EXERCISE 8.1

1. Classify the following matrices:

(i) $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$

Solution:
It is square matrix of order 2

(ii) $[2 \;\; 3 \;\; - \;\; 7]$

Solution:
It is row matrix of order 1 × 3

(iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

Solution:
It is column matrix of order 3 × 1

(iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

Solution:
It is a matrix of order 3 × 2

(v) $\begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$

Solution:

It is a matrix of order 2 × 3

(vi) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Solution:
It is zero matrix of order 2 × 3

2. (i) If a matrix has 4 elements, what are the possible order it can have?

Solution:
It can have 1 × 4, 4 × 1 or 2 × 2 order.

(ii) If a matrix has 4 elements, what are the possible orders it can have?

Solution:
It can have 1 × 8, 8 × 1, 2 × 4 or 4 × 2 order.

3. Construct a 2 × 2 matrix whose elements a_{ij} are given by
(i) a_{ij} = 2i – j
(ii) a_{ij} =i.j

Solution:
(i) Given a_{ij} = 2i – j
Therefore matrix of order 2 × 2 is
$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

(ii) Given a_{ij} =i.j
Therefore matrix of order 2 × 2 is
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

4. Find the values of x and y if:

$$\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now by comparing the corresponding elements,

2x + y = 5 ….. i

3x – 2y = 4 ….ii

Multiply (i) by 2 and (ii) by 1 we get

4x + 2y = 10 and 3x – 2y = 4

By adding we get

7x = 14

x = 14/7

x = 2

Substituting the value of x in (i)

4 + y = 5

y = 5 – 4

y = 1

Hence x = 2 and y = 1

5. Find the value of x if

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Comparing the corresponding terms of given matrix we get

-y = 2

Therefore y = -2

Again we have

3x + y = 1
3x = 1 – y
Substituting the value of y we get
3x = 1 – (-2)
3x = 1 + 2
3x = 3
x = 3/3
x = 1
Hence x = 1 and y = -2

6. If

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Find the values of x and y.

Solution:
Given

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Comparing the corresponding terms, we get
x + 3 = 5
x = 5 – 3
x = 2
Again we have
y – 4 = 3
y = 3 + 4
y = 7
Hence x = 2 and y = 7

7. Find the values of x, y and z if

$$\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$$

Comparing the corresponding elements of given matrix, then we get

x + 2 = -5

x = -5 – 2

x = -7

Also we have 5z = -20

z = -20/5

z = - 4

Again from given matrix we have

$y^2 + y - 6 = 0$

The above equation can be written as

$y^2 + 3y - 2y - 6 = 0$

y (y + 3) – 2 (y + 3) = 0

y + 3 = 0 or y – 2 = 0

y = -3 or y = 2

Hence x = -7, y = -3, 2 and z = -4

8. Find the values of x, y, a and b if

$$\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Comparing the corresponding elements

x – 2 = 3 and y = 1

x = 2 + 3

x = 5

again we have

a + 2b = 5….. i

3a – b = 1 ……ii

Multiply (i) by 1 and (ii) by 2

a + 2b = 5

6a – 2b = 2

Now by adding above equations we get

7a = 7

a = 7/7

a = 1

Substituting the value of a in (i) we get

1 + 2b = 5

2b = 5 -1

2b = 4

b = 4/2

b = 2

9. Find the values of a, b, c and d if

$$\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Comparing the corresponding terms, we get

3 = d

d = 3

Also we have

5 + c = -1

c = -1 – 5

c = -6

Also we have,

a + b = 6 and a b = 8

we know that,

$(a – b)^2 = (a + b)^2 – 4$ ab

$(6)^2 – 32 = 36 – 32 = 4 = (\pm 2)^2$

a – b = ± 2

If a – b = 2
a + b = 6
Adding the above two equations we get
2a = 4
a = 4/2
a = 2
b = 6 – 4
b = 2
Again we have a – b = -2
And a + b = 6
Adding above equations we get
2a = 4
a = 4/2
a = 2
Also, b = 6 – 2 = 4
a = 2 and b = 4

