}+\frac{cz-ax}{(c+a)(ck-ak)}$$

 $By\ further\ calculation$ 



$$= \frac{a^2k - b^2k}{(a+b)k(a-b)} + \frac{b^2k - c^2k}{(b+c)k(b-c)} + \frac{c^2k - a^2k}{(c+a)k(c-a)}$$

 $Taking\ common\ terms$ 

$$= \frac{k(a^2 - b^2)}{k(a^2 - b^2)} + \frac{k(b^2 - c^2)}{k(b^2 - c^2)} + \frac{k(c^2 - a^2)}{k(c^2 - a^2)}$$

So we get

= 1 + 1 + 1= 3= RHS

Therefore, LHS = RHS.

17. If 
$$a/b = c/d = e/f$$
 prove that:  
(i)  $(b^2 + d^2 + f^2) (a^2 + c^2 + e^2) = (ab + cd + ef)^2$   
(ii)  $\frac{(a^3 + c^3)^2}{(b^3 + d^3)^2} = \frac{e^6}{f^6}$   
(iii)  $\frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2} = \frac{ac}{bd} + \frac{ce}{df} + \frac{ae}{df}$   
(iv)  $bdf[\frac{a + b}{b} + \frac{c + d}{d} + \frac{c + f}{f}]^3 = 27(a + b)(c + d)(e + f)$   
Solution:

Consider a/b = c/d = e/f = kSo we get a = bk, c = dk, e = fk(i) LHS =  $(b^2 + d^2 + f^2) (a^2 + c^2 + e^2)$ We can write it as  $= (b^2 + d^2 + f^2) (b^2k^2 + d^2k^2 + f^2k^2)$ Taking out the common terms  $= (b^2 + d^2 + f^2) k^2 (b^2 + d^2 + f^2)$ So we get  $= k^2 (b^2 + d^2 + f^2)$ 

RHS =  $(ab + cd + ef)^2$ We can write it as =  $(b. kb + dk. d + fk. f)^2$ So we get =  $(kb^2 + kd^2 + kf^2)$ Taking out common terms =  $k^2 (b^2 + d^2 + f^2)^2$ 



Therefore, LHS = RHS.

$$(ii)LHS = \frac{(a^3 + c^3)^2}{(b^3 + d^3)^2}$$

 $It \ can \ be \ written \ as$ 

$$= \frac{(b^3k^3 + d^3k^3)^2}{(b^3 + d^3)^2}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{[k^{3}(b^{3} + d^{3})]^{2}}{(b^{3} + d^{3})^{2}}$$
So we get
$$= \frac{k^{6}(b^{3} + d^{3})^{2}}{(b^{3} + d^{3})^{2}}$$

$$= k^{6}$$
RHS =  $\frac{e^{6}}{f^{6}}$ 
We get
$$= \frac{f^{6}k^{6}}{f^{6}}$$

$$= k^{6}$$
Therefore, LHS = RHS.

$$(iii)LHS = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

It can be written as

$$=\frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$$

So we get

 $=k^{2}+k^{2}+k^{2}$ 

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 $= 3k^{2}$ 

$$RHS = \frac{ac}{bd} + \frac{ce}{df} + \frac{ae}{bf}$$

It can be written as

$$= \left[\frac{bk.dk}{bd} + \frac{dk.fk}{df} + \frac{bk.fk}{bf}\right]$$

So we get

$$=k^{2}+k^{2}+k^{2}$$

$$= 3k^2$$
  
Therefore, LHS = RHS.

$$(iv)LHS = bdf[\frac{a+b}{b} + \frac{c+d}{d} + \frac{e+f}{f}]^3$$

 $It \ can \ be \ written \ as$ 

$$= bdf \left[\frac{bk+b}{b} + \frac{dk+d}{d} + \frac{fk+f}{f}\right]^3$$

Taking out the common terms

$$= bdf \left[\frac{b(k+1)}{b} + \frac{d(k+1)}{d} + \frac{f(k+1)}{f}\right]^{3}$$

So we get = bdf  $(k + 1 + k + 1 + k + 1)^3$ By further calculation = bdf  $(3k + 3)^3$ = 27 bdf  $(k + 1)^3$ 

RHS = 27 (a + b) (c + d) (e + f) It can be written as = 27 (bk + b) (dk + d) (fk + f) Taking out the common terms = 27 b (k + 1) d (k + 1) f (k + 1) So we get = 27 bdf (k + 1)<sup>3</sup>

Therefore, LHS = RHS.

### 18. If ax = by = cz; prove that



 $\frac{\mathbf{x}^2}{\mathbf{y}\mathbf{z}} + \frac{\mathbf{y}^2}{\mathbf{z}\mathbf{x}} + \frac{\mathbf{z}^2}{\mathbf{x}\mathbf{y}} = \frac{\mathbf{b}\mathbf{c}}{\mathbf{a}^2} + \frac{\mathbf{c}\mathbf{a}}{\mathbf{b}^2} + \frac{\mathbf{a}\mathbf{b}}{\mathbf{c}^2}$ Solution:

Consider ax = by = cz = kIt can be written as x = k/a, y = k/b, z = k/c

$$LHS = \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$$

Substituting the values

$$= \frac{\frac{k^2}{a^2}}{\frac{k}{b} \cdot \frac{k}{c}} + \frac{\frac{k^2}{b^2}}{\frac{k}{c} \cdot \frac{k}{a}} + \frac{\frac{k^2}{c^2}}{\frac{k}{a} \cdot \frac{k}{b}}$$

By further calculation

$$= \frac{\frac{k^2}{a^2}}{\frac{k^2}{bc}} + \frac{\frac{k^2}{b^2}}{\frac{k^2}{ca}} + \frac{\frac{k^2}{c^2}}{\frac{k^2}{ab}}$$

It can be written as

$$= \frac{k^2}{a^2} \times \frac{bc}{k^2} + \frac{k^2}{b^2} \times \frac{ca}{k^2} + \frac{k^2}{c^2} \times \frac{al}{k^2}$$

So we get

$$= \frac{bc}{a^2} + \frac{ca}{b^2} + \frac{ab}{c^2}$$

= RHS

19. If a, b, c and d are in proportion, prove that: (i) (5a + 7v) (2c - 3d) = (5c + 7d) (2a - 3b)(ii) (ma + nb): b = (mc + nd): d(iii)  $(a^4 + c^4): (b^4 + d^4) = a^2c^2: b^2d^2$ (iv)  $\frac{a^2 + ab}{c^2 + cd} = \frac{b^2 - 2ab}{d^2 - 2cd}$ (v)  $\frac{(a + c)^3}{(b + d)^3} = \frac{a(a - c)^2}{b(b - d)^2}$ (vi)  $\frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$ 



$$\begin{split} (vii) \frac{a^2 + b^2}{c^2 + d^2} &= \frac{ab + ad - bc}{bc + cd - ad} \\ (viii) abcd[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}] = a^2 + b^2 + c^2 + d^2 \end{split}$$

Solution:

It is given that a, b, c, d are in proportion Consider a/b = c/d = ka = b, c = dk

(i) LHS = (5a + 7b) (2c - 3d)Substituting the values = (5bk + 7b) (2dk - 3d)Taking out the common terms = k (5b + 7b) k (2d - 3d)So we get =  $k^2 (12b) (-d)$ =  $- 12 bd k^2$ 

RHS = (5c + 7d) (2a - 3b)Substituting the values = (5dk + 7d) (2kb - 3b)Taking out the common terms = k (5d + 7d) k (2b - 3b)So we get =  $k^2 (12d) (-b)$ =  $- 12 bd k^2$ 

Therefore, LHS = RHS.

(ii) (ma + nb): b = (mc + nd): d We can write it as  $\frac{ma + nb}{b} = \frac{mc + nd}{d}$ 

Consider

$$LHS = \frac{mbk + nb}{b}$$

Taking out the common terms

$$=\frac{b(mk+n)}{b}$$
$$=mk+n$$



$$RHS = \frac{mc + nd}{d}$$

It can be written as

$$=\frac{mdk+nd}{d}$$

 $Taking \ out \ the \ common \ terms$ 

$$=\frac{d(mk+n)}{d}$$

= mk + n

Therefore, LHS = RHS.

(iii)(a<sup>4</sup> + c<sup>4</sup>): (b<sup>4</sup> + d<sup>4</sup>) = a<sup>2</sup>c<sup>2</sup>: b<sup>2</sup>d<sup>2</sup>  
We can write it as  
$$\frac{a^4 + c^4}{b^4 + d^4} = \frac{a^2c^2}{b^2d^2}$$

Consider

$$LHS = \frac{a^4 + c^4}{b^4 + d^4}$$

 $Substituting \ the \ values$ 

 $= \frac{b^4k^4 + d^4k^4}{b^4 + d^4}$ 

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{k^4(b^4 + d^4)}{(b^4 + d^4)}$$
$$= k^4$$

$$RHS = \frac{a^2c^2}{b^2d^2}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{k^2 b^2 k^2 d^2}{b^2 d^2}$$
$$= k^4$$



Therefore, LHS = RHS.

$$(iv)LHS = \frac{a^2 + ab}{c^2 + cd}$$

 $It\ can\ be\ written\ as$ 

$$=\frac{k^2b^2+bk.b}{k^2d^2+dk.d}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{kb^2(k+1)}{d^2k(k+1)}$$
$$= \frac{b^2}{d^2}$$
$$RHS = \frac{b^2 - 2ab}{d^2 - 2cd}$$

It can be written as

$$= \frac{b^2 - 2bkb}{d^2 - 2dkd}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{b^2(1-2k)}{d^2(1-2k)} \\ = \frac{b^2}{d^2}$$

Therefore, LHS = RHS.

$$(v)LHS = \frac{(a+c)^3}{(b+d)^3}$$

 $We\ can\ write\ it\ as$ 

$$= \frac{(bk+dk)^3}{(b+d)^3}$$
$$= k^3$$

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$$RHS = \frac{a(a-c)^2}{b(b-d)^2}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{bk(bk - dk)^2}{b(b - d)^2}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{bk \cdot k^2 (b-d)^2}{b(b-d)^2}$$
$$= k^3$$

Therefore, LHS = RHS.

$$(vi)LHS = \frac{a^2 + ab + b^2}{a^2 - ab + b^2}$$

 $We\ can\ write\ it\ as$ 

$$= \frac{b^2k^2 + bk.b + b^2}{b^2k^2 - bk.b + b^2}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{b^2(k^2 + k + 1)}{b^2(k^2 - k + 1)}$$