EXERCISE 8.2

1. Given that M = $\begin{bmatrix}2 & 0\\1 & 2\end{bmatrix}$ and N = $\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}$, find M + 2N

Solution:

Given

$M = \begin{bmatrix}2 & 0\\1 & 2\end{bmatrix}$

$N = \begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}$

Now we have to find M + 2N

$M + 2N = \begin{bmatrix}2 & 0\\1 & 2\end{bmatrix} + 2\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}$

On simplifying we get,

$= \begin{bmatrix}2 & 0\\1 & 2\end{bmatrix} + \begin{bmatrix}4 & 0\\-2 & 4\end{bmatrix}$

$= \begin{bmatrix}2+4 & 0+0\\1-2 & 2+4\end{bmatrix} = \begin{bmatrix}6 & 0\\-1 & 6\end{bmatrix}$

2. If $A = \begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}$ and $B = \begin{bmatrix}0 & 1\\-2 & 3\end{bmatrix}$

find 2A – 3B

Solution:

Given

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

Now we have to find,

$$\therefore 2A - 3B = 2\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

3. Simplify:

$$sinA \begin{bmatrix} sinA & -cosA \\ cosA & sinA \end{bmatrix} + cosA \begin{bmatrix} cosA & sinA \\ -sinA & cosA \end{bmatrix}$$

Solution:

Given,

$$sinA \begin{bmatrix} sinA & -cosA \\ cosA & sinA \end{bmatrix} + cosA \begin{bmatrix} cosA & sinA \\ -sinA & cosA \end{bmatrix}$$

$On\ simplification, we\ get$

$$= \begin{bmatrix} sin^2A & -sinA\,cosA \\ sinA\,cosA & sin^2A \end{bmatrix} + \begin{bmatrix} cos^2A & cosA\,sinA \\ -cosA\,sinA & cos^2A \end{bmatrix}$$

$$= \begin{bmatrix} sin^2A + cos^2A & -sinA\,cosA + cosA\,sinA \\ sinA\,cosA - cosA\,sinA & sin^2A + cos^2A \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{(Since, } \sin^2A + \cos^2A = 1)$$

4.

A = $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, C = $\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Find A + 2B – 3C

Solution:

Given

A = $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, C = $\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Now we have to find A + 2B – 3C

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 1-4-0 & 2-2-9 \\ -2+2-6 & 3+4+3 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

5. If A = $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

Find the matrix X if:

(i) 3A + X = B

(ii) X – 3B = 2A

Solution:

Given

A = $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

B = $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

Now we have to find

(i) 3A + X = B

X = B – 3A

Substituting the values we get

$$X = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & 2+3 \\ -1-3 & 1-6 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -4 & -5 \end{bmatrix}$$

(ii) X – 3B = 2A

X = 2A + 3B

Now substituting the values A and B we get

$$X = 2\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3 & -2+6 \\ 2-3 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$$

6. Solve the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

On rearranging we get

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} = 3X$$

On simplification we get

$$X = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

7. If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M

Solution:

Given,

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$

$$2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \text{ (After further simplification)}$$

$$= \begin{bmatrix} 9-1 & 6-4 \\ 0-(-2) & -9-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix} \text{ (After subtraction of matrices)}$$

$$\therefore M = \frac{1}{2}\begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix} \text{ (Dividing by 2)}$$

Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

8. Find the matrix X such that A + 2X = 2B + C

Solution:

$A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

let $X = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$

$A + 2X = 2B + C$ (Given condition)

$2X = 2B + C - A$

$$2\begin{bmatrix} x & y \\ z & t \end{bmatrix} = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} \text{ (On further calculation)}$$

$$= \begin{bmatrix} -6+4-2 & 4+0+6 \\ 8+0-2 & 0+2-0 \end{bmatrix} = \begin{bmatrix} -4 & -10 \\ 6 & 2 \end{bmatrix} \text{ (Addition and subtraction of matrices)}$$

$$\therefore 2\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix} \text{ (Dividing by 2)}$$

9. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Solution:

Given,

$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$.....(i)

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.......(ii)

Adding (*i*) and (*ii*) we get,

$$2x = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore x = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Now,
Subtracting (*ii*) from (*i*), we get

$$2y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow 2y = \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\therefore\ y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

10. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$ Find the values of x and y

Solution:

Given,

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \quad \text{(Adition of matrices)}$$

On comparing the corresponding elements, we have
8 + y = 0
Then, y = -8
And, 2x + 1 = 5
2x = 5 – 1 = 4
x = 4/2 = 2
Therefore, x = 2 and y = -8

11. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$ Find the values of x and y

Solution:

Given,

$$2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}z & 0\\10 & 5\end{bmatrix}$$

$$\begin{bmatrix}6 & 8\\10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}z & 0\\10 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}6+1 & 8+y\\10+0 & 2x+1\end{bmatrix}=\begin{bmatrix}z & 0\\10 & 5\end{bmatrix} \text{ (Addition of matrices)}$$

$$\Rightarrow \begin{bmatrix}7 & 8+y\\10 & 2x+1\end{bmatrix}=\begin{bmatrix}z & 0\\10 & 5\end{bmatrix}$$

On comparing the corresponding terms, we have

2x + 1 = 5

2x = 5 -1 = 4

x = 4/2 = 2

And,

8 + y = 0

y = -8

And, z = 7

Therefore, x = 2, y = -8 and z = 7.

12. If $\begin{bmatrix}5 & 2\\-1 & y+1\end{bmatrix}-2\begin{bmatrix}1 & 2x-1\\3 & -2\end{bmatrix}=\begin{bmatrix}3 & -8\\-7 & 2\end{bmatrix}$ Find the values of x and y

Solution:

Given,

$$\begin{bmatrix}5 & 2\\-1 & y+1\end{bmatrix}-2\begin{bmatrix}1 & 2x-1\\3 & -2\end{bmatrix}=\begin{bmatrix}3 & -8\\-7 & 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}5 & 2\\-1 & y+1\end{bmatrix}-\begin{bmatrix}2 & 4x-2\\6 & -4\end{bmatrix}=\begin{bmatrix}3 & -8\\-7 & 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}5-2 & 2-4x+2\\-1-6 & y+1+4\end{bmatrix}=\begin{bmatrix}3 & -8\\-7 & 2\end{bmatrix} \text{ (Subtraction of matrices)}$$

$$\Rightarrow \begin{bmatrix}3 & 4-4x\\-7 & y+5\end{bmatrix}=\begin{bmatrix}3 & -8\\-7 & 2\end{bmatrix}$$

Now, comparing the corresponding terms, we get

4 – 4x = -8

4 + 8 = 4x
12 = 4x
x = 12/4
x = 3
And, y + 5 = 2
y = 2 – 5 =
y = -3
Therefore, x = 3 and y = -3

13. If $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$
Find the value of a, b and c.
Solution:

Given,
$\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 2-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} a+1 & b+2 \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$ (After further calculations)
Next, on comparing the corresponding terms, we have
a + 1 = 5 ⇒ a = 4
b + 2 = 0 ⇒ b = -2
-c = 3 ⇒ c = -3
Therefore, the value of a, b and c are 4, -2 and -3 respectively.

14. If A = $\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, C = $\begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$ **and 5A + 2B = C,**
find the values of a, b and c.
Solution:

Given,

A $= \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ and B $= \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, C $= \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$ and 5A + 2B = C

So, we have

$$5\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} + 2\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 5a \\ -15 & 25 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 14 & 2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 10-4 & 5a+6 \\ -15+14 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 5a+6 \\ -1 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

On comparing the corresponding terms, we get

5a + 6 = 9

5a = 9 – 6

5a = 3

a = 3/5

And,

25 + 2b = -11

2b = -11 – 25

2b = -36

b = -36/2

b = -18

And, c = 6

Therefore, the value of a, b and c are 3/5, -18 and 6 respectively.