So we get

$$= \frac{(k^2 + k + 1)}{(k^2 - k + 1)}$$
$$RHS = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$$

 $We\ can\ write\ it\ as$ 

$$= \frac{d^2k^2 + dkd + d^2}{d^2k^2 - dkd + d^2}$$



 $Taking \ out \ the \ common \ terms$ 

$$= \frac{d^2(k^2 + k + 1)}{d^2(k^2 - k + 1)}$$

So we get

$$=\frac{(k^2+k+1)}{(k^2-k+1)}$$

Therefore, LHS = RHS.

$$(vii)LHS = \frac{a^2 + b^2}{c^2 + d^2}$$

 $We \ can \ write \ it \ as$ 

$$=\frac{b^2k^2+b^2}{d^2k^2+d^2}$$

 $Taking out the \ common \ terms$ 

$$= \frac{b^2(k^2+1)}{d^2(k^2+1)}$$
$$= \frac{b^2}{d^2}$$
$$BHS = \frac{ab+ad-bc}{ab+ad-bc}$$

bc + cd - adWe can write it as

w e can write it as

$$= \frac{bk.b + bk.d - b.dk}{bk.d + dk.d - bk.d}$$

 $Taking \ out \ the \ common \ terms$ 

$$=\frac{k(b^2+bd-bd)}{k(bd+d^2-bd)}$$
$$=\frac{b^2}{d^2}$$

Therefore, LHS = RHS.



$$(viii)LHS = abcd[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}]$$

 $We \ can \ write \ it \ as$ 

$$= bk.b.dk.d[\frac{1}{b^2k^2} + \frac{1}{b^2} + \frac{1}{d^2k^2} + \frac{1}{d^2}]$$

By further calculation

$$=k^{2}b^{2}d^{2}\left[\frac{d^{2}+d^{2}k^{2}+b^{2}+b^{2}k^{2}}{b^{2}d^{2}k^{2}}\right]$$

$$= d^{2} (1 + k^{2}) + b^{2} (1 + k^{2})$$
$$= (1 + k^{2}) (b^{2} + d^{2})$$

RHS =  $a^2 + b^2 + c^2 + d^2$ We can write it as =  $b^2k^2 + b^2 + d^2k^2 + d^2$ Taking out the common terms =  $b^2 (k^2 + 1) + d^2 (k^2 + 1)$ =  $(k^2 + 1) (b^2 + d^2)$ 

Therefore, LHS = RHS.

#### 20. If x, y, z are in continued proportion, prove that: $(x + y)^2/(y + z)^2 = x/z$ . Solution:

It is given that x, y, z are in continued proportion Consider x/y = y/z = k So we get y = kz x = yk = kz × k = k<sup>2</sup>z LHS =  $\frac{(x + y)^2}{(y + z)^2}$ 

 $We \ can \ write \ it \ as$ 

$$=\frac{(k^2z+kz)^2}{(kz+z)^2}$$

 $Taking \ out \ the \ common \ terms$ 

$$=\frac{[kz(k+1)]^2}{[z(k+1)]^2}$$



So we get

$$= \frac{k^2 z^2 (k+1)^2}{z^2 (k+1)^2}$$
$$= k^2$$

$$RHS = \frac{x}{z}$$

 $We\ can\ write\ it\ as$ 

$$=\frac{k^2z}{z}$$
$$=k^2$$

Therefore, LHS = RHS.

21. If a, b, c are in continued proportion, prove that:  $\frac{pa^2 + qab + rb^2}{pb^2 + qbc + rc^2} = \frac{a}{c}$ Solution:

It is given that a, b, c are in continued proportion  $\frac{pa^2 + qab + rb^2}{pb^2 + qbc + rc^2} = \frac{a}{c}$ 

Consider a/b = b/c = kSo we get a = bk and b = ck ..... (1) From equation (1)  $a = (ck) k = ck^2$  and b = ck

We know that

$$LHS = \frac{pa^2 + qab + rb^2}{pb^2 + qbc + rc^2}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{p(ck^2)^2 + q(ck^2)ck + r(ck)^2}{p(ck)^2 + q(ck)c + rc^2}$$

 $By \ further \ calculation$


$$=\frac{pc^2k^4 + qc^2k^3 + rc^2k^2}{pc^2k^2 + qc^2k + rc^2}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{c^2k^2}{c^2} [\frac{pk^2 + qk + r}{pk^2 + qk + r}] = k^2$$

$$RHS = \frac{a}{c}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{ck^2}{c}$$
$$= k^2$$

Therefore, LHS = RHS.

### 22. If a, b, c are in continued proportion, prove that:

(i) 
$$\frac{\mathbf{a} + \mathbf{b}}{\mathbf{b} + \mathbf{c}} = \frac{\mathbf{a}^2(\mathbf{b} - \mathbf{c})}{\mathbf{b}^2(\mathbf{a} - \mathbf{b})}$$
  
(ii)  $\frac{1}{\mathbf{a}^3} + \frac{1}{\mathbf{b}^3} + \frac{1}{\mathbf{c}^3} = \frac{\mathbf{a}}{\mathbf{b}^2\mathbf{c}^2} + \frac{\mathbf{b}}{\mathbf{c}^2\mathbf{a}^2} + \frac{\mathbf{c}}{\mathbf{a}^2\mathbf{b}^2}$   
(iii) a:  $\mathbf{c} = (\mathbf{a}^2 + \mathbf{b}^2)$ :  $(\mathbf{b}^2 + \mathbf{c}^2)$   
(iv)  $\mathbf{a}^2\mathbf{b}^2\mathbf{c}^2(\mathbf{a}^{-4} + \mathbf{b}^{-4} + \mathbf{c}^{-4}) = \mathbf{b}^{-2}(\mathbf{a}^4 + \mathbf{b}^4 + \mathbf{c}^4)$   
(v)  $\mathbf{abc} (\mathbf{a} + \mathbf{b} + \mathbf{c})^3 = (\mathbf{ab} + \mathbf{bc} + \mathbf{ca})^3$   
(vi)  $(\mathbf{a} + \mathbf{b} + \mathbf{c}) (\mathbf{a} - \mathbf{b} + \mathbf{c}) = \mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2$   
Solution:

It is given that a, b, c are in continued proportion So we get a/b = b/c = k

$$(i)LHS = \frac{a+b}{b+c}$$

 $We \ can \ write \ it \ as$ 

$$=\frac{ck^2 + ck}{ck + c}$$



 $Taking \ out \ the \ common \ terms$ 

$$=\frac{ck(k+1)}{c(k+1)}$$

$$= k$$

$$RHS = \frac{a^2(b-c)}{b^2(a-b)}$$

 $We\ can\ write\ it\ as$ 

$$=\frac{(ck^2)^2(ck-c)}{(ck)^2(ck^2-ck)}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{c^2 k^4 c(k-1)}{c^2 k^2 c k(k-1)}$$

So we get

$$= \frac{c^3 k^4 (k-1)}{c^3 k^3 (k-1)} - k$$

Therefore, LHS = RHS.

$$(ii)LHS = \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{1}{(ck^2)^3} + \frac{1}{(ck)^3} + \frac{1}{c^3}$$
$$= \frac{1}{c^3k^6} + \frac{1}{c^3k^3} + \frac{1}{c^3}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{1}{c^3} [\frac{1}{k^6} + \frac{1}{k^3} + \frac{1}{1}]$$



$$RHS = \frac{a}{b^2c^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{ck^2}{(ck)^2c^2} + \frac{ck}{c^2(ck^2)^2} + \frac{c}{(ck^2)^2(ck)^2}$$
$$= \frac{ck^2}{c^4k^2} + \frac{ck}{c^4k^4} + \frac{c}{c^4k^6}$$

So we get

$$=\frac{1}{c^3}+\frac{1}{c^3k^3}+\frac{1}{c^3k^6}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{1}{c^3} \left[ 1 + \frac{1}{k^3} + \frac{1}{k^6} \right]$$
$$= \frac{1}{c^3} \left[ \frac{1}{k^6} + \frac{1}{k^3} + 1 \right]$$

Therefore, LHS = RHS.

(iii) a: 
$$c = (a^2 + b^2)$$
:  $(b^2 + c^2)$   
We can write it as  
$$\frac{a}{c} = \frac{a^2 + b^2}{b^2 + c^2}$$

We know that

$$LHS = \frac{a}{c} = \frac{ck^2}{c} = k^2$$
$$RHS = \frac{a^2 + b^2}{b^2 + c^2}$$

 $We\ can\ write\ it\ as$ 

$$= \frac{(ck^2)^2 + (ck)^2}{(ck)^2 + c^2}$$

So we get



$$=\frac{c^2k^4+c^2k^2}{c^2k^2+c^2}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{c^2 k^2 (k^2 + 1)}{c^2 (k^2 + 1)}$$
$$= k^2$$

Therefore, LHS = RHS.

(iv)  $a^{2}b^{2}c^{2} (a^{-4} + b^{-4} + c^{-4}) = b^{-2} (a^{4} + b^{4} + c^{4})$  $LHS = a^{2}b^{2}c^{2}(a^{-4}b^{-4}c^{-4})$ 

We can write it as

$$= a^2 b^2 c^2 \left[\frac{1}{a^4} + \frac{1}{b^4} + \frac{1}{c^4}\right]$$

So we get

$$= \frac{a^2b^2c^2}{a^4} + \frac{a^2b^2c^2}{b^4} + \frac{a^2b^2c^2}{c^4}$$

 $By \ further \ calculation$ 

$$= \frac{b^2c^2}{a^2} + \frac{c^2a^2}{b^2} + \frac{a^2b^2}{c^2}$$

It can be written as

$$= \frac{(ck)^2 c^2}{(ck^2)^2} + \frac{c^2 (ck^2)^2}{(ck)^2} + \frac{(ck^2)^2 (ck)^2}{c^2}$$
$$= \frac{c^2 k^2 c^2}{c^2 k^4} + \frac{c^2 c^2 k^4}{c^2 k^2} + \frac{c^2 k^4 c^2 k^2}{c^2}$$

 $On \ further \ simplification$ 

$$= \frac{c^2}{k^2} + \frac{c^2k^2}{1} + \frac{c^2k^6}{1}$$
  
Taking out the common terms

$$= c^{2} \left[ \frac{1}{k^{2}} + k^{2} + k^{6} \right]$$



$$=\frac{c^2}{k^2}[1+k^4+k^8]$$

 $RHS = b^{-2}[a^4 + b^4 + c^4]$ 

 $We \ can \ write \ it \ as$ 

$$=rac{1}{b^2}[a^4+b^4+c^4]$$

So we get

$$= \frac{1}{(ck)^2} [(ck^2)^4 + (ck)^4 + c^4]$$

By further calculation

$$=\frac{1}{c^2k^2}[c^4k^8 + c^4k^4 + c^4]$$

Taking out the common terms

$$= \frac{c^4}{c^2 k^2} [k^8 + k^4 + 1]$$
$$= \frac{c^2}{k^2} [1 + k^4 + k^8]$$

Therefore, LHS = RHS.

(v) LHS = abc  $(a + b + c)^3$ We can write it as = ck<sup>2</sup>. ck. c  $[ck^2 + ck + c]^3$ Taking out the common terms = c<sup>3</sup> k<sup>3</sup> [c  $(k^2 + k + 1)]^3$ So we get = c<sup>3</sup> k<sup>3</sup>. c<sup>3</sup>  $(k^2 + k + 1)^3$ = c<sup>6</sup> k<sup>3</sup>  $(k^2 + k + 1)^3$ 

RHS =  $(ab + bc + ca)^3$ We can write it as =  $(ck^2. ck + ck. c + c. ck^2)^3$ So we get =  $(c^2k^3 + c^2k + c^2k^2)^3$ =  $(c^2k^3 + c^2k^2 + c^2k)^3$ Taking out the common terms =  $[c^2k (k^2 + k + 1)]^3$ 

## ML Aggarwal Solutions for Class 10 Maths Chapter 7 -Ratio and Proportion



 $= c^{6}k^{3}(k^{2}+k+1)^{3}$ 

Therefore, LHS = RHS.

(vi) LHS = (a + b + c) (a - b + c)We can write it as =  $(ck^2 + ck + c) (ck^2 - ck + c)$ Taking out the common terms =  $c (k^2 + k + 1) c (k^2 - k + 1)$ =  $c^2 (k^2 + k + 1) (k^2 - k + 1)$ So we get =  $c^2 (k^4 + k^2 + 1)$ 

 $RHS = a^{2} + b^{2} + c^{2}$ We can write it as  $= (ck^{2})^{2} + (ck)^{2} + (c)^{2}$ So we get  $= c^{2}k^{4} + c^{2}k^{2} + c^{2}$ Taking out the common terms  $= c^{2} (k^{4} + k^{2} + 1)$ 

Therefore, LHS = RHS.

23. If a, b, c, d are in continued proportion, prove that:

(i)  $\frac{a^3 + b^3 + c^3}{b^3 + c^3 + d^3} = \frac{a}{d}$ (ii)  $(a^2 - b^2) (c^2 - d^2) = (b^2 - c^2)^2$ (iii)  $(a + d) (b + c) - (a + c) (b + d) = (b - c)^2$ (iv) a: d = triplicate ratio of (a - b): (b - c)(v)  $(\frac{a - b}{c} + \frac{a - c}{b})^2 - (\frac{d - b}{c} + \frac{d - c}{b})^2 = (a - d)^2 (\frac{1}{c^2} - \frac{1}{b^2})$ Solution:

It is given that a, b, c, d are in continued proportion Here we get a/b = b/c = c/d = k  $c = dk, b = ck = dk \cdot k = dk^2$   $a = bk = dk^2 \cdot k = dk^3$  $(i)LHS = \frac{a^3 + b^3 + c^3}{b^3 + c^3 + d^3}$ 

We can write it as

$$= \frac{(dk^3)^3 + (dk^2)^3 + (dk)^3}{(dk^2)^3 + (dk)^3 + d^3}$$

So we get



$$= \frac{d^3k^9 + d^3k^6 + d^3k^3}{d^3k^6 + d^3k^3 + d^3}$$

Taking out the common terms

$$= \frac{d^3k^3(k^6 + k^3 + 1)}{d^3(k^6 + k^3 + 1)}$$
$$= k^3$$

$$RHS = \frac{a}{d} = \frac{dk^3}{d} = k^3$$

Therefore, LHS = RHS.

(ii) LHS = 
$$(a^2 - b^2) (c^2 - d^2)$$
  
We can write it as  
=  $[(dk^3)^2 - (dk^2)^2] [(dk)^2 - d^2]$   
By further calculation  
=  $(d^2k^6 - d^2k^4) (d^2k^2 - d^2)$   
Taking out the common terms  
=  $d^2k^4 (k^2 - 1) d^2 (k^2 - 1)$   
=  $d^4k^4 (k^2 - 1)^2$ 

RHS =  $(b^2 - c^2)^2$ We can write it as =  $[(dk^2)^2 - (dk)^2]^2$ By further calculation =  $[d^2k^4 - d^2k^2]^2$ Taking out the common terms =  $[d^2k^2 (k^2 - 1)]^2$ =  $d^4 k^4 (k^2 - 1)^2$ 

Therefore, LHS = RHS.

(iii) LHS = (a + d) (b + c) - (a + c) (b + d)We can write it as =  $(dk^3 + d) (dk^2 + dk) - (dk^3 + dk) (dk^2 + d)$ Taking out the common terms =  $d (k^3 + 1) dk (k + 1) - dk (k^2 + 1) d (k^2 + 1)$ By further simplification =  $d^2k (k + 1) (k^3 + 1) - d^2k (k^2 + 1) (k^2 + 1)$ So we get =  $d^2k (k^4 + k^3 + k + 1 - k^4 - 2k^2 - 1)$ =  $d^2k (k^3 - 2k^2 + k)$ Taking k as common =  $d^2k^2 (k^2 - 2k + 1)$ =  $d^2k^2 (k - 1)^2$ RHS =  $(b - c)^2$ 



We can write it as =  $(dk^2 - dk)^2$ Taking out the common terms =  $d^2k^2 (k-1)^2$ 

Therefore, LHS = RHS.

(iv) a: d = triplicate ratio of (a - b):  $(b - c) = (a - b)^3$ :  $(b - c)^3$ We know that  $LHS = a : d = \frac{a}{d}$ 

It can be written as

$$= \frac{dk^3}{d}$$
$$= k^3$$

$$RHS = \frac{(a-b)^3}{(b-c)^3}$$

We can write it as

$$= \frac{(dk^3 - dk^2)^3}{(dk^2 - dk)^3}$$

Taking out the common terms

$$= \frac{d^3k^6(k-1)^3}{d^3k^3(k-1)^3}$$
$$= k^3$$

Therefore, LHS = RHS.

(v)  

$$LHS = (\frac{a-b}{c} + \frac{a-c}{b})^2 - (\frac{d-b}{c} + \frac{d-c}{b})^2$$

 $We \ can \ write \ it \ as$ 

$$=(\frac{dk^{3}-dk^{2}}{dk}+\frac{dk^{3}-dk}{dk^{2}})^{2}-(\frac{d-dk^{2}}{dk}+\frac{d-dk}{dk^{2}})^{2}$$

 $Taking \ out \ the \ common \ terms$ 



$$= \left(\frac{dk^2(k-1)}{dk} + \frac{dk(k^2-1)}{dk^2}\right)^2 - \left(\frac{d(1-k^2)}{dk} + \frac{d(1-k)}{dk^2}\right)^2$$

 $By \ further \ calculation$ 

$$= (k(k-1) + \frac{(k^2-1)}{k})^2 - (\frac{1-k^2}{k} + \frac{(1-k)}{k^2})^2$$

Taking LCM we get

$$=(\frac{k^2(k-1)+(k^2-1)}{k})^2-(\frac{k(1-k^2)+1-k}{k^2})^2$$

So we get

$$= \left(\frac{k^3 - k^2 + k^2 - 1}{k}\right)^2 - \left(\frac{k - k^3 + 1 - k}{k^2}\right)^2$$
$$= \frac{(k^3 - 1)^2}{k^2} - \frac{(-k^3 + 1)^2}{k^4}$$
$$= \frac{(k^3 - 1)^2}{k^2} - \frac{(1 - k^3)^2}{k^4}$$

 $On \ further \ simplification$ 

$$= (\frac{k^3 - 1}{k^2})^2 (1 - \frac{1}{k^2})$$
  
We get  
$$= \frac{(k^3 - 1)^2 (k^2 - 1)}{k^4}$$
  
RHS =  $(a - d)^2 (\frac{1}{c^2} - \frac{1}{b^2})$ 

 $We\ can\ write\ it\ as$ 

$$= (dk^3 - d)^2 (\frac{1}{d^2k^2} - \frac{1}{d^2k^4})$$

So we get

$$= d^{2}(k^{3} - 1)^{2}(\frac{k^{2} - 1}{d^{2}k^{4}})$$
$$= \frac{(k^{3} - 1)^{2}(k^{2} - 1)}{k^{4}}$$

Therefore, LHS = RHS.



## **EXERCISE 7.3**

1. If a: b :: c: d, prove that (i)  $\frac{2a + 5b}{2a - 5b} = \frac{2c + 5d}{2c - 5d}$ (ii)  $\frac{5a + 11b}{5c + 11d} = \frac{5a - 11b}{5c - 11d}$ (iii) (2a + 3b) (2c - 3d) = (2a - 3b) (2c + 3d) (iv) (la + mb): (lc + mb) :: (la - mb): (lc - mb) Solution:

(i) We know that If a: b :: c: d we get a/b = c/dBy multiplying 2/52a/5b = 2c/5dBy applying componendo and dividendo (2a + 5b)/(2a - 5b) = (2c + 5d)/(2c - 5d)

(ii) We know that If a: b :: c: d we get a/b = c/dBy multiplying 5/11 5a/11b = 5c/11dBy applying componendo and dividendo (5a + 11b)/(5a - 11b) = (5c + 11d)/(5c - 11d)Now by applying alternendo (5a + 11b)/(5c + 11d) = (5a - 11b)/(5c - 11d)

(iii) We know that If a: b :: c: d we get a/b = c/dBy multiplying 2/32a/3b = 2c/3dBy applying componendo and dividendo (2a + 3b)/(2a - 3b) = (2c + 3d)/(2c - 3d)By cross multiplication (2a + 3b)(2c - 3d) = (2a - 3b)(2c + 3d)

(iv) We know that If a: b :: c: d we get a/b = c/dBy multiplying l/m la/mb = lc/mdBy applying componendo and dividendo (la + mb)/(la - mb) = (lc + md)/(lc - md)Now by applying alternendo (la + mb)/(lc + md) = (la - mb)/(lc - md)So we get (la + mb): (lc + md) :: (la - mb): (lc - md)





$$\begin{split} (i) If \ &\frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}, \ show that \ &\frac{x}{y} = \frac{u}{v}. \\ (ii) \frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}, \ prove that \ &\frac{a}{b} = \frac{c}{d}. \\ Solution: \end{split}$$

 $(i)\frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}$ 

By applying alternendo

 $\frac{5x+7y}{5x-7y}=\frac{5u+v}{5u-7v}$ 

Now by applying componendo and dividendo

 $\frac{5x + 7y + 5x - 7y}{5x + 7y - 5x + 7y} = \frac{5u + 7v + 5u - 7v}{5u + 7v - 5u + 7v}$ 

By further calculation

 $\frac{10x}{14y} = \frac{10u}{14v}$ Dividing by  $\frac{10}{14}$  $\frac{x}{y} = \frac{u}{v}$ 

Therefore, it is proved.

$$(ii)\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$$

By applying alternendo

 $\frac{8a+5b}{8a-5b} = \frac{8c+5d}{8c-5d}$ 

Now by applying componendo and dividendo

 $\frac{8a+5b+8a-5b}{8a+5b-8a+5b} = \frac{8c+5d+8c-5d}{8c+5d-8c+8d}$ 



 $By\ further\ calculation$ 

 $\frac{16a}{10b} = \frac{16c}{10d}$   $Dividing \ by \ \frac{16}{10}$   $\frac{a}{b} = \frac{c}{d}$ 

Therefore, it is proved.