EXERCISE 8.3

1. If A = $\begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible ? Give a reason. If yes, find AB.

Solution:
Yes, the product is possible because of number of column in A = number of row in B
That is order of matrix is 2 × 1

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3\times2+5\times4 \\ 4\times2+(-2)\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+20 \\ 8-8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

2. If A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$,B = $\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$, find AB and BA, Is AB = BA ?

Solution:
Given

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix},$$
$$B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

Now we have to find A × B

$$\therefore A \times B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\times\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-15 & -2+10 \\ 1-9 & -1+6 \end{bmatrix} = \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

Again have to find B × A

$$B \times A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-3 \\ -6+2 & -15+6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Hence AB is not equal to BA

3. If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Find AB – 5C

Solution:

Given

$A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$

On simplification we get

$$= \begin{bmatrix} 3\times0+7\times5 & 3\times2+7\times3 \\ 2\times0+4\times5 & 2\times2+4\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A(BA)

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A\,(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} = \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

5. Given matrices:

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}, C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

Find the products of

(i) ABC

(ii) ACB and state whether they are equal.

Solution:

Given

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix},$$

$$C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

Now consider,

$$ABC = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$$

$$ACB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -18+0 & -24+0 \\ -36+0 & -48+0 \end{bmatrix} = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$$

$\therefore ABC \neq ACB.$

6. **Evaluate** : $\begin{bmatrix} 4\sin 30^o & 2cos60^o \\ sin90^o & 2cos0^o \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

Solution:

Given

$$\begin{bmatrix} 4\sin 30^o & 2cos60^o \\ sin90^o & 2cos0^o \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$sin30^o = \frac{1}{2}, cos60^o = \frac{1}{2}$

$\sin 90^o = 1$ and $\cos 0^o = 1$

$$\therefore \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+1\times5 & 2\times5+1\times4 \\ 1\times4+2\times5 & 1\times5+2\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

7. If $A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$ find the matrix AB + BA

Solution:

Given

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -2-12 & 3-18 \\ 4-16 & -6-24 \end{bmatrix} = \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-6 & 6-12 \\ 4-12 & -12-24 \end{bmatrix} = \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$$

$\therefore$ AB + BA

$$= \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix} + \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$$

$$= \begin{bmatrix} -14-8 & -15-6 \\ -12-8 & -30-36 \end{bmatrix} = \begin{bmatrix} -22 & -21 \\ -20 & -66 \end{bmatrix}$$

8. If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

Solution:

Given,

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$2B = 2\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix}$$

Now,

$$A^2 = A \times A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & -2+2 \\ 2-2 & -4+1 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\therefore 2B - A^2 = \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-3) & 4-0 \\ -4-0 & 2-(-3) \end{bmatrix} = \begin{bmatrix} 6+3 & 4 \\ -4 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ -4 & 5 \end{bmatrix}$$

9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$, compute

(i) A(B + C) **(ii) (B + C)A**

Solution:

Given,

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$,

$C = \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$

(i) A(B + C)

$$A(B + C) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\left[\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2+5 & 1+1 \\ 4+7 & 2+4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7+22 & 2+12 \\ 21+44 & 6+24 \end{bmatrix} = \begin{bmatrix} 29 & 14 \\ 65 & 30 \end{bmatrix}$$

$$\text{(ii)}\ (B + C)A = \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7+6 & 14+8 \\ 11+18 & 22+24 \end{bmatrix} = \begin{bmatrix} 13 & 22 \\ 29 & 46 \end{bmatrix}$$

10. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

Find the matrix C(B – A).

Solution:

Given,

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

Now,

$$B - A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$C(B - A) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & -1-3 \\ 3+1 & -3-1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}$$

11. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

Find $A^2 + AB + B^2$.