3. If (4a + 5b) (4c - 5d) = (4a - 5d) (4c + 5d), prove that a, b, c, d are in proportion. Solution:

It is given that (4a + 5b) (4c - 5d) = (4a - 5d) (4c + 5d)We can write it as  $\frac{4a + 5b}{4a - 5b} = \frac{4c + 5d}{4c - 5d}$ 

By applying componendo and dividendo

 $\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$ 

So we get

 $\frac{8a}{10b} = \frac{8a}{10b}$   $Dividing by \frac{8}{10}$   $\frac{a}{b} = \frac{c}{d}$ 

Therefore, it is proved that a, b, c, d are in proportion.

4. If (pa + qb): (pc + qd) :: (pa – qb): (pc – qd) prove that a: b :: c: d. Solution:

It is given that (pa + qb): (pc + qd) :: (pa - qb): (pc - qd)We can write it as



 $\frac{pa+qb}{pc+qd} = \frac{pa-qb}{pc-qd}$  $\frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$ 

By applying componendo and dividendo

 $\frac{pa+qb+pa-qb}{pa+qb-pa+qb} = \frac{pc+qd+pc-qd}{pc+qd-pc+qd}$ So we get

$$\begin{split} \frac{2pa}{2qb} &= \frac{2pc}{2qd} \\ Dividing \ by \ \frac{2p}{2q} \\ \frac{a}{b} &= \frac{c}{d} \end{split}$$

Therefore, it is proved that a: b :: c: d.

# 5. If (ma + nb): b :: (mc + nd): d, prove that a, b, c, d are in proportion. Solution:

It is given that (ma + nb): b :: (mc + nd): dWe can write it as (ma + nb)/b = (mc + nd)/dBy cross multiplication mad + nbd = mbc + nbd Here mad = mbc ad = bc By further calculation a/b = c/d

Therefore, it is proved that a, b, c, d are in proportion.

6. If  $(11a^2 + 13b^2) (11c^2 - 13d^2) = (11a^2 - 13b^2) (11c^2 + 13d^2)$ , prove that a: b :: c: d. Solution:

It is given that  $(11a^2 + 13b^2)(11c^2 - 13d^2) = (11a^2 - 13b^2)(11c^2 + 13d^2)$ We can write it as



 $\frac{11a^2+13b^2}{11a^2-13b^2}=\frac{11c^2+13d^2}{11c^2-13d^2}$ 

By applying componendo and dividendo

 $\frac{11a^2 + 13b^2 + 11a^2 - 13b^2}{11a^2 + 13b^2 - 11a^2 + 13b^2} = \frac{11c^2 + 13d^2 + 11c^2 - 13d^2}{11c^2 + 13d^2 - 11c^2 + 13d^2}$ 

So we get

 $\frac{22a^2}{26b^2} = \frac{22c^2}{26d^2}$  $Dividing by \frac{22}{26}$  $\frac{a^2}{b^2} = \frac{c^2}{d^2}$  $\frac{a}{b} = \frac{c}{d}$ 

Therefore, it is proved that a: b :: c: d.

7.  
If 
$$\mathbf{x} = \frac{\mathbf{2ab}}{\mathbf{a} + \mathbf{b}}$$
 find the value of  $\frac{\mathbf{x} + \mathbf{a}}{\mathbf{x} - \mathbf{a}} + \frac{\mathbf{x} + \mathbf{b}}{\mathbf{x} - \mathbf{b}}$   
Solution:

It is given that

 $\begin{aligned} x &= \frac{2ab}{a+b} \\ \frac{x}{a} &= \frac{2b}{a+b} \end{aligned}$ 

By applying componendo and dividendo

$$\frac{x+a}{x-a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}.....(1)$$

Similarly

 $\frac{x}{b} = \frac{2a}{a+b}$ 

By applying componendo and dividendo



$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}....(2)$$

Now adding both the equations

$$\frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

By further calculation

$$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$$

So we get

$$= \frac{-a - 3b + 3a + b}{a - b}$$
$$= \frac{2a - 2b}{a - b}$$

Taking 2 as common= 2(a - b)/ (a - b) = 2

If  $\mathbf{x} = \frac{\mathbf{8}\mathbf{a}\mathbf{b}}{\mathbf{a} + \mathbf{b}}$  find the value of  $\frac{\mathbf{x} + 4\mathbf{a}}{\mathbf{x} - 4\mathbf{a}} + \frac{\mathbf{x} + 4\mathbf{b}}{\mathbf{x} - 4\mathbf{b}}$ Solution:

It is given that

 $x = \frac{8ab}{a+b}$  $\frac{x}{4a} = \frac{2b}{a+b}$ 

 $By \ applying \ componendo \ and \ dividendo$ 

$$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}.....(1)$$

Similarly

 $\frac{x}{4b} = \frac{2a}{a+b}$ 

By applying componendo and dividendo

 $\frac{x+b}{x-b}=\frac{2a+a+b}{2a-a-b}=\frac{3a+b}{a-b}....(2)$ 



 $Now \ adding \ both \ the \ equations$ 

$$\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

 $By\ further\ calculation$ 

$$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$$

So we get

$$= \frac{-a - 3b + 3a + b}{a - b}$$
$$= \frac{2a - 2b}{a - b}$$

Taking 2 as common= 2(a - b)/ (a - b) = 2

If 
$$x = \frac{4\sqrt{6}}{\sqrt{2} + \sqrt{3}}$$
 find the value of  $\frac{x + 2\sqrt{2}}{x - 2\sqrt{2}} + \frac{x + 2\sqrt{3}}{x - 2\sqrt{3}}$   
Solution:

It is given that

$$x = \frac{4\sqrt{6}}{\sqrt{2} + \sqrt{3}}$$

We can write it as

$$x = \frac{4\sqrt{2} \cdot \sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

Here

$$\frac{x}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

By applying componendo and dividendo

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}-\sqrt{3}}$$

By further calculation

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \dots \dots (1)$$



Similarly

$$\frac{x}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{2} + \sqrt{3}}$$

By applying componendo and dividendo

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

By further calculation

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}\dots(2)$$

By adding both the equations

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

We can write it as

$$=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

So we get

$$=\frac{3\sqrt{3} + \sqrt{2} - 3\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}}$$
$$=\frac{2\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

Taking out 2 as common

$$=\frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}}$$

**10.** Using properties of properties, find **x** from the following equations:



(i) 
$$\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}} = 3$$
  
(ii)  $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \frac{5}{2}$   
(iii)  $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{a}{b}$   
(iv)  $\frac{\sqrt{12x+1}+\sqrt{2x-3}}{\sqrt{12x+1}-\sqrt{2x-3}} = \frac{3}{2}$   
(v)  $\frac{3x+\sqrt{9x^2+5}}{3x-\sqrt{9x^2+5}} = 5$ 

(vi) 
$$\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} = \frac{c}{d}$$

Solution:

$$(i)\frac{\sqrt{2-x} + \sqrt{2+x}}{\sqrt{2-x} - \sqrt{2+x}} = 3$$

By applying componendo and dividendo

$$\frac{\sqrt{2-x} + \sqrt{2+x} + \sqrt{2-x} - \sqrt{2+x}}{\sqrt{2-x} + \sqrt{2+x} - \sqrt{2-x} + \sqrt{2+x}} = \frac{3+1}{3-1}$$

On further calculation

 $\frac{2\sqrt{2-x}}{2\sqrt{2+x}} = \frac{4}{2}$  $\frac{\sqrt{2-x}}{\sqrt{2-x}} = \frac{2}{1}$ 

 $By \ squaring \ on \ both \ sides$ 

 $\frac{2-x}{2+x}=\frac{4}{1}$ 

By cross multiplication 8 + 4x = 2 - xSo we get 4x + x = 2 - 8 5x = -6x = -6/5



$$(ii)\frac{\sqrt{x+4} + \sqrt{x-10}}{\sqrt{x+4} - \sqrt{x-10}} = \frac{5}{2}$$

By applying componendo and dividendo

$$\frac{\sqrt{x+4} + \sqrt{x-10} + \sqrt{x+4} - \sqrt{x-10}}{\sqrt{x+4} + \sqrt{x-10} - \sqrt{x+4} + \sqrt{x-10}} = \frac{5+2}{5-2}$$

 $On\ further\ calculation$ 

 $\frac{2\sqrt{x+4}}{2\sqrt{x-10}} = \frac{7}{3}$   $\frac{\sqrt{x+4}}{\sqrt{x-10}} = \frac{7}{3}$ By squaring on both sides  $\frac{x+4}{x-10} = \frac{49}{9}$ By cross multiplication 49x - 490 = 9x + 36 49x - 9x = 36 + 490So we get 40x = 526 x = 526/40 x = 263/20



$$(iii)\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x}} - \sqrt{1-x} = \frac{a}{b}$$

 $By\ applying\ componendo\ and\ dividendo$ 

$$\frac{\sqrt{1+x} + \sqrt{1-x} + \sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x} - \sqrt{1+x} + \sqrt{1-x}} = \frac{a+b}{a-b}$$

On further calculation

$$\frac{2\sqrt{1+x}}{2\sqrt{1-x}} = \frac{a+b}{a-b}$$
$$\frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{a+b}{a-b}$$

By squaring on both sides

$$\frac{1+x}{1-x} = \frac{(a+b)^2}{(a-b)^2}$$

 $By \ applying \ componendo \ and \ dividendo$ 

$$\frac{1 + x + 1 - x}{1 + x - 1 + x} = \frac{(a + b)^2 + (a - b)^2}{(a + b)^2 - (a - b)^2}$$

By further calculation

 $\frac{2}{2x} = \frac{2(a^2 + b^2)}{4ab}$  $\frac{1}{x} = \frac{a^2 + b^2}{2ab}$ 

So we get

$$x = \frac{2ab}{a^2 + b^2}$$





$$(iv)\frac{\sqrt{12x+1} + \sqrt{2x-3}}{\sqrt{12x+1} - \sqrt{2x-3}} = \frac{3}{2}$$

By applying componendo and dividendo

$$\frac{\sqrt{12x+1} + \sqrt{2x-3} + \sqrt{12x+1} - \sqrt{2x-3}}{\sqrt{12x+1} + \sqrt{2x-3} - \sqrt{12x+1} + \sqrt{2x-3}} = \frac{3+2}{3-2}$$

 $On\ further\ calculation$ 

$$\frac{2\sqrt{12x+1}}{2\sqrt{2x-3}} = \frac{5}{1}$$
$$\frac{\sqrt{12x+1}}{\sqrt{2x-3}} = \frac{5}{1}$$

 $By\ squaring\ on\ both\ sides$ 

$$\frac{12x+1}{2x-3} = \frac{25}{1}$$

By cross multiplication 50x - 75 = 12x + 1 50x - 12x = 1 + 75So we get 38x = 76x = 76/38 = 2

$$(v)\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = \frac{5}{1}$$

 $By \ applying \ componendo \ and \ dividendo$ 

$$\frac{3x + \sqrt{9x^2 - 5} + 3x - \sqrt{9x^2 - 5}}{3x + \sqrt{9x^2 - 5} - 3x + \sqrt{9x^2 - 5}} = \frac{5+1}{5-1}$$

 $On\ further\ calculation$ 

$$\frac{6x}{2\sqrt{9x^2 - 5}} = \frac{6}{4}$$
$$\frac{3x}{\sqrt{9x^2 - 5}} = \frac{3}{2}$$



 $By\ squaring\ on\ both\ sides$ 

$$\frac{9x^2}{9x^2 - 5} = \frac{9}{4}$$

By cross multiplication  $81x^2 - 45 = 36x^2$   $81x^2 - 36x^2 = 45$ So we get  $45x^2 = 45$   $x^2 = 1$   $x = \pm 1$ x = 1, -1

Verification: (i) If x = 1  $\frac{3 \times 1 + \sqrt{9 \times 1 - 5}}{3 \times 1 - \sqrt{9 \times 1 - 5}} = \frac{3 + \sqrt{4}}{3 - \sqrt{4}}$ 

So we get

 $= \frac{3+2}{3-2}$  $= \frac{5}{1}$ Hence, x = 1.

(ii) If x = -1  $\frac{3 \times (-1) + \sqrt{9 \times (-1)^2 - 5}}{3 \times (-1) - \sqrt{9 \times (-1)^2 - 5}} = \frac{-3 + \sqrt{9 - 5}}{-3 - \sqrt{9 - 5}}$ 

By further calculation

$$=\frac{-3+\sqrt{4}}{-3-\sqrt{4}}$$

So we get

$$= \frac{-3+2}{-3-2}$$
$$= \frac{-1}{-5}$$
$$= \frac{1}{5}$$



Here  $1/5 \neq 5/1$ x = -1 is not the solution

Therefore, x = 1.

$$(vi)\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} = \frac{c}{d}$$

By applying componendo and dividendo

$$\frac{\sqrt{a+x}+\sqrt{a-x}+\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}-\sqrt{a+x}+\sqrt{a-x}}=\frac{c+d}{c-d}$$

 $On \ further \ calculation$ 

$$\frac{2\sqrt{a+x}}{2\sqrt{a-x}} = \frac{c+d}{c-d}$$
$$\frac{\sqrt{a+x}}{\sqrt{a-x}} = \frac{c+d}{c-d}$$

 $By\ squaring\ on\ both\ sides$ 

 $\frac{a+x}{a-x}=\frac{(c+d)^2}{(c-d)^2}$ 

By applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{(c+d)^2 + (c-d)^2}{(c+d)^2 - (c-d)^2}$$

 $By \ further \ calculation$ 

 $\frac{2a}{2x} = \frac{2(c^2 + d^2)}{4cd}$  $\frac{a}{x} = \frac{c^2 + d^2}{2cd}$ 

 $By \ cross \ multiplication$ 

$$x(c^2 + d^2) = 2acd$$

$$x = \frac{2acd}{c^2 + d^2}$$

#### 11. Using properties of proportion solve for x. Given that x is positive.

$$\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = 5$$
(i) 
$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$
Solution:



 $\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = \frac{5}{1}$ 

By applying componendo and dividendo

 $\frac{3x + \sqrt{9x^2 - 5} + 3x - \sqrt{9x^2 - 5}}{3x + \sqrt{9x^2 - 5} - 3x + \sqrt{9x^2 - 5}} = \frac{5 + 1}{5 - 1}$ 

On further calculation

 $\frac{6x}{2\sqrt{9x^2-5}} = \frac{6}{4}$   $\frac{3x}{\sqrt{9x^2-5}} = \frac{3}{2}$ By squaring on both sides  $\frac{9x^2}{9x^2-5} = \frac{9}{4}$ By cross multiplication  $81x^2-45 = 36x^2$   $81x^2-36x^2 = 45$ So we get  $45x^2 = 45$ So we get  $45x^2 = 45$   $x^2 = 1$   $x = \pm 1$  x = 1, -1Verification:
(i) If x = 1

(i) If x = 1  $\frac{3 \times 1 + \sqrt{9 \times 1 - 5}}{3 \times 1 - \sqrt{9 \times 1 - 5}} = \frac{3 + \sqrt{4}}{3 - \sqrt{4}}$ 

So we get

$$= \frac{3+2}{3-2}$$
$$= \frac{5}{1}$$

Hence, x = 1.



$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

#### (ii) $2x^{-1}$ Solution:

 $\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = \frac{4}{1}$ By applying componendo and dividendo, we get  $\frac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \frac{4 + 1}{4 - 1}$ On further calculation, we have 4x5 $=\frac{3}{3}$  $2\sqrt{4x^2-1}$ 5 $=\frac{3}{3}$ - 1 On squaring on both sides, we get  $4x^2$ 25 $\frac{1}{1} = \frac{1}{2}$  $4x^2 - 1$ 9 Upon cross multiplication,  $3\hat{6}x^2 = 25(4x^2 - 1)$  $36x^2 = 100x^2 - 25$  $64x^2 = 25$  $x^2 = 25/64$ Taking square root on both sides,  $x = \sqrt{25/64}$  $x = \pm 5/8$ Given that, x is positive Thus the value of x = 5/8. 12. Solve  $\frac{1+\mathbf{x}+\mathbf{x}^2}{1-\mathbf{x}+\mathbf{x}^2} = \frac{62(1+\mathbf{x})}{63(1-\mathbf{x})}$ Solution:



 $\frac{1+x+x^2}{1-x+x^2} = \frac{62(1+x)}{63(1-x)}$ 

 $We \ can \ write \ it \ as$ 

$$\frac{(1-x)(1+x+x^2)}{(1+x)(1-x+x^2)} = \frac{62}{63}$$
$$\frac{(1+x)(1-x+x^2)}{(1-x)(1+x+x^2)} = \frac{63}{62}$$
$$\frac{1+x^3}{1-x^3} = \frac{63}{62}$$

By applying componendo and dividendo

 $\frac{1+x^3+1-x^3}{1+x^3-1+x^3}=\frac{63+62}{63-62}$ 

 $On\ further\ calculation$ 

 $\frac{2}{2x^3} = \frac{125}{1}$  $\frac{1}{x^3} = \frac{125}{1}$ So we get

 $x^3 = (\frac{1}{5})^3$ 

$$x = 1/5$$

13. Solve for x:  $16(\frac{\mathbf{a}-\mathbf{x}}{\mathbf{a}+\mathbf{x}})^3 = \frac{\mathbf{a}+\mathbf{x}}{\mathbf{a}-\mathbf{x}}$  Solution:

 $16(\frac{a-x}{a+x})^3 = \frac{a+x}{a-x}$ 

 $We \ can \ write \ it \ as$ 



$$(\frac{a+x}{a-x}) \times (\frac{a+x}{a-x})^3 = 16$$
  
 $(\frac{a+x}{a-x})^4 = 16 = (\pm 2)^4$ 

Here

$$\frac{a+x}{a-x} = \pm 2$$

$$If \frac{a+x}{a-x} = \frac{2}{1}$$

 $By \ applying \ componendo \ and \ dividendo$ 

 $\frac{a+x+a-x}{a+x-a+x}=\frac{2+1}{2-1}$ 

 $On \ further \ calculation$ 

 $\frac{2a}{2x} = \frac{3}{1}$  $\frac{a}{x} = \frac{3}{1}$ So we get3x = ax = a/3

 $If \, \frac{a+x}{a-x} = \frac{-2}{1}$ 

By applying componendo and dividendo

 $\frac{a+x+a-x}{a+x-a+x} = \frac{-2+1}{-2-1}$ 

On further calculation

 $\frac{2a}{2x} = \frac{-1}{-3}$  $\frac{a}{x} = \frac{1}{3}$ So we get x = 3a



Therefore, x = a/3, 3a.

14.