Solution:

Given,

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

Now,

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times2 & 1\times0+0\times1 \\ 2\times1+1\times2 & 2\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+0\times-1 & 1\times3+0\times0 \\ 2\times2+1\times-1 & 2\times3+1\times0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times(-1) & 2\times3+3\times0 \\ -1\times2+0\times(-1) & -1\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times(-1) & 2\times3+3\times0 \\ -1\times2+0\times(-1) & -1\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

Hence,

$$A^2+AB+B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+1 & 0+3+6 \\ 4+3-2 & 1+6+-3 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$

12. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$, **find $A^2 + AC - 5B$.**

Solution:

Given,

$A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$

Now,

$$A^2 + AC - 5B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} - 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} + \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} - 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$ (Substituting the values from given)

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$

13. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ $and\ C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, **find $AC + B^2 - 10C$.**

Solution:

Given,

Given,

$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ $and\ C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

Now,

$$AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$$

$$10C = 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

Hence,

$$AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4-10 & 12+28-0 \\ -2-7+10 & 28+45-40 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 40 \\ -1 & 33 \end{bmatrix}$$

14. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find A^2 and A^3. Also state that which of these is equal to A.

Solution:

Given,

$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Next,

$A^3 = A^2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

From above, its clearly seen that $A^3 = A$.

15. If $X = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, show that $6X – X^2 = 9I$ where I is the unit matrix.

Solution:

Given,

$X = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

Now,

$X^2 = X \times X = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$

Taking L.H.S, we have

$6X - X^2 = 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 24-15 & 6-6 \\ -6-6 & 12-3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I = \text{R.H.S.}$

- Hence proved

16. Show that $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is a solution of the matrix equation $X^2 - 2X - 3I = 0$, where I is the unit matrix of order 2.

Solution:

Given,

$X^2 - 2X - 3I = 0$

Solution $= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\therefore X^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

Now, $X^2 - 2X - 3I$

$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 5-2-3 & 4-4+0 \\ 4-4-0 & 5-2-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore X^2 - 2X - 3I = 0$ Hence proved.

17. Find the matrix 2 × 2 which satisfies the equation

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Solution:

Given,

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$2X = -\begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix} = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

18. If $A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$, find the value of x, so that A^2 – 0

Solution:

Given,

$$A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix} \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$$

$$= \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix}$$

But, $A^2 = 0$

$$\begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing,

1 + x = 0

∴ x = -1

19.

(i) Find x and y if $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

(ii) Find x and y if $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -3x & 4 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -3x & +4 \\ & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

Comparing the corresponding elements,

- 3x + 4 = -5

-3x = -5 – 4 = -9

x = -9/-3 = 3

Therefore, x = 3 and y = -10.

(ii) $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \times 3 + x \times 2 \\ y \times 3 + 3y \times 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

Comparing, we get

8x = 16

⇒ x = 16/8 = 2

And, 9y = 9

y = 9/9 = 1

20. Find the values of x and y if $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Solution:

Given,

$$\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+2y & -y \\ 4x & -x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & +y \\ 3x & +y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

On comparing the corresponding elements, we have

2x + y = 3 … (i)

3x + y = 2 … (ii)

Subtracting, we get

-x = 1 ⇒ x = -1

Substituting the value of x in (i),

2(-1) + y = 3

-2 + y = 3

y = 3 + 2 = 5

Therefore, x = -1 and y = 5.

CHAPTER TEST

1. Find the values of a and b if $\begin{bmatrix} a+3 & b^2+2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a+1 & 3b \\ 0 & b^2-5b \end{bmatrix}$

Solution:

Given

$$\begin{bmatrix} a+3 & b^2+2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a+1 & 3b \\ 0 & b^2-5b \end{bmatrix}$$

comparing the corresponding elements

a + 3 = 2a + 1

⇒ 2a – a =3 – 1

⇒ a = 2 b^2 + 2 = 3b

⇒ b^2 – 3b + 2 = 0

⇒ b^2 – b – 2b + 2 = 0

⇒ b (b – 1) – 2 (b – 1) = 0

⇒ (b – 1) (b – 2) = 0.