 $If \ x = \frac{\sqrt{a+x} + \sqrt{a-1}}{\sqrt{a+1} - \sqrt{a-1}}, \ using \ properties \ of \ proportion, \ show \ that \ x^2 - 2ax + 1 = 0$ 

Solution:

It is given that  

$$x = \frac{\sqrt{a+x} + \sqrt{a-1}}{\sqrt{a+1} - \sqrt{a-1}}$$

We can write it as

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

By applying componendo and dividendo

 $\frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$ 

On further calculation

 $\frac{(x+1)^2 + (x-1)^2}{(x+1)^2 - (x-1)^2} = \frac{2a}{2}$ 

By applying componendo and dividendo

 $\frac{x^2 + 1 + 2x + x^2 + 1 - 2x}{x^2 + 1 + 2x - x^2 - 1 + 2x} = a$ 

So we get

$$\frac{2x^2+2}{4x} = a$$

Taking out common terms

$$\frac{2(x^2+1)}{4x} = a \\ \frac{x^2+1}{2x} = a$$

We get  $2ax = x^2 + 1$ 



 $x^2 - 2ax + 1 = 0$ 

Therefore, it is proved.

15.

 $\label{eq:Give x} \text{Give } \mathbf{x} = \frac{\sqrt{\mathbf{a}^2 + \mathbf{b}^2} + \sqrt{\mathbf{a}^2 - \mathbf{b}^2}}{\sqrt{\mathbf{a}^2 + \mathbf{b}^2} - \sqrt{\mathbf{a}^2 - \mathbf{b}^2}} \text{ use componendo and dividendo to prove that } \mathbf{b}^2 = \frac{2\mathbf{a}^2\mathbf{x}}{\mathbf{x}^2 + 1}$ 

Solution:

 $\frac{x}{1} = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$ 

By applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2} + \sqrt{a^2+b^2} - \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} + \sqrt{a^2-b^2} - \sqrt{a^2+b^2} + \sqrt{a^2-b^2}}$$

On further calculation

$(x+1)$ _	$2\sqrt{a^2+b^2}$
(x-1)	$\overline{2\sqrt{a^2+b^2}}$
(x+1)	$\sqrt{a^2 + b^2}$
$(x-1)^{-}$	$\sqrt{a^2 + b^2}$

By squaring both sides

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2}$$

Expanding the equations

 $\frac{x^2 + 1 + 2x}{x^2 + 1 - 2x} = \frac{a^2 + b^2}{a^2 - b^2}$ 

By applying componendo and dividendo

 $\frac{x^2 + 1 + 2x + x^2 + 1 - 2x}{x^2 + 1 + 2x - x^2 - 1 + 2x} = \frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - a^2 + b^2}$ 

So we get

$$\frac{2x^2 + 2}{4x} = \frac{2a^2}{2b^2}$$

Taking out common terms

$$\frac{2(x^2+1)}{4x} = \frac{2a^2}{2b^2}$$



$$\frac{x^2 + 1}{2x} = \frac{a^2}{b^2}$$

So we get

$$b^2 = \frac{2a^2x}{x^2+1}$$

16.

Given that  $\frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62}$ . Using componendo and dividendo find a : b. Solution:

 $\frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62}$ 

By applying componendo and dividendo

$$\frac{a^3 + 3ab^2 + b^3 + 3a^2b}{a^3 + 3ab^2 - b^3 - 3a^2b} = \frac{63 + 62}{63 - 62} = \frac{125}{1}$$

On further calculation

 $\frac{(a+b)^3}{(a-b)^3}=(\frac{5}{1})^3$ 

So we get

 $\frac{(a+b)}{(a-b)} = 5$ 

By cross multiplication a + b = 5a - 5bWe can write it as 5a - a - 5b - b = 0 4a - 6b = 0 4a = 6bWe get. a/b = 6/4 a/b = 3/2 $\therefore$  a: b = 3: 2

17.



Give  $\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}.$  Using componendo and dividendo find x:y. Solution:

It is given that  

$$\frac{x^3 + 12x}{6x^2 + 8} = \frac{y^3 + 27y}{9y^2 + 27}$$

By applying componendo and dividendo

 $\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8}=\frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$ 

 $On \ further \ calculation$ 

$$\frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

 $We \ can \ write \ it \ as$ 

$$\left(\frac{x+2}{x-2}\right)^3 = \left(\frac{y+3}{y-3}\right)^3$$

So we get

 $\frac{x+2}{x-2} = \frac{y+3}{y-3}$ 

By applying componendo and dividendo

 $\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$ 

By further calculation 2x/4 = 2y/3 x/2 = y/3By cross multiplication x/y = 2/3

Hence, the required ratio x: y is 2: 3.

18. Using the properties of proportion, solve the following equation for x; given  $\frac{x^3+3x}{3x^2+1}=\frac{341}{91}$ 

**Solution:** It is given that



 $\frac{x^3+3x}{3x^2+1}=\frac{341}{91}$ 

By applying componendo and dividendo

 $\frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1}=\frac{341+91}{341-91}$ 

On further calculation

 $\frac{x^3+3x^2+3x+1}{x^3-3x^2+3x-1}=\frac{432}{250}=\frac{216}{125}$ 

 $We \ can \ write \ it \ as$ 

$$\frac{(x+1)^3}{(x-1)^3} = \frac{216}{125} = (\frac{6}{5})^3$$

So we get

 $\frac{x+1}{x-1} = \frac{6}{5}$ 

By cross multiplication 6x - 6 = 5x + 5 6x - 5x = 5 + 6x = 11

19.

If  $\frac{x+y}{ax+by} = \frac{y+z}{ay+bz} = \frac{z+x}{az+bx}$ , prove that each of these ratio is equal to  $\frac{2}{a+b}$  unless x+y+z=0. Solution:

It is given that

$$\frac{x+y}{ax+by} = \frac{y+z}{ay+bz} = \frac{z+x}{az+bx}$$

 $By \ addition$ 

$$=\frac{x+y+y+z+z+x}{ax+by+ay+bz+az+bx}$$

 $By \ further \ calculation$ 



$$= \frac{2(x+y+z)}{x(a+b) + y(a+b) + z(a+b)}$$

So we get

$$= \frac{2(x+y+z)}{(a+b)(x+y+z)}$$
$$= \frac{2}{a+b}$$
If x + y + z \neq 0

Therefore, it is proved.





# **CHAPTER TEST**

1. Find the compound ratio of  $(a + b)^2$ :  $(a - b)^2$ ,  $(a^2 - b^2)$ :  $(a^2 + b^2)$  and  $(a^4 - b^4)$ :  $(a + b)^4$ Solution:

 $(a + b)^{2}: (a - b)^{2}$   $(a^{2} - b^{2}): (a^{2} + b^{2})$   $(a^{4} - b^{4}): (a + b)^{4}$ We can write it as  $= \frac{(a + b)^{2}}{(a - b)^{2}} \times \frac{a^{2} - b^{2}}{a^{2} + b^{2}} \times \frac{a^{4} - b^{4}}{(a + b)^{4}}$ 

By further calculation

$$= \frac{(a+b)^2}{(a-b)^2} \times \frac{(a+b)(a-b)}{a^2+b^2} \times \frac{(a^2+b^2)(a+b)(a-b)}{(a+b)^4}$$

So we get

$$=\frac{1}{1}$$
  
= 1 : 1

2. If (7p + 3q): (3p – 2q) = 43: 2, find p: q. Solution:

It is given that (7p + 3q): (3p - 2q) = 43: 2We can write it as (7p + 3q)/(3p - 2q) = 43/2By cross multiplication 129p - 86q = 14p + 6q 129p - 14p = 6q + 86qSo we get 115p = 92qBy division p/q = 92/115 = 4/5

Hence, p: q = 4: 5.

# 3. If a: b = 3: 5, find (3a + 5b): (7a - 2b). Solution:

It is given that a: b = 3: 5 We can write it as a/b = 3/5



Here (3a + 5b): (7a - 2b)Now dividing the terms by b  $3 \times \frac{a}{b} + 5: 3 \times \frac{a}{b} - 2$ 

Substituting the values of  $\frac{a}{b}$ 

$$3 \times \frac{3}{5} + 5 : 3 \times \frac{3}{5} - 2$$

 $By \ further \ calculation$ 

$$\left(\frac{9}{5}+5\right):\left(\frac{21}{5}-2\right)$$

Taking LCM

 $\frac{9+25}{5}:\frac{21-10}{5}$ 

So we get

$$\frac{34}{5}:\frac{11}{5}$$

Here (3a + 5b): (7a - 2b) = 34: 11

# 4. The ratio of the shorter sides of a right angled triangle is 5: 12. If the perimeter of the triangle is 360 cm, find the length of the longest side. Solution:

Consider the two shorter sides of a right-angled triangle as 5x and 12x So the third longest side

$$= \sqrt{(5x)^2 + (12x)^2}$$
$$= \sqrt{25x^2 + 144x^2}$$
$$= \sqrt{169x^2}$$
$$= 13x$$

It is given that 5x + 12x + 13x = 360 cm By further calculation 30x = 360



We get x = 360/30 = 12

Here the length of the longest side = 13xSubstituting the value of x =  $13 \times 12$ = 156 cm

5. The ratio of the pocket money saved by Lokesh and his sister is 5: 6. If the sister saves Rs 30 more, how much more the brother should save in order to keep the ratio of their savings unchanged? Solution:

Consider 5x and 6x as the savings of Lokesh and his sister.

Lokesh should save Rs y more Based on the problem (5x + y)/(6x + 30) = 5/6By cross multiplication 30x + 6y = 30x + 150By further calculation 30x + 6y - 30x = 150So we get 6y = 150y = 150/6 = 25

Therefore, Lokesh should save Rs 25 more than his sister.

6. In an examination, the number of those who passed and the number of those who failed were in the ratio of 3: 1. Had 8 more appeared, and 6 less passed, the ratio of passed to failures would have been 2: 1. Find the number of candidates who appeared. Solution:

Consider the number of passed = 3xNumber of failed = xSo the total candidates appeared = 3x + x = 4x

In the second case Number of candidates appeared = 4x + 8Number of passed = 3x - 6Number of failed = 4x + 8 - 3x + 6 = x + 14Ratio = 2: 1

Based on the condition (3x-6)/(x+14) = 2/1By cross multiplication 3x-6 = 2x + 28 3x-2x = 28 + 6x = 34

Here the number of candidates appeared =  $4x = 4 \times 34 = 136$


7. What number must be added to each of the numbers 15, 17, 34 and 38 to make them proportional? Solution:

Consider x be added to each number So the numbers will be 15 + x, 17 + x, 34 + x and 38 + xBased on the condition (15 + x)/(17 + x) = (34 + x)/(38 + x)By cross multiplication (15 + x) (38 + x) = (34 + x) (17 + x)By further calculation  $570 + 53x + x^2 = 578 + 51x + x^2$ So we get  $x^2 + 53x - x^2 - 51x = 578 - 570$ 2x = 8x = 4

Hence, 4 must be added to each of the numbers.