Either b – 1 = 0,

then b = 1 or b – 2 = 0,

then b = 2

Hence a = 2, b = 2 or 1

2. Find a, b, c and d if $3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$

Solution:

Given

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$$

Now comparing the corresponding elements

3a = 4 + a

a – a = 4
2a = 4
Therefore, a = 2
3b = a + b + 6
3b – b = 2 + 6
2b = 8
Therefore, b = 4
3d = 3 + 2d
3d – 2d = 3
Therefore, d = 3
3c = c + d – 1
3c – c = 3 – 1
2c = 2
Therefore, c = 1
Hence a = 2,b = 4, c = 1 and d= 3

3. Determine the matrices A and B when

$A + 2B = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}$

Solution:

Given,

$A + 2B = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$ .....(i)

$2A - B = \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}$ ....(ii)

Multiplying (i) by 1 and (ii) by 2

$A + 2B = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

$4A - 2B = 2\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -2 \end{bmatrix}$

Now, adding we get

$5A = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 10 & -5 \end{bmatrix}$

$A = \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 10 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

From (*i*) $A + 2B = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} + 2B = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

$2B = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix}$

$\therefore B = \frac{1}{2}\begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

Thus, $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

4.

(i) Find the matrix B if $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and $A^2 = A + 2B$

(ii) If $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$ find $A(4B - 3C)$

Solution:

Given,

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$\text{let } B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 16+2 & 4+3 \\ 8+6 & 2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 4+2a & 1+2b \\ 2+2c & 3+2d \end{bmatrix}$$

As, $A^2 = A + 2B$

$$\Rightarrow \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} = \begin{bmatrix} 4+2a & 1+2b \\ 2+2c & 3+2d \end{bmatrix}$$

Comparing the corresponding elements, we have

4 + 2a = 18

2a = 18 – 4 = 14

a = 14/2

⇒ a = 7

1 + 2b = 7

2b = 7 – 1 = 6

b = 6/2

⇒ b = 3

2 + 2c = 14

2c = 14 – 2 = 12

2c = 12

c = 12/2

⇒ c = 6

3 + 2d = 11

2d = 11 – 3

d = 8/2

⇒ d = 4

Therefore, a = 7, b = 3, c = 6 and d = 4.

$$\therefore B = \begin{bmatrix} 7 & 3 \\ 6 & 4 \end{bmatrix}$$

(ii) $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$

$$4B - 3C = 4\begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix} - 3\begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 \\ -8 & 20 \end{bmatrix} - \begin{bmatrix} -6 & 0 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-(-6) & 4-0 \\ -8-(-3) & 20-3 \end{bmatrix} = \begin{bmatrix} 0+6 & 4-0 \\ -8+3 & 20-3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ -5 & 17 \end{bmatrix}$$

Now, $A(4B - 3C) = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 6 & 4 \\ -5 & 17 \end{bmatrix}$

$$= \begin{bmatrix} 1\times 6 + 2(-5) & 1\times 4 + 2\times 17 \\ -3\times 6 + 4\times(-5) & -3\times 4 + 4\times 17 \end{bmatrix}$$

$$= \begin{bmatrix} 6-10 & 4+34 \\ -18-20 & -12+68 \end{bmatrix} = \begin{bmatrix} -4 & 38 \\ -38 & 56 \end{bmatrix}$$

5. If $A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, find the each of the following and state it they are equal:

(i) (A + B) (A - B)

(ii) $A^2 - B^2$

Solution:

Given,

$A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

(*i*) (A + B) (A – B)

$$= \left\{\begin{bmatrix}3 & 2\\0 & 5\end{bmatrix}+\begin{bmatrix}1 & 0\\1 & 2\end{bmatrix}\right\} \times \left\{\begin{bmatrix}3 & 2\\0 & 5\end{bmatrix}-\begin{bmatrix}1 & 0\\1 & 2\end{bmatrix}\right\}$$

$$= \begin{bmatrix}3+1 & 2+0\\0+1 & 5+2\end{bmatrix} \times \begin{bmatrix}3-1 & 2-0\\0-1 & 5-2\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2\\1 & 7\end{bmatrix} \times \begin{bmatrix}2 & 2\\-1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}8-2 & 8+6\\2-7 & 2+21\end{bmatrix} = \begin{bmatrix}6 & 14\\-5 & 23\end{bmatrix}$$