8. If (a + 2b + c), (a - c) and (a - 2b + c) are in continued proportion, prove that b is the mean proportional between a and c. Solution:

It is given that (a + 2b + c), (a - c) and (a - 2b + c) are in continued proportion We can write it as (a + 2b + c)/(a - c) = (a - c)/(a - 2b + c)By cross multiplication  $(a + 2b + c)(a - 2b + c) = (a - c)^2$ On further calculation  $a^2 - 2ab + ac + 2av - 4b^2 + 2bc + ac - 2bc + c^2 = a^2 - 2ac + c^2$ So we get  $a^2 - 2ab + ac + 2ab - 4b^2 + 2bc + ac - 2bc + c^2 - a^2 + 2ac - c^2 = 0$   $4ac - 4b^2 = 0$ Dividing by 4  $ac - b^2 = 0$  $b^2 = ac$ 

Therefore, it is proved that b is the mean proportional between a and c.

## 9. If 2, 6, p, 54 and q are in continued proportion, find the values of p and q. Solution:

It is given that 2, 6, p, 54 and q are in continued proportion We can write it as 2/6 = 6/p = p/54 = 54/q

(i) We know that 2/6 = 6/p

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By cross multiplication 2p = 36p = 18

(ii) We know that p/54 = 54/qBy cross multiplication  $pq = 54 \times 54$ Substituting the value of p  $q = (54 \times 54)/18 = 162$ 

Therefore, the values of p and q are 18 and 162.

10. If a, b, c, d, e are in continued proportion, prove that:  $a: e = a^4: b^4$ . Solution:

It is given that a, b, c, d, e are in continued proportion We can write it as a/b = b/c = c/d = d/e = kd = ek,  $c = ek^2$ ,  $b = ek^3$  and  $a = ek^4$ 

```
Here

LHS = a/e

Substituting the values

= ek^4/e

= k^4
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\begin{split} RHS &= a^4/b^4\\ Substituting the values \\ &= (ek^4)^4/(ek^3)^4\\ So we get \\ &= e^4k^{16}/e^4k^{12}\\ &= k^{16-12}\\ &= k^4 \end{split}
```

Hence, it is proved that  $a: e = a^4: b^4$ .

## 11. Find two numbers whose mean proportional is 16 and the third proportional is 128. Solution:

Consider x and y as the two numbers Mean proportion = 16 Third proportion = 128  $\sqrt{xy} = 16$ xy = 256Here  $x = 256/y \dots (1)$  $y^2/x = 128$ Here

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 $x = y^2/128 \dots (2)$ 

Using both the equations  $256/y = y^3/128$ By cross multiplication  $y^3 = 256 \times 128 = 32768$   $y^3 = 32^3$ y = 32

Substituting the value of y in equation (1) x = 256/ySo we get x = 256/32 = 8

Hence, the two numbers are 8 and 32.

12. If q is the mean proportional between p and r, prove that:  $p^2 - 3q^2 + r^2 = q^4(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2})$ 

#### Solution:

It is given that q is the mean proportional between p and r  $q^2 = pr$ 

Here

LHS =  $p^2 - 3q^2 + r^2$ We can write it as =  $p^2 - 3pr + r^2$ RHS =  $q^4(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2})$ 

 $We \ can \ write \ it \ as$ 

$$= (q^2)^2 (\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2})$$

Substituting the value of q

$$= (pr)^2 \left(\frac{1}{p^2} - \frac{3}{pr} + \frac{1}{r^2}\right)$$

Taking LCM

$$= p^2 r^2 (\frac{r^2 - 3pr + p^2}{p^2 r^2})$$
  
So we get

 $= r^2 - 3pr + p^2$ 



Here LHS = RHS

Therefore, it is proved.

13. If 
$$a/b = c/d = e/f$$
, prove that each ratio is  
(i)  $\sqrt{\frac{3a^2 - 5c^2 + 7e^2}{3b^2 - 5d^2 + 7f^2}}$   
(ii)  $[\frac{2a^3 + 5c^3 + 7e^3}{2b^3 + 5d^3 + 7f^3}]^{\frac{1}{3}}$   
Solution:

It is given that a/b = c/d = e/f = kSo we get a = k, c = dk, e = fk

$$(i)\sqrt{\frac{3a^2-5c^2+7e^2}{3b^2-5d^2+7f^2}}$$

 $Substituting \ the \ values$ 

$$= \sqrt{\frac{3b^2k^2 - 5d^2k^2 + 7f^2k^2}{3b^2 - 5d^2 - 7f^2}}$$

 $Taking \ k \ as \ common$ 

$$= k \sqrt{\frac{3b^2 - 5d^2 + 7f^2}{3b^2 - 5d^2 + 7f^2}}$$
  
= k

Therefore, it is proved.

$$(ii)\left[\frac{2a^3+5c^3+7e^3}{2b^3+5d^3+7f^3}
ight]^{\frac{1}{3}}$$

 $Substituting \ the \ values$ 

$$= \left[\frac{2b^3k^3 + 5d^3k^3 + 7f^3k^3}{2b^3 + 5d^3 + 7f^3}\right]^{\frac{1}{3}}$$



 $Taking \ k \ as \ common$ 

$$= k \left[\frac{2b^3 + 5d^3 + 7f^3}{2b^3 + 5d^3 + 7f^3}\right]^{\frac{1}{3}}$$
  
= k

Therefore, it is proved.

 $\frac{14. \text{ If } x/a = y/b = z/c, \text{ prove that}}{3x^3 - 5y^3 + 4z^3} = (\frac{3x - 5y + 4z}{3a - 5b + 4c})^3$ Solution:

It is given that x/a = y/b = z/c = kSo we get x = ak, y = bk, z = ck

Here

$$LHS = \frac{3x^3 - 5y^3 + 4z^3}{3a^3 - 5b^3 + 4c^3}$$

Substituting the values

$$=\frac{3a^3k^3-5b^3k^3+4c^3k^3}{3a^3-5b^3+4c^3}$$

Taking out the common terms

 $=\frac{k^3(3a^3-5b^3+4c^3)}{3a^3-5b^3+4c^3}\\=\mathbf{k}^3$ 

$$RHS = (\frac{3x - 5y + 4z}{3a - 5b + 4c})^3$$

Substituting the values

$$= (\frac{3ak - 5bk + 4ck}{3a - 5b + 4c})^3$$

 $Taking \ out \ the \ common \ terms$ 

$$= \left(\frac{k(3a-5b+4c)}{3a-5b+4}\right)^3$$



 $= k^3$ 

Hence, LHS = RHS.

15. If x: a = y: b, prove that  

$$\frac{x^4 + a^4}{x^3 + a^3} + \frac{y^4 + b^4}{y^3 + b^3} = \frac{(x + y)^4 + (a + b)^4}{(x + y)^3 + (a + b)^3}$$
Solution:

We know that x/a = y/b = kSo we get x = ak, y = bk

Here

$$LHS = \frac{x^4 + a^4}{x^3 + a^3} + \frac{y^4 + b^4}{y^3 + b^3}$$

 $Substituting \ the \ values$ 

$$= \frac{a^4k^4 + a^4}{a^3k^3 + a^3} + \frac{b^4k^4 + b^4}{b^3k^3 + b^3}$$

 $Taking \ out \ the \ common \ terms$ 

$$= \frac{a^4(k^4+1)}{a^3(k^3+1)} + \frac{b^4(k^4+1)}{b^3(k^3+1)}$$

 $We \ get$ 

$$= \frac{a(k^4+1)}{k^3+1} + \frac{b(k^4+1)}{k^3+1}$$

 $We \ can \ write \ it \ as$ 

$$= \frac{a(k^4+1)+b(k^4+1)}{k^3+1}$$
$$= \frac{(k^4+1)(a+b)}{k^3+1}$$

$$RHS = \frac{(x+y)^4 + (a+b)^4}{(x+y)^3 + (a+b)^3}$$

Substituting the values



$$=\frac{(ak+bk)^4+(a+b)^4}{(ak+bk)^3+(a+b)^3}$$

 $Taking \ out \ the \ common \ terms$ 

$$=\frac{k^4(a+b)^4+(a+b)^4}{k^3(a+b)^3(a+b)^3}$$

We get

$$=\frac{(a+b)^4(k^4+1)}{(a+b)^3(k^3+1)}$$

 $We \ can \ write \ it \ as$ 

$$=\frac{(a+b)(k^4+1)}{k^3+1}$$

Here LHS = RHS

Therefore, it is proved.

#### 16.

If  $\frac{\mathbf{x}}{\mathbf{b} + \mathbf{c} - \mathbf{a}} = \frac{\mathbf{y}}{\mathbf{c} + \mathbf{a} - \mathbf{b}} = \frac{\mathbf{z}}{\mathbf{a} + \mathbf{b} - \mathbf{c}}$  prove that each ratio's equal to :  $\frac{\mathbf{x} + \mathbf{y} + \mathbf{z}}{\mathbf{a} + \mathbf{b} + \mathbf{c}}$  Solution:

Consider

 $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = k$ So we get x = k (b+c-a)y = k (c+a-b)z = k (a+b-a)

#### Here

$$\frac{x+y+z}{a+b+c} = \frac{k(b+c-a) + k(c+a-b) + k(a+b-c)}{a+b+c}$$

 $By\ further\ calculation$ 

$$=\frac{k(b+c-a+c+a-b+a+b-c)}{a+b+c}$$

So we get



$$=\frac{k(a+b+c)}{a+b+c}$$

Therefore, it is proved.

17. If a: b = 9: 10, find the value of (i) $\frac{5a + 3b}{5a - 3b}$ (ii) $\frac{2a^2 - 3b^2}{2a^2 + 3b^2}$ Solution:

It is given that a: b = 9: 10So we get a/b = 9/10

$$(i)\frac{5a+3b}{5a-3b} = \frac{\frac{5a}{b} + \frac{3b}{b}}{\frac{5a}{b} - \frac{3b}{b}}$$

 $By\ further\ calculation$ 

$$=\frac{\frac{5a}{b}+3}{\frac{5a}{b}-3}$$

Substituting the values of  $\frac{a}{b}$ 

$$=\frac{5\times\frac{9}{10}+3}{5\times\frac{9}{10}-3}$$

So we get

$$=\frac{\frac{9}{2}+3}{\frac{9}{2}-3}\\=\frac{\frac{15}{2}}{\frac{3}{2}}$$

 $By\ further\ simplification$ 

$$=\frac{15}{2}\times\frac{2}{3}$$
$$=5$$



# $(ii)\frac{2a^2-3b^2}{2a^2+3b^2}$

 $Dividing \ by \ b^2$ 

$$= \frac{\frac{2a^2}{b^2} - \frac{3b^2}{b^2}}{\frac{2a^2}{b^2} + \frac{3b^2}{b^2}}$$

By further calculation

$$=\frac{2(\frac{a}{b})^2-3}{2(\frac{a}{b})^2+3}$$

Substituting the values of  $\frac{a}{b}$ 

$$=\frac{2(\frac{9}{10})^2-3}{2(\frac{9}{10})^2+3}$$

So we get

$$= \frac{2 \times \frac{81}{100} - 3}{2 \times \frac{81}{100} + 3}$$
$$= \frac{\frac{81}{50} - 3}{\frac{81}{50} + 3}$$

By further simplification

$$=\frac{\frac{81-150}{50}}{\frac{81+150}{50}}$$

We get

 $= \frac{-69}{50} \times \frac{50}{231}$  $= \frac{-69}{231}$  $= \frac{-23}{77}$ 

18. If  $(3x^2 + 2y^2)$ :  $(3x^2 - 2y^2) = 11$ : 9, find the value of  $3x^4 + 25y^4$   $\overline{3x^4 - 25y^4}$ Solution:

It is given that  $(3x^2 + 2y^2)$ :  $(3x^2 - 2y^2) = 11:9$ We can write it as ML Aggarwal Solutions for Class 10 Maths Chapter 7 -

**Ratio and Proportion** 



 $\frac{3x^2+2y^2}{3x^2-2y^2}=\frac{11}{9}$ 

By applying componendo and dividendo

$$\frac{3x^2 + 2y^2 + 3x^2 - 2y^2}{3x^2 + 2y^2 - 3x^2 + 2y^2} = \frac{11 + 9}{11 - 9}$$

 $By\ further\ calculation$ 

$$\frac{6x^2}{4y^2} = \frac{20}{2} \\ \frac{3x^2}{2y^2} = 10$$

 $We \ can \ write \ it \ as$ 

$$\frac{x^2}{y^2} = 10 \times \frac{2}{3} = \frac{20}{3}$$

Here

$$\frac{3x^4 + 25y^4}{3x^4 - 25y^4} = \frac{\frac{3x^4}{y^4} + \frac{25y^4}{y^4}}{\frac{3x^4}{y^4} - \frac{25y^4}{y^4}}$$

 $We \ can \ write \ it \ as$ 

$$=\frac{3(\frac{x^2}{y^2})^2+25}{3(\frac{x^2}{y^2})^2-25}$$

By substituing the values

$$=\frac{3(\frac{20}{3})^2+25}{3(\frac{20}{3})^2-25}$$

 $By \ further \ calculation$ 

$$=\frac{3\times\frac{400}{9}+25}{3\times\frac{400}{9}-25}$$

Taking LCM



$$=\frac{\frac{400+75}{3}}{\frac{400-75}{3}}$$

So we get

$$= \frac{475}{3} \times \frac{3}{325}$$
$$= \frac{19}{13}$$

19. If  $\mathbf{x} = \frac{2\mathbf{m}\mathbf{a}\mathbf{b}}{\mathbf{a} + \mathbf{b}}$ , find the value of  $\frac{\mathbf{x} + \mathbf{m}\mathbf{a}}{\mathbf{x} - \mathbf{m}\mathbf{a}} + \frac{\mathbf{x} + \mathbf{m}\mathbf{b}}{\mathbf{x} - \mathbf{m}\mathbf{b}}$ . Solution:

It is given that

 $x = \frac{2mab}{a+b}$ 

 $We \ can \ write \ it \ as$ 

 $= \frac{x}{ma} + \frac{2b}{a+b}$ 

 $By\ applying\ componendo\ and\ dividendo$ 

 $\frac{x+ma}{x-ma} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}....(1)$ 

Similarly

 $\frac{x}{mb} = \frac{2a}{a+b}$ 

 $By \ applying \ componendo \ and \ dividendo$ 

 $\frac{x+mb}{x-mb}=\frac{2a+a+b}{2a-a-b}=\frac{3a+b}{a-b}....(2)$ 

Now adding both the equations

$$\frac{x+ma}{x-ma} + \frac{x+mb}{x-mb} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

 $By \ further \ calculation$ 

$$=\frac{-3b+a}{a-b}+\frac{3a+b}{a-b}$$



So we get

$$= \frac{-3b - a + 3a + b}{a - b}$$
$$= \frac{2a - 2b}{a - b}$$

Taking out 2 as common

$$=\frac{2(a-b)}{a-b}$$
$$=2$$

20.

 $=rac{2(\mathbf{a}^2-\mathbf{b}^2)}{\mathbf{b}^2}$ If  $\mathbf{x} = \frac{\mathbf{pab}}{\mathbf{a} + \mathbf{b}}$ , prove that  $\frac{\mathbf{x} + \mathbf{pa}}{\mathbf{x} - \mathbf{pa}} - \frac{\mathbf{x} + \mathbf{pb}}{\mathbf{x} - \mathbf{pb}}$ ab Solution:

It is given that

$$x = \frac{pab}{a+b}$$

We can write it as

$$=\frac{x}{pa}+\frac{b}{a+b}$$

By applying componendo and dividendo

$$\frac{x+pa}{x-pa} = \frac{b+a+b}{b-a-b} = \frac{a+2b}{-a}....(1)$$

Similarly

$$\frac{x}{pb} = \frac{a}{a+b}$$

By applying componendo and dividendo

$$\frac{x+pb}{x-pb} = \frac{a+a+b}{a-a-b} = \frac{2a+b}{-b}....(2)$$

We know that

 $LHS = \frac{x + pa}{x - pa} - \frac{x + pb}{x - pb}$ 

Using both the equations



# $= \frac{a+2b}{-a} - \frac{2a+b}{-b}$ $= \frac{a+2b}{-a} + \frac{2a+b}{b}$

Taking LCM

$$= \frac{ab + 2b^2 - 2a^2 - ab}{-ab} = \frac{2b^2 - 2a^2}{-ab}$$

So we get

$$=\frac{-2a^2+2b^2}{-ab}$$

 $Taking \ out \ 2 \ as \ common$ 

$$= \frac{-2(a^2 - b^2)}{-ab}$$
$$= \frac{2(a^2 - b^2)}{ab}$$
$$= RHS$$

21.

#### $\sqrt{a^2}$ $=\frac{\mathbf{b}}{\mathbf{x}}$ . $+\mathbf{x}+$ а Find x from the equation $\mathbf{x}^2$

Solution:

It is given that  $\frac{a + x + \sqrt{a^2 - x^2}}{a + x - \sqrt{a^2 - x^2}} = \frac{b}{x}$ 

By applying componendo and dividendo

$$\frac{a+x+\sqrt{a^2-x^2}+a+x-\sqrt{a^2-x^2}}{a+x+\sqrt{a^2-x^2}-a-x+\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$$

By further calculation

$$\frac{2(a+x)}{2\sqrt{a^2 - x^2}} = \frac{b+x}{b-x}$$

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### ML Aggarwal Solutions for Class 10 Maths Chapter 7 -**Ratio and Proportion**



 $Dividing \ by \ 2$ 

$$\frac{(a+x)}{\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$$

 $By\ squaring\ on\ both\ sides$ 

$$\frac{(a + x)^2}{a^2 - x^2} = \frac{(b + x)^2}{(b - x)^2}$$

 $We\ can\ write\ it\ as$ 

$$\frac{(a+x)^2}{(a+x)(a-x)} = \frac{(b+x)^2}{(b-x)^2}$$
$$\frac{a+x}{a-x} = \frac{(b+x)^2}{(b-x)^2}$$

 $By \ applying \ componendo \ and \ dividendo$ 

$$\frac{a+x+a-x}{a+x-a+x} = \frac{(b+x)^2 + (b-x)^2}{(b+x)^2 - (b-x)^2}$$

 $By\ further\ calculation$ 

$$\frac{2a}{2x} = \frac{2(b^2 + x^2)}{4bx}$$

 $Dividing \ by \ 2$ 

$$\frac{a}{x} = \frac{(b^2 + x^2)}{2bx}$$

 $By \ cross \ multiplication$ 

$$2abx = x(b^{2} + x^{2})$$
$$2ab = b^{2} + x^{2}$$
$$x^{2} = 2ab - b^{2}$$
$$x = \sqrt{2ab - b^{2}}$$



22.

If 
$$\mathbf{x} = \frac{\sqrt[3]{a+1} + \sqrt[3]{a-1}}{\sqrt[3]{a+1} - \sqrt[3]{a-1}}$$
, prove that :  $\mathbf{x}^3 - 3\mathbf{a}\mathbf{x}^2 + 3\mathbf{x} - \mathbf{a} = \mathbf{0}$ .  
Solution:

It is given that  $x = \frac{\sqrt[3]{a+1} + \sqrt[3]{a-1}}{\sqrt[3]{a+1} - \sqrt[3]{a-1}}$ 

By applying componendo and dividendo

 $\frac{x+1}{x-1} = \frac{\sqrt[3]{a+1} + \sqrt[3]{a-1} + \sqrt[3]{a+1} - \sqrt[3]{a-1}}{\sqrt[3]{a+1} - \sqrt[3]{a-1} - \sqrt[3]{a+1} + \sqrt[3]{a-1}}$ 

 $On \ further \ calculation$ 

$$\frac{x+1}{x-1} = \frac{2\sqrt[3]{a+1}}{2\sqrt[3]{a-1}}$$
$$\frac{x+1}{x-1} = \frac{\sqrt[3]{a+1}}{\sqrt[3]{a-1}}$$

By cubing on both sides

 $\frac{(x+1)^3}{(x-1)^3} = \frac{a+1}{a-1}$ 

By applying componendo and dividendo

 $\frac{(x+1)^3+(x-1)^3}{(x+1)^3-(x-1)^3}=\frac{a+1+a-1}{a+1-a+1}$ 

By further calculation

$$\frac{2(x^3+3x)}{2(3x^2+1)} = \frac{2a}{2}$$

Dividing by 2

 $\frac{(x^3 + 3x)}{(3x^2 + 1)} = \frac{a}{1}$ By cross multiplication  $x^3 + 3x = 3ax^2 + a$  $x^3 - 3ax^2 + 3x - a = 0$ 



Therefore, it is proved.



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