(*ii*) $A^2 - B^2$

$$= \begin{bmatrix}3 & 2\\0 & 5\end{bmatrix} \times \begin{bmatrix}3 & 2\\0 & 5\end{bmatrix} - \begin{bmatrix}1 & 0\\1 & 2\end{bmatrix} \times \begin{bmatrix}1 & 0\\1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}9+0 & 6+10\\0+0 & 0+25\end{bmatrix} - \begin{bmatrix}1+0 & 0+0\\1+2 & 0+4\end{bmatrix}$$

$$= \begin{bmatrix}9 & 16\\0 & 25\end{bmatrix} - \begin{bmatrix}1 & 0\\3 & 4\end{bmatrix} = \begin{bmatrix}9-1 & 16-0\\0-3 & 25-4\end{bmatrix}$$

$$= \begin{bmatrix}8 & 16\\-3 & 21\end{bmatrix}$$

Hence, its clearly seen that (A + B) (A - B) ≠ $A^2 - B^2$.

6. If A = $\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}$, find $A^2 - 5A - 14I$, where I is unit matrix of order 2 × 2.

Solution:

Given,

$$A = \begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$$

$$\therefore\ A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -20 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20+0 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

7. If $A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$ and $A^2 = 0$, find p and q.

Solution:

Given

$$A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}\begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$$

$$= \begin{bmatrix} 9+3p & 9+3q \\ 3p+pq & 3p+q^2 \end{bmatrix}$$

But $A^2 = 0$

$$\therefore \begin{bmatrix} 9+3p & 9+3q \\ 3p+pq & 3p+q^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing the corresponding elements, we have

9 + 3p = 0

3p = -9

p = -9/3

p = -3

And,

9 + 3q = 0
3q = -9
q = -9/3
q = -3
Therefore, p = -3 and q = -3.

8. If $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **a, b, c and d.**

Solution:

Given,

Given,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a+0 & -b+0 \\ 0+c & 0+d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a & -b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

On comparing the corresponding elements, we have
-a = 1 ⇒ a = -1
-b = 0 ⇒ b = 0
c = 0 and d = -1
Therefore, a = -1, b = 0, c = 0 and = -1.

9. Find a and b if $\begin{bmatrix} a-b & b-4 \\ b+4 & a-2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$
Solution:

Given

$$\begin{bmatrix} a-b & b-4 \\ b+4 & a-2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-2b+0 & 0+2a-2b \\ 2b+8+0 & 0+2a-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2a-2b & 2a-2b \\ 2b+8 & 2a-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

On comparing the corresponding terms, we have

2a – 4 = 0

2a = 4

a = 4/2

a = 2

And, 2a – 2b = -2

2(2) – 2b = -2

4 – 2b = -2

2b = 4 + 2

b = 6/2

b = 3

Therefore, a = 2 and b = 3.

10. If $A = \begin{bmatrix} sec60^o & cos90^o \\ -3tan45^o & sin90^o \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & cos45^o \\ -2 & 3sin90^o \end{bmatrix}$

Find (i) 2A – 3B (ii) A^2 (iii) BA

Solution:

Given,

A = $\begin{bmatrix} sec60^o & cos90^o \\ -3tan45^o & sin90^o \end{bmatrix}$ and B = $\begin{bmatrix} 0 & cos45^o \\ -2 & 3sin90^o \end{bmatrix}$

$A = \begin{bmatrix} \sec 60° & \cos 90° \\ -3\tan 45° & \sin 90° \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ ($\because$ sec 60° = 2, cos 90° = 0, tan 45° = 1, sin 90° =1)

$B = \begin{bmatrix} 0 & \cot 45° \\ -2 & 3\sin 90° \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ ($\because$ cot 45° = 1)

(*i*) 2 A – 3 B

$$= 2\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

(*ii*) $A^2 = A \times A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 4+0 & 0+0 \\ -6-3 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 1 \end{bmatrix}$$

(*iii*) $BA = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0-3 & 0+1 \\ -4-9 & 0+3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -13 & 3 \end{bmatrix}$